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PROCEEDINGS
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October 1895.

I. *The Development of Arbitrary Functions.*
By Prof. John Perry *and* Mr. H. F. Hunt.[*]

Students at the Finsbury Technical College have for some years carried out the Fourier development idea of the late Professor Clifford by practical Geometrical methods. The curve which graphically represents the function is supposed to be wrapped round a circular cylinder, and the areas of its projections upon two diametral planes enable the coefficients to be found. In 'The Electrician' of June 28th, 1895, one of us gave detailed instructions for carrying out the process. We do not know of any method of development in a Fourier series which can compare with this in either rapidity or accuracy.

In the discussion of Prof. Henrici's paper, April 13th, 1894, one of us described a graphical method of developing any arbitrary function in a series of other normal forms than sines and cosines, such as Bessels or Zonal Spherical Harmonics. It consisted in wrapping the curve which represents the function round a specially shaped cylinder, not circular, and projecting it upon a certain plane. One of us has wasted many months in finding with great exactness a sufficient number of coordinates of the trace of the cylinder suitable for a Zeroth Bessel development. The labour was unnecessary because the coordinate most troublesome to calculate is not really needed: that is, the actual shape of the cylinder is not needed, and indeed it is better to put aside altogether the idea of a curve being wrapped round a cylinder.

---

[*] Read October 25, 1895.

To develop any arbitrary function of x (let us call it y) in normal forms, the real difficulty consists in finding the value of an integral such as $\int_0^a y \, . \, Q(x) \, . \, dx$, where $Q(x)$ is some tabulated function. If now z is another tabulated function which is the integral of $Q(x)$ the required integral is $\int y \, . \, dz$.

Suppose the values of y for 25 equidistant values of x to be known, from $x=0$ to $x=a$ (that is, a is divided into 24 equal parts). Let the corresponding values of z be also tabulated, and let a curve be drawn with the values of y as ordinates and the values of z as abscissæ; the area between this curve and the axis of z gives the value of the integral required.

To illustrate the method we shall show how it is applied in a well-known kind of problem. An arbitrary function $y=f(r)$ of r from $r=0$ to $r=a$ is to be developed in Zeroth Bessels; that is, we must determine the constants A_1, A_2, &c., in

$$f(r)=A_1J_0(\mu_1 r)+A_2J_0(\mu_2 r)+\&c.+A_sJ_0(\mu_s r)+\&c., \quad (1)$$

where μ_1, μ_2, &c., are the successive roots of

$$J_0(\mu a)=0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Now it is well known that

$$A_s=\frac{2}{a^2[J_1(\mu_s a)]^2}\int_0^a rf(r)J_0(\mu_s r) \, . \, dr; \quad . \quad . \quad (3)$$

or, letting $\mu_s r=x$, (3) becomes

$$A_s=\frac{2}{a^2[J_1(\mu_s a)]^2}\frac{1}{\mu_s^2}\int_0^{\mu_s a} xf(x)J_0(x) \, . \, dx. \quad . \quad (4)$$

Now

$$\int x \, . \, J_0(x) \, . \, dx=x \, . \, J_1(x),$$

which is easily found for any value of x.

To evaluate the integral in (4), take the 25 equidistant values of our function from 0 to a and use them as the ordinates of points in a curve. The values of the corresponding abscissæ are obtained in the following way. The values of $\mu_1 a$ &c. are the successive roots of $J_0(\mu a)=0$, and are

$\mu_1 a=$ 2·40483
$\mu_2 a=$ 5·52008
$\mu_3 a=$ 8·65373
$\mu_4 a=$ 11·79153
and so on.

Take 25 equidistant values of x beginning with 0 and ending with the value $\mu_s a$: the corresponding values of $xJ_1(x)$ are the abscissæ of points in the curve required to find A_s.

The area of the curve is easily obtained by means of the planimeter. Having found the 25 abscissæ once for all, they may be used for the development of any arbitrary function of r of which 25 equidistant values are given. We now publish four tables giving these abscissæ, which we have carefully calculated for the first four terms of any such development.

In the column headed P we give the number of the point in the curve of which $xJ_1(x)$ is the abscissa, there being 25 such points altogether. In Table I. the point 24 corresponds

TABLE I.

P.	$x\,.\,J_1(x)$.
0	0·0000
1	·0050
2	·0200
3	·0450
4	·0787
5	·1216
6	·1727
7	·2312
8	·2962
9	·3667
10	·4416
11	·5197
12	·5999
13	·6807
14	·7609
15	·8392
16	·9142
17	·9845
18	1·0490
19	1·1062
20	1·1552
21	1·1949
22	1·2242
23	1·2423
24	1·2485

TABLE II.

P.	$x\,.\,J_1(x)$.
0	0·0000
1	+ ·0267
2	·1010
3	·2242
4	·3800
5	·5578
6	·7458
7	·9191
8	1·0706
9	1·1825
10	1·2417
11	1·2160
12	1·1141
13	·9418
14	·7052
15	·4132
16	+ ·0791
17	− ·2801
18	·6455
19	·9963
20	1·3118
21	1·5723
22	1·7605
23	1·8626
24	1·8783
10 A	1·2485

TABLE III.

P.	$x \cdot J_1(x)$.
0	0·0000
1	+ ·0640
2	·2434
3	·5036
4	·7921
5	·9485
6	1·2135
7	1·2395
8	1·0986
9	·8366
10	+ ·3353
11	− ·2107
12	·7807
13	1·2957
14	1·6788
15	1·8667
16	1·8195
17	1·5279
18	1·0163
19	− ·3817
20	+ ·4154
21	1·1561
22	1·7807
23	2·2002
24	2·3490
6 A	+1·2485
15 A	−1·8783

TABLE IV.

P.	$x \cdot J_1(x)$.
0	0·0000
1	+ ·1171
2	·4268
3	·8168
4	1·1373
5	1·2468
6	1·0555
7	+ ·5601
8	− ·1542
9	·9246
10	1·5554
11	1·8658
12	1·7443
13	1·1825
14	− ·2851
15	+ ·7485
16	1·6705
17	2·2544
18	2·3065
19	1·8114
20	+ ·8497
21	− ·3674
22	1·5548
23	2·4214
24	2·7410
4 A	+1·2485
10 A	−1·8783
17 A	+2·3490

to $x=2·40483$. In Table II. the point named 24 corresponds to $x=5·52008$, and so on.

In Table II. an additional abscissa is given corresponding to some unknown point between numbers 10 and 11; this is the extreme abscissa of the curve, and the corresponding ordinate would be a tangent to the curve. In Table III. there are two additional or bounding abscissæ and in Table IV. there are three of them.

As an example we have taken a function which has the value 1 at the centre of a circle and the value 0 at the circumference, and which at any place is a linear function of the radius. The outer radius a of the circle is 1.

In fig. 1 the function is shown graphically by the straight line MDD, and the numbered points in this line correspond to 25 equidistant ordinates. In our actual drawing, of which fig. 1 is a reduction, the scale is such that one unit of value of the function corresponds to 10 inches.

Fig. 1.

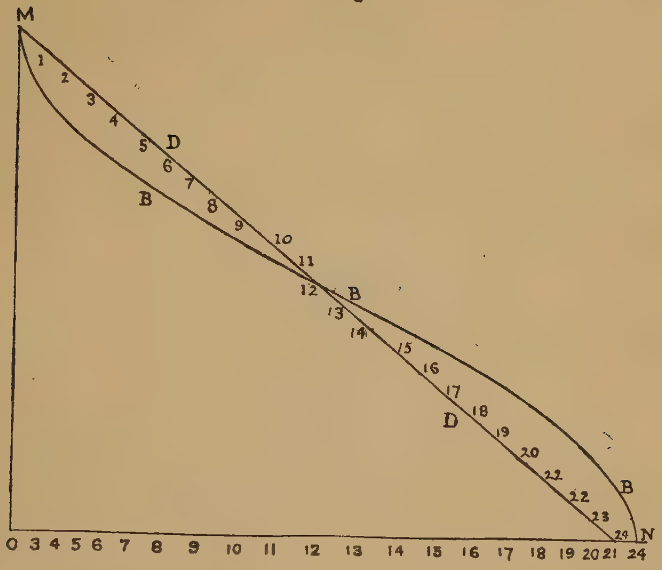
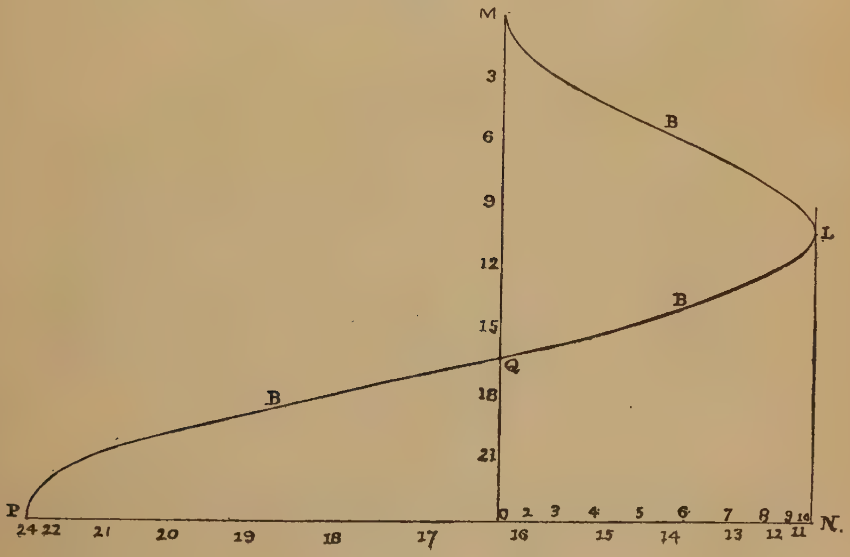

M
D
B
B
D
B
N
0 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 24

Fig. 2.


M
3
6
9
12
15
18
21
B
L
B
Q
B
P
0 2 3 4 5 6 7 8 9 10
N.
24 22 21 20 19 18 17 16 15 14 13 12 11

The distances from 0 to any number in the axis of abscissæ ON represented in inches ten times the corresponding values of $xJ_1(x)$ of Table I. Projecting vertically from any point in ON, and horizontally from the corresponding point in DD we obtained points in the curve MBB. We use inches because British planimeters usually measure areas in square inches. We found the area OMBBNO to be 61·17 square inches, and as both coordinates were magnified ten times, the value of the integral in (4) or

$$\int_0^{2\cdot40483} xf(x)\, J_0(x)\,.dx = 0\cdot6117,$$

and hence by (4), as $J_1(2\cdot40483) = 0\cdot519$,

$$A_1 = \frac{2 \times 0\cdot6117}{(0\cdot519)^2 (2\cdot405)^2} = 0\cdot7852.$$

To find A_2. The 25 equidistant ordinates of the original curve (straight line) are the ordinates of points in the curve MBLBBP (fig. 2), the abscissæ being taken from Table II. We found it convenient to multiply the abscissæ by 5 and represent in inches. Our ordinates were multiplied by 10 and represented in inches. The actual area was 2·36 square inches and one-fiftieth of this, or 0·0472, is the value of the integral in (4).

Hence, as $J_1(5\cdot5201) = \cdot3403$,

$$A_2 = \frac{2 \times 0\cdot0472}{(0\cdot3403)^2 \times (5\cdot5201)^2} = 0\cdot0268.$$

We need not show the curves used in finding A_3 and A_4. The area of fig. 2 is the positive area MLNOM minus the area LQPNL, but one need not think about whether an area is positive or negative. It is only necessary to start the planimeter-tracer from the point numbered 0 in every case, and go from 0 to M and along the curve in the direction of the increasing numbers to 24, then along the axes of abscissæ, ending at the point 0 from which we started.

DISCUSSION.

Prof. HENRICI said the method was a new departure, since in the place of an instrument of complicated design, the authors only used a planimeter and pencil and paper, and obtained the same degree of accuracy. The fact that the series employed to test the method consisted of a finite number of terms seemed to him an objection. Prof. Karl Pearson had, in a recent conversation, informed him of a method for the development of functions which he (Prof. Pearson) had recently discovered. This method was not, however, so simple, at least in most cases, as that of the authors.

Prof. MINCHIN thought it would add to the intelligibility of the paper if it were stated that the method was similar to that employed when expanding in terms of a Fourier series or in spherical harmonics. In these cases you have a function which, when multiplied by other functions of different orders, kills all the terms except one. Graphic methods ought, in his opinion, to be very much oftener employed, and he considered that there was no problem in physical mathematics of which the solution could not be obtained by graphic methods. He would like to know if Prof. Perry had obtained a graphic method of calculating Bessels.

Mr. TROTTER agreed with Prof. Minchin as to the neglect of graphic methods. He regretted that Prof. Perry did not continue to consider the method as the projection from a cylinder, as he had found the method of wrapping curves round a cylinder most useful.

Prof. PERRY, in his reply, said he had adopted the expansion they had employed under the impression that the test was a particularly severe one. He had not discovered a graphic method of calculating Bessels. The reason they gave up the cylinder was the immense labour involved in calculating the y co-ordinates of the trace, which would afterwards be of no use in the development of the function.

II. *The Radial Cursor: a new addition to the Slide-Rule.*

By F. W. LANCHESTER*.

THE slide-rule in its ordinary form is chiefly useful in working simple problems involving multiplication and division, also involution and evolution where the indices occurring are integers. When an expression involves fractional indices its solution is only possible, on a slide-rule of ordinary construction, by employing the scale of logarithms on the back of the slide in the same manner as a table of logarithms, when the process becomes so clumsy as to practically destroy its utility.

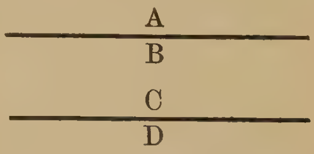
The object of the present improvement is to enable the operator to solve problems of the latter class with the same ease and degree of accuracy as that only previously attainable in connexion with the simpler cases.

In thermodynamic problems such as arise in dealing theoretically with gas- and other heat-engines, the need of some method of computation more rapid than that of ordinary arithmetic involving many references to tables becomes evident, and it was in connexion with work of this description that the author was led to devise the instrument which is the subject of the present paper.

In the case of the isothermal curve of a perfect gas, the ordinary slide-rule is all that can be required, for the curve is represented by the simple expression

$$pv = \text{constant}.$$

For the sake of brevity let us name the scales in a rule of the ordinary "Gravet" type thus



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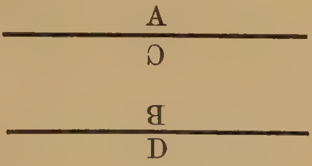
* Read October 25, 1895.

becoming when the slide is inverted,



Then with the slide in the inverted position the corresponding pressures and volumes of an isothermal can be read off directly on the scales A and ꓭ.

In the case of an adiabatic curve, however, we have an expression of the form

$$pv^{\gamma} = \text{const.},$$

where γ has a value 1·4 more or less according to the physical properties of the gas or mixture of gases we are dealing with. In order to deal with an expression of this form, it is necessary to provide some ready means of dividing the scales A and ꓭ proportionately to the value of γ which corresponds to the division and multiplication of the respective logarithms of the quantities dealt with, in the proportion of the indices of p and v, that is $1 : \gamma$.

This proportionate division of the scales is effected in the new Cursor by a radial index-arm, indicated in fig. 1 by lines f and g: this is arranged to swing about a stud which is itself carried on a sliding bar running in guides at right angles to the rule. All readings are taken at the points of intersection of the line with the edges of the slide. In the actual instrument the radial arm is made of a transparent slip of celluloid on which is scratched a straight line true to the centre of the stud-hole. The back view shows (fig. 2) the transverse bar graduated to give the indices to which it is desired to work. In fig. 2 this transverse bar is shown set to 1·408, the γ of a perfect gas.

A few words may be added with regard to the method of graduation of the transverse bar. Referring to fig. 3, in which M N is the width of the slide, and O is the centre of

Fig. 1.

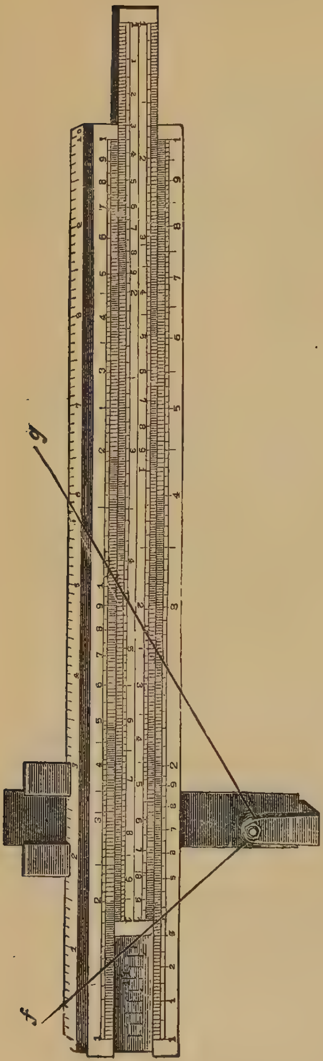
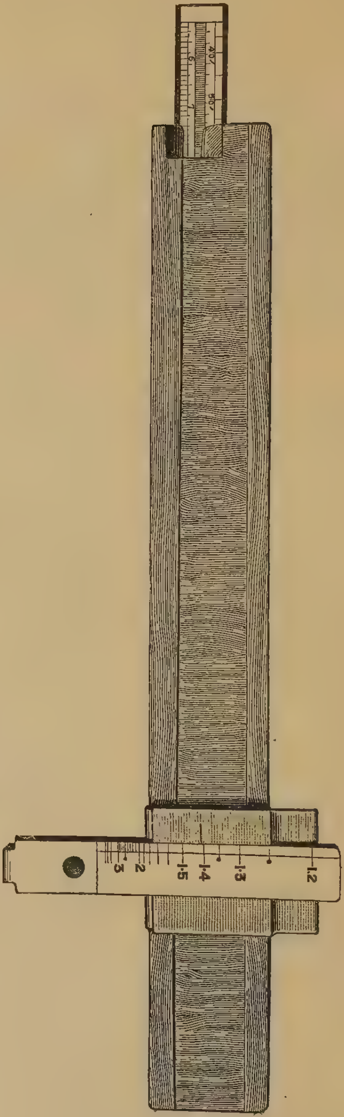


Fig. 2.



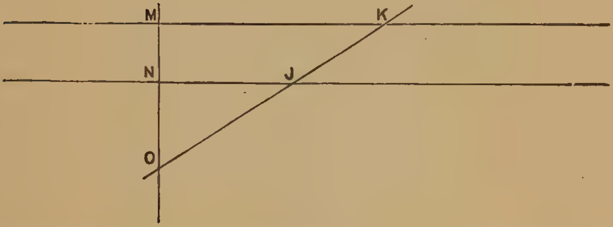
the radial arm stud-hole, for any index γ, we have

$$\gamma=\frac{MK}{NJ}=\frac{MN+NO}{NO}=\frac{MN}{NO}+1,$$

where OK is any position of the index line, or

$$NO=\frac{MN}{\gamma-1}.$$

Fig. 3.



The following examples will serve as an illustration of the class of calculation for which the instrument is designed.

1. *To determine the maximum pressure of an adiabatic compression.*

Set the slide on back of cursor to 1·408 (γ). Suppose, for example, that the initial pressure be 15 lbs. per square inch, and that 100 vols. be compressed to 26 vols.

Invert slide. Set index line of cursor to 15 on A, then set 1 (100) on g to index line of cursor. Now set index line to 26 on g and read on scale A the corresponding pressure =100 lbs. absolute, 85 above atmosphere (figs. 1 and 2).

2. The work done in any adiabatic compression can easily be calculated from the above by the following formula :—

$$W=\frac{1}{\gamma-1}\,p_1V;$$

where W = work done in ft.-lbs., V = final vol. of the gas (after compression) in units of 12 cub. in. (to harmonize with ft.-lbs. and pressure in pounds per square inch), and p_1 = difference of pressure between adiabatic and isothermal maximum pressure for the given ratio of compression (fig. 4).

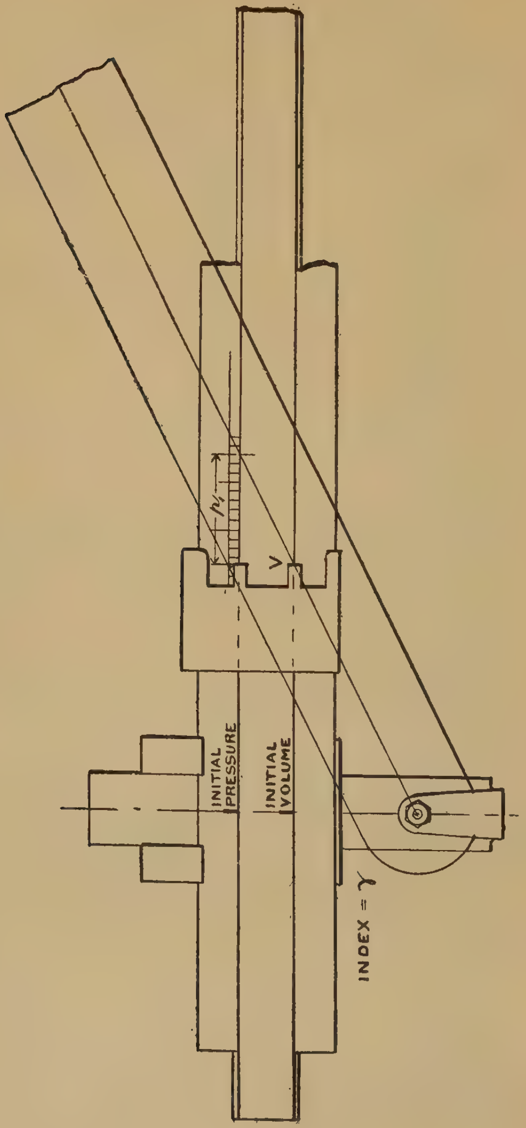


Fig. 4.

Work done $= \frac{1}{\gamma - 1} p_1 V$.

3. The temperature of an adiabatic at any point can be determined from either pressure or volume by using suitable indices for

$$v \, . \, T^{\frac{1}{\gamma-1}} = \text{constant};$$

also

$$p \, . \, T^{\frac{\gamma}{1-\gamma}} = \text{constant}.$$

In determining temperatures from volumes or *vice versâ* it is only necessary to proceed as before (1). In dealing, however, with pressures and temperatures the index is negative (where γ is $>$ unity), the interpretation of this being that an increase of pressure is accompanied by an increase in the temperature. The slide therefore does not require to be reversed as in pv and vT calculations; but with a rule of the Gravet type, owing to the scales C and D being the square roots of A and B, it is necessary to use a fictitious index obtained by dividing $\frac{\gamma}{1-\gamma}$ by the logarithmic ratio of the scales, giving $\frac{\gamma}{2(1-\gamma)}$. It would also be quite possible to graduate one edge of the index slide with fictitious values to compensate for the ratio of the scale-readings.

4. In the process described in (1) we have the means of plotting a series of adiabatics or isentropics, and each position of the radial cursor on the rule gives in its readings on the index line a corresponding isentropic, and a movement of the cursor to the left corresponds to a gain in entropy and *vice versâ*, and it can be shown that equal increments of linear movement of the cursor correspond to equal increments of entropy. In other words, the scale of inches or centimetres on the edge of the rule may be regarded as a scale of entropy in arbitrary units.

5. The following is a very good example of the use of the radial cursor in practice.

The theoretical efficiency of an Otto cycle gas-engine is independent of the working temperatures and is given by the expression $1-\left(\frac{v_2}{v_1}\right)^{\gamma-1}$, where v_1 is the total effective volume of the cylinder and compression-space, and v_2 = volume of compression-space alone.

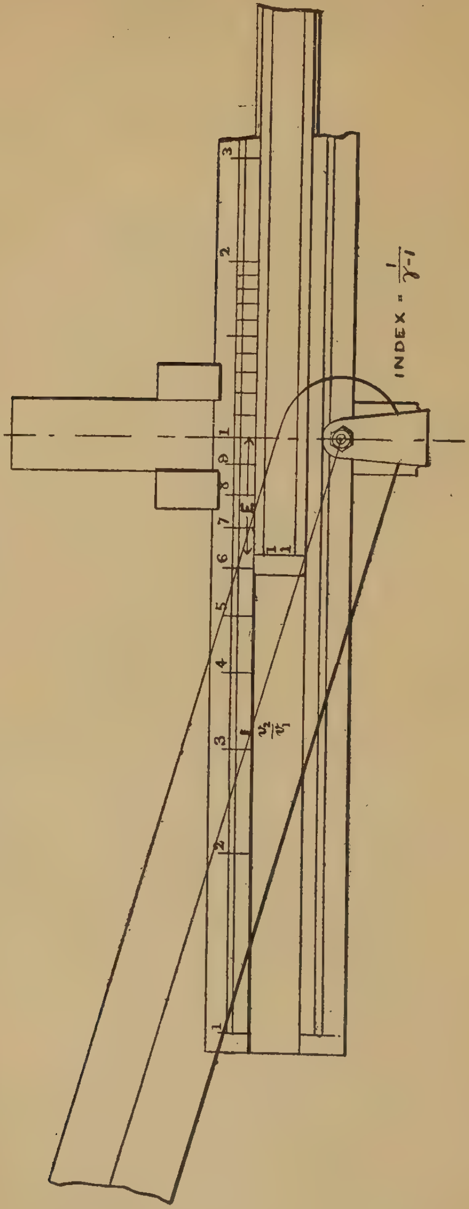


Fig. 5.

Efficiency of Otto Cycle Gas Engine $= 1 - \left(\frac{v_2}{v_1}\right)^{\gamma-1}$.

Taking $\gamma = 1{\cdot}408$, $\gamma - 1 = {\cdot}408$. This is beyond the range of the instrument, so we will employ $\frac{1}{{\cdot}408} = 2{\cdot}45$.

Let us suppose that in the case we are dealing with $\frac{v_2}{v_1}$ comes out at ·32 (an actual example, see 'The Engineer,' April 6th, 1894).

Set the index slide to 2·45 and place cursor accurately opposite middle 1 on scale A (use one of end lines on slide to centre with), and set index line to ·32 $\left(\frac{v_2}{v_1}\right)$ on scale A (over to the left, fig. 5).

Now using one of the end lines on slide read on scale A opposite to intersection of index line on scale D, *but read backwards from the centre* 1. This will be found to give ·37, or the theoretical efficiency of an Otto engine having ratio of expansion ·32 is ·37.

6. To determine x in such an expression as

$$7^x = 3.$$

Here x will be a fractional index, and we shall first find $\frac{1}{x}$.

Set cursor to 1 in centre of scale A.

Set index line to 7 on scale A, and keeping it there move slide of cursor till index line intersects scale D opposite figure 3 on scale A (use end line of slide).

Now read result on index-scale

$$= 1{\cdot}76,$$

Or
$$x = \frac{1}{1{\cdot}76} = {\cdot}568.$$

DISCUSSION.

Mr. C. V. Boys said that owing to the kindness of the author he had been able to try the cursor and had found it of great service in dealing with questions of adiabatic expansion. The new addition to the slide-rule suffers under the same disadvantage as the rule itself, namely, that a verbal or written description seems so very much more complex than is the actual operation when using the rule. The author's device might be described as an indiarubber slide-rule, for it performed the function of a slide-rule, in which the graduations of the slide were made on indiarubber, so that the ratio of the length of the scale on the rule to the length of the scale on the slide might be altered at will, and thus involution and evolution with fractional indices performed.

Prof. S. P. THOMPSON and Mr. TROTTER expressed their admiration for the author's method of "stretching" the scale.

Mr. BURSTALL said he had attempted to apply a similar method to the Fuller rule, but did not succeed, since in this rule there was only one scale. He hoped the author's method could be applied in a form such that a greater accuracy than 1 in 300 could be obtained.

Mr. BOURNE thought the fact that the point of intersection of two lines inclined at an acute angle had to be read was likely to limit the accuracy.

III. *The Latent Heat of Evaporation of Benzene.* *By* E. H. GRIFFITHS, *M.A., F.R.S., Sidney Sussex College, Cambridge, and* Miss DOROTHY MARSHALL, *B.Sc., University College, London**.

1. THE method of experiment and the nature of the apparatus employed have been fully described in previously published papers. Although it appears somewhat presumptuous to assume, on the part of the reader, a knowledge of such communications, it would on the other hand seem redundant to devote many pages to the mere repetition of what has already appeared in print. We therefore propose to give an explanatory statement of the theory and methods employed, but to omit all detailed description of the apparatus. Full references to former communications will be given when necessary, so that all particulars can be ascertained by any who desire them.

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* Read November 8, 1895.

As reference has frequently to be made to the following papers (by E. H. Griffiths) they are, for convenience, denoted as follows :—

Paper J. "The Mechanical Equivalent of Heat," Phil. Trans. vol. clxxxiv. 1893, A, pp. 361–504.

Paper A. "The Influence of Temperature upon the Specific Heat of Aniline," Phil. Mag. Jan. 1895.

Paper W. "The Latent Heat of Evaporation of Water," Phil. Trans. vol. clxxxvi. 1895, A, pp. 261–341.

2. *Description of the Method.*

In order to render our account of these experiments intelligible, we find it necessary to give the following somewhat lengthy extract from Paper W, pp. 270–273 :—

"The method adopted was of such a nature that the results would not be appreciably affected by

(1) errors in thermometry ;
(2) changes in the specific heat of water ;
(3) the capacity for heat of the calorimeter ;
(4) loss or gain of heat by radiation, &c. ;

and if these points are borne in mind, they may serve to explain some of the contrivances which might otherwise appear uncalled for.

"If the vessel in which the evaporation is taking place is kept at a constant temperature, we are independent of the capacity for heat of it and its contents ; we also dispense with the measurements of changes of temperature. Thus, if matters be so arranged that the loss and gain of heat throughout an experiment are balanced, many fruitful causes of error are avoided. Of course, the actual temperature of the calorimeter during evaporation must be determined, but a small error here is of little consequence. The change in the value of L is small as compared with the changes in θ. In fact, an accuracy of an order of $\frac{1}{10}$ of a degree would be sufficient when determining the actual elevation.

"The heat was supplied to the calorimeter by means of a wire whose ends were kept at a constant potential-difference.

The thermal balance could be maintained in one of two ways:—

(1) If the heat-supply was too great, the electric current could be temporarily stopped, or, the rate of evaporation of the water increased. (The latter was the method generally adopted.)

(2) If the cooling was too rapid, the only mode of maintaining the balance was (in the apparatus about to be described) to reduce the rate of evaporation.

"The liquid to be evaporated was contained in a small silver flask, connected with which was a spiral coil of silver tubing 18 feet in length. Both flask and spiral were within the calorimeter, and the water-vapour, after passing through the spiral, emerged from the apparatus at the temperature of the calorimeter. Surrounding the flask, and between it and the spiral, a coil of platinum-silver wire was arranged, and flask, spiral, and coil were entirely immersed in a certain singularly limpid oil consisting of hydrocarbons only.

"The calorimeter (which was filled to the roof with the oil, and the equality of temperature maintained by rapid stirring) was suspended by glass tubes within a steel chamber, whose walls were maintained at a constant temperature. So long, therefore, as the calorimeter and the surrounding walls were at equal temperatures, there was no loss or gain by radiation, &c. If during an experiment the temperature of the surrounding walls changed, the method of experiment involved a corresponding change in the temperature of the calorimeter, and, therefore, some loss or gain of heat would be experienced. The apparatus was so designed that any such change in temperature was extremely small (in no case amounting to $\frac{1}{100}°$), yet, in order to estimate the loss or gain, it was necessary to know approximately the capacity for heat of the calorimeter and contents.

"Small differences between the temperature of the calorimeter and the surrounding walls would, during an experiment, be of no consequence provided that the oscillations were of such a nature that the mean temperature of the calorimeter was that of the surrounding space, and it will be found that this condition was fulfilled.

"In addition to the heat supplied by the electric current, there is also a supply due to the work done by the stirrer, and it was in the estimation of this 'stirring supply' that the greatest difficulties were encountered. Fortunately the heat thus generated was only about $\frac{1}{100}$ of the heat supplied by the current, and thus any small error in that portion of the work becomes of little account.

"Of the accuracy with which the electrical supply could be measured there is no question; and even if the value of the E.M.F. of the Clark cells, or the absolute resistance of the box-coils given by the standardizations performed during the determinations of J, is in any way inaccurate, such errors would now eliminate, since the value of J was determined by means of the same standards as those by which the quantity of heat developed in these experiments was determined. Hence, by assuming the value of J obtained by the use of these standards, we get the comparison in terms of a thermal unit at 15° C., independently of the numerical value assumed in the reductions.

"One further correction remains to be noticed. The temperature of the calorimeter has been referred to as oscillating about the exterior temperature, and it might happen that at the close of an experiment this difference was not the same as that at the commencement—if any such difference existed. The magnitude of this correction depended, of course, on the ability of the observer to maintain the thermal balance. In these experiments the correction was usually small, and in any case could be determined with great accuracy.

"Having indicated the nature of the observations, we proceed to state the relation between the various sources of loss or gain of heat.

"Let Q_E be the thermal units per second due to the electrical supply;
Q_S be the thermal units per second due to the mechanical supply;
Σq be the total heat-supply during an experiment from any other causes.

"Then, if M be the mass of water evaporated, L the latent heat of evaporation at temperature θ, and if the electrical supply is maintained for a time t_E, and the mechanical for a time t_S,

$$ML = Q_E t_E + Q_S t_S + \Sigma q. \quad . \quad . \quad . \quad . \quad (1)$$

"Now the D.P. at the ends of the coil was always some integral multiple of the D.P. of a Clark cell.

"Let e be the D.P. of a Clark cell, n the number of cells, and R_1 the resistance of the coil at the temperature θ_1, then

$$Q_E = \frac{e^2 n^2}{R_1 J}. \quad \ldots \ldots \ldots \quad (2)$$

"If the calorimeter at the commencement and end of an experiment was at exactly the same temperature as the surrounding walls, then, if their temperature was unchanged, the term Σq would vanish; but although this term throughout these experiments was of small dimensions, it could not be entirely ignored.

"Let θ_0' and θ_0'' be the temperature of the surrounding walls at the beginning and end of an experiment; suppose the calorimeter temperature (θ_1) to exceed the surrounding temperature by d' at the commencement and d'' at the end of an experiment. Then fall in temperature of calorimeter

$$=(\theta_0'+d')-(\theta_0''+d'').$$

Hence the heat given out by the calorimeter in consequence of this fall in temperature is

$$C_{\theta_1}\{(\theta_0'+d')-(\theta_0''+d'')\},$$

where C_{θ_1} is the capacity for heat of calorimeter and contents at the temperature θ_1.

"If we neglect any small loss by radiation, &c., due to the differences d' and d'' between the temperature of the calorimeter and the surrounding walls, we may conclude that the whole of the heat thus evolved by the calorimeter was expended in the evaporation of water, hence

$$\Sigma q = C_{\theta_1}\{(\theta_0'-\theta_0'')+(d'-d'')\}^*. \quad \ldots \quad (3)$$

Hence

$$ML = \frac{e^2 n^2 \times t_E}{R_1 J} + Q_S \times t_S + C_{\theta_1}\{(\theta_0'-\theta_0'')+(d'-d'')\}. \quad (4)"$$

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* This apparently clumsy method of representing the quantity of heat evolved or taken up by the calorimeter was adopted because, as the method of experiment involved separate determinations of θ_0', θ_0'', d', and d'', the actual temperature of the calorimeter at any time could only be obtained in this manner.

3. The following table shows the *comparative* mean values of the terms in equation (4) resulting from our experiments on benzene[*].

TABLE I.

No. of Cells.	$Q_E t_E$.	$Q_S t_S$.	Σq.
5	1	·008	±·005
4	1	·011	±·009
3	1	·015	±·001

We found that the thermal balance was most easily maintained when the potential-difference was that of 4 cells, therefore the majority of our experiments were performed with that potential-difference, the experiments with 5 and 3 cells being used as a check upon the results. To secure an accuracy of (say) 1 in 1000 in the total heat-supply, the above table shows that it was necessary to measure

$Q_E t_E$ to (say) 1 part in 2000,
$Q_S t_S$ „ 1 „ 15,
Σq „ 1 „ 10;

but as the sign of the last term was in some experiments positive, in others negative, the above degree of accuracy was not essential, since, when taking the mean of a number of observations, the effect of any error in Σq would be greatly diminished. We believe that the above order of accuracy was exceeded in the measurement of the respective terms. In the case of the experiments with 3 cells at 20° an order of accuracy of 1 in 50 would be required in the value of $Q_S t_S$ to secure the same standard; hence we do not regard our determinations at that temperature as possessing an equal authority with those at higher temperatures.

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[*] (Experiments Preliminary, XIX. *a*, XXXV., and XXXVI. were not included when calculating this table, as they are in several respects exceptional.)

4. *The Method of maintaining the Space surrounding the Calorimeter at a Constant Temperature* θ_0.

A full description of the somewhat elaborate apparatus designed for this purpose will be found on pp. 374–378 of Paper J, and some improvements which were subsequently added are described on pp. 274–276 of Paper W. This portion of the apparatus did not, for some unknown reason, appear to be working with the same perfection this year as it did in the summer of 1894. Oscillations in θ_0 (the external temperature), amounting to $\frac{1}{100}$° C., were on several occasions observed, the greatest change during the whole series being ·0145°. This, however, was exceptional and, as an inspection of our final tables will show, the change in the course of an experiment (*i. e.* in about half an hour) was usually only about a few thousandths of a degree at temperatures varying from 20° to 50° C.; the constancy of θ_0 was therefore sufficient for our purpose.

Had it been possible, we should have preferred to determine θ_0 by means of a platinum-thermometer, as we could thus have detected smaller changes; but a third observer would, in that case, have been required, and circumstances did not permit of this addition*.

The mercury-thermometers used for indicating the temperature of the steel chamber in which the calorimeter was suspended were graduated in millimetres, and had been carefully calibrated and compared with two Tonnelot thermometers standardized at the Bureau International†. The stems of these thermometers (where they projected above the tank-lid) were surrounded by glass tubes up which a stream of tank-water was forced by means of a pump driven by a water-motor: thus the stem-temperature was always that (or nearly that) of the tank, and the readings were not affected by changes of temperature in the room. The actual value of θ_0 was of small consequence, an accuracy of $\frac{1}{10}$° C. being sufficient; but it was necessary to read *changes* of temperature during an experiment to nearly 0°·001 C., and this we believe we were able to do. A full description of the various precautions taken will be found in Paper A, pp. 55 and 56, and Paper W, pp. 275, 276.

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* We believe that such irregularities as present themselves in our final results are partly due to the employment of mercury-thermometers for the determination of the changes in θ_0.

† See "The Measurement of Temperature," Science Progress, Sept. 1894.

The observations were taken by means of a reading-telescope fitted with a micrometer-eyepiece, which directly divided 1 millim. on the thermometer-stem into 10 parts: thus 0·1 of a millimetre could be directly read, and ·01 could be estimated. There is no doubt that the actual readings could be taken to ·025 millim., that is about 0°·001 C.

Thermometer A had a range of 16° to 26° C., and about 27 millim. were equivalent to 1° C.

Thermometer II. had a range of 28° to 53° C., and about 20 millim. were equivalent to 1° C.

All temperatures, both when obtained from these and from the platinum-thermometers, are throughout this Paper expressed in terms of the nitrogen-scale.

5. *The Calorimeter and its Connexions.*

A full description will be found on pp. 276–281 of Paper W, and drawings and sections on plates 5, 6 of the same paper. The brief account given in the description of the method *supra* is sufficient to indicate the nature of the arrangements to those who are not familiar with that paper.

The *differences* of temperature between the calorimeter and surrounding walls were determined by means of differential platinum-thermometers. A description of the method of standardization and observation of those thermometers is given on pp. 52–56 of Paper A, and some further details on pp. 285–290 of Paper W. Experimental evidence is there adduced in support of the following statement:—"It follows that differences of temperature could be determined to ·0004°, and differences of ·0001° could be detected."

The following Table shows the difference in temperature corresponding to a difference of 1 mean millim. of the bridge-wire whose opposite ends were connected with the thermometers:—

TABLE II.

Temp.	$\Delta\theta$ for difference of 1 mean millim. bridge-wire.
20°	0°·009046
30	0·009073
40	0·009101
50	0·009128

By experiments repeated this summer we found that no change had taken place in the value of the mean bridge-wire millimetre. A slight change (particulars of which are given in a subsequent section) had, however, occurred in the position of the null-point, *i. e.* in the reading on the bridge-wire when the temperature of the two thermometers was identical.

6. *Brief Description of the Method of obtaining the Value of the Terms in the Expression*

$$\text{Total heat} = Q_E t_E + Q_S t_S + \Sigma q.$$

$$Q_E t_E = \frac{(ne)^2 \times t_E}{R_I \times J}.$$

The ends of the platinum-silver coil (immersed in the oil surrounding the evaporating-flask and spiral) were kept at a constant potential-difference by means of the arrangements described in Paper J, pp. 382–388. This potential-difference was always some integral multiple (n) of the potential-difference of a Clark cell (e). During the spring of this year the Clark cells used were again compared with the Cavendish standard (R_I), which has shown no signs of change since its standardization by Lord Rayleigh in 1883 and by Messrs. Glazebrook and Skinner in 1891.

The mean value of the whole set of 30 cells differs from R_I by 0·00004 volt only; and although individual cells show larger discrepancies than in previous years, their mean potential-difference at 15° C. may be taken as 1·4342 volts (see Paper W, p. 297). A number of these cells were always placed in parallel arc: thus when n is given as 4, we were really using 12 cells as four files of 3 each. The arrangement for keeping the temperature of these cells at nearly 15° C. has been described in Paper J, p. 385. During the period covered by our experiments, some of the days were extremely hot, and the tap-water became so warm that when turned on by the regulator it was unable to keep down the temperature sufficiently. The extreme range in the temperature of the cell-tank during these experiments was from 14°·8 to 16°·3 C. As, however, the movement was always extremely slow, it is probable that the tank-temperature closely corresponded to the effective temperature of the cells, and hence the correction

$$e = 1{\cdot}4342\{1 + (15 - \theta) \times {\cdot}00077\}$$

applied by us gave the value of e with sufficient accuracy.

The time t_E was determined by means of a chronograph controlled by an electric clock, whose gaining rate is now less than 1 in 20,000. Any movement of the keys by which the current was switched on to the calorimeter-coil was automatically recorded on the tape; and thus personal errors were eliminated. The times could have been read to $\frac{1}{100}$ of a second, but it was considered unnecessary to read to nearer than $\frac{1}{10}$, *i. e.* about 1 in 7000 of t_E.

The value of R_1 is expressed in terms of the "true ohm" as given by the B.A. standards, with which (by kind permission of Mr. Glazebrook) the coils used by us have been directly compared (Paper J, pp. 407–410).

The increase in R_1 due to the rise of temperature caused by the current was determined in the manner described in Paper J, pp. 404–407 (see also Paper W, p. 296).

The value of J assumed by us was 4·199 (Paper W, p. 314). We would emphasize the fact to which attention has been previously directed (section 2, p. 3), namely, that even if, in consequence of errors in the standards &c., this value of J is incorrect, it is still the right value to use for the reduction of these observations, for, provided that no change has taken place during the past three years in the standards used (and direct comparisons show no signs of any change which would affect the results), the values of L obtained by us are independent of the numerical values of J and R when expressed in terms of the same units as those assumed during the determination of the mechanical equivalent.

The value of $Q_E t_E$ is thus expressed in terms of a "thermal unit at 15° C."

7. $Q_S t_S$.

The method adopted for finding Q_S (the thermal units per second due to the work done by stirring) has been fully described in Paper W, pp. 290–293. It was there shown that the value of Q_S varied approximately as r^4 (where r was the rate of revolution of the stirrer), and that this relation was sufficiently close for the reduction of experiments in which the value of r was somewhere between 5 and 6 per second.

The value of Q_S was found to increase rapidly as θ_1 (the temperature of the calorimeter) diminished. Our recent redeterminations of Q_S have explained a discrepancy which caused an uncertainty in the former determination of Q_S at 20° and 50°, and we now find that it was due to an arithmetical error in the reduction of the observations. Fortunately, the resulting correction in no way affects the conclusions arrived at in Paper W; and our present investigation confirms the accuracy of the experimental determinations of Q_S obtained in 1894. As the extreme differences between any of our determinations of Q_S at temperatures above 20° (after the arithmetical corrections at the 20° and 50° points) do not exceed 1 part in 70, even if we include the values found in 1894, and as the mean probable error at any temperature is much below 1 in 100, it is obvious that the values of Q_S have been ascertained with more than sufficient precision.

8. $\Sigma q = C_{\theta_1}\{(\theta_0' - \theta_0'') + (d' - d'')\}$.

We have previously indicated (sect. 4, p. 6) the manner in which θ_0' and θ_0'' (the initial and final temperature of the walls of the surrounding chamber) were determined by the direct observation of mercury-thermometers. The value of d (the difference between θ_1 and θ_0, *i. e.* the calorimeter-temperature and that of the surrounding walls) was ascertained by the differential platinum-thermometers previously referred to, and was always small—rarely greater than ·01° C. It was usually determined by the reading of the galvanometer-swings without altering the position of the contact-maker on the bridge-wire.

When evaporating water, the values of d were so small that an approximate value of the swing (in terms of a length of the bridge-wire) was sufficient. During these benzene experiments, however, we found it impossible to maintain the thermal balance with such perfection. The only values of d which were of consequence were d' and d'' (the initial and final values), and, especially in the earlier experiments, the internal temperature (θ_1) rose so rapidly as soon as the last drop of benzene had evaporated, that it was found impossible to switch off the electric current from the calorimeter-coil at the precise moment necessary to reduce d'' to negligible dimensions. As a consequence, the final swings (which were read by means of a micrometer-eyepiece) were often considerable, and therefore it was necessary to determine their value on each occasion; for, although throughout a series of experiments at the same temperature the value varied but little, the changes were sufficient to affect the resulting value of Σq in exceptional cases. In the earlier experiments, especially from Preliminary to No. VII., when the difficulty of the final adjustment had not been fully realized, the values of d'' were unduly large, and their equivalent degree-measurements are therefore somewhat doubtful, owing to insufficient determinations of the value of the galvanometer-swings. We feel sure, however, that from No. VII. onwards any errors due to this cause must be very small—certainly not so great as 1 in 50; for the value of a swing of 100 (in terms of a millim. of the bridge-wire) would, throughout a group of experiments at the same temperature, vary (for example) from 1·31 to 1·34 millim.* This change in value would produce no appreciable effect on Σq when the difference between the initial and final swings did not exceed 50 or 60, as was the case in most of our later, and better, experiments.

The value of C_{θ_1} (capacity for heat of calorimeter and contents) remains the same as last year with the exception that 0·1 grm. of the oil was removed at the commencement of July when withdrawing the platinum thermometer for purposes of re-standardization. As the value of C_{θ_1} varied with θ_1 from 304 to 323, this loss was negligible.

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* 1 millim. of the bridge-wire indicated a temperature-difference of about 0°·009 C.

The quantity Σq represents the heat absorbed by the calorimeter and contents, and it should be remembered that it is by the measurement of a similar quantity that the majority of the determinations of thermal constants have been made by previous observers. We were, however, anxious to diminish the importance of this term as much as possible, for we wished our values of L to be independent of any thermometric errors. As shown by Table II., the average value of Σq was only about $\frac{1}{200}$ of the total heat-supply. In the experiments in which the value of Σq is less than 1 (of which there are many examples), we may say that our final results are independent of such errors, for, as pointed out in Paper W, the value of Q_S is independent of temperature measurements, since it depends on the *ratio* of the rate of rise due to the mechanical supply to the rate of rise due to an electrical together with a mechanical supply (Paper W, p. 331).

The values of C_{θ_1} at 20° and 50° used in this Paper differ somewhat from those given in Paper W. This difference is due to the error, previously referred to, in the value of Q_S at those temperatures. Thus

the value of C_{θ_1} at 20° is reduced from 307·5 to 305·2,
and „ „ 50° „ „ 323·1 to 322·0.

This, however, in no way affects the values of L in Paper W, nor, except in the case of two experiments (viz., Preliminary and XIX. *a.*, *infra*), would it, if left uncorrected, have affected the values of L as given in this communication.

We believe that the values of Σq are correct to better than 1 in 100; and Table I. (p. 21) shows that an accuracy of 1 in 10 would have been sufficient.

9. *Measurement of Mass.*

The actual measurement of the mass of benzene evaporated presented several difficulties which were not encountered when working with water. A tube resembling a weight-thermometer was filled with benzene, placed in an air-tight case just large enough to contain it, and then weighed against a precisely similar case used as a tare. The weight-thermometer (termed by us a "dropper") narrowed at its open extremity to a capillary tube, which was doubled back on itself for rather over 1 cm., and again bent near the open end, so that the last 1 or 2 mm. were horizontal. These droppers varied in capacity from about 4 to 6½ c.c., and were filled in the following manner:—The dropper (point uppermost) was lowered by a fine wire to the bottom of a tube about 4½ ft. long, of which the lower 10 inches or so were filled with benzene, while the upper 3 ft. were surrounded by a "condenser-tube" through which tap-water was continually passing. The lower end, containing the benzene, was transferred at regular intervals from a vessel of water at about 86° C. to a vessel of cold water; thus the benzene was alternately boiled and cooled without any escape of vapour into the room. Five or six such transferences were generally required to completely fill the dropper. The containing tube was then placed in a bath at about 65° C. until the temperature of the benzene was steady. The dropper was now removed and allowed to stand in the open air for some time, in order to get rid of any benzene adhering to its surface. Although simple and effective, these operations occupied a considerable time, and, as a rule, the whole of the morning had to be devoted to the filling of the droppers required for the experiments, which were usually performed at night.

Before an experiment the dropper was lowered, by means of a thread passed through a platinum wire sealed into the closed end, into the calorimeter, where it stood in a vertical position. In Paper W (p. 307) it was shown that, when filled with water, the evaporation through the capillary opening between the time of weighing and the commencement of an experiment might be neglected. In the case of benzene, however, it was found that, for several reasons, loss of this kind could not be disregarded:—

(1) The vapour-pressure of benzene so greatly exceeds that of water at corresponding temperatures that the loss by diffusion through the capillary was appreciable.

(2) The surface-tension of benzene is so great that the liquid crept up the sides of the capillary to the opening, and the consequent loss by evaporation was increased.

(3) In order to supply the air necessary for starting the

boiling when the exterior pressure was removed, a capillary tube, closed at one end, had been sealed within each dropper. It was found that this answered very well during some preliminary trials, when the dropper was placed within a glass tube connected with an exhaust pump so that its manner of discharging could be watched; but on a second filling with our purest sample (the first filling having thoroughly cleaned the interior surface of the droppers) no action took place even when the surrounding pressure was reduced to a few millimetres. In order, therefore, that the expulsion of the benzene from the dropper should commence as soon as the external pressure was reduced to the right amount, it was found necessary to leave a very considerable air-bubble within the tube. Precautions had to be taken to prevent the expulsion of the liquid by the alternate contraction and expansion of the air-bubble when the temperature was changed from that of the balance-case to that of the tank.

These difficulties were surmounted in the following manner.

During the process of filling, as above described, the droppers stood in a vertical position, with the doubled over and open end uppermost, and were never inverted after their removal from the filling-tube, at a temperature of about 65° C., until their insertion into the calorimeter. The coefficient of expansion of benzene is very large, and, on cooling to the room-temperature, the whole of the upper bend together with a couple of centimetres of the neck between it and the body of the dropper was found to be free from benzene. The dropper was then placed in its case and left in a vertical position in the balance-case for, as a rule, some hours, when the small air-space in the enclosing-case no doubt became saturated with the vapour and thus further evaporation ceased. After being weighed, the dropper and case were lowered into a large tube placed within the tank and left until they acquired the tank temperature θ_0; they were then rapidly withdrawn and for the first time inverted—the air-bubble rising to the closed end. The case was opened for a moment and the dropper at once lowered into place by means of the previously attached thread: this operation only occupied a couple of seconds. As the dropper left the case the latter was instantly closed by a second operator to prevent the escape of any benzene vapour left within it. Thus from the time of inversion no change took place in the temperature, and therefore in the volume of the air-bubble; also no appreciable change in the temperature of the calorimeter (θ_1) was caused by the introduction of the dropper and contents. However, the mere act of opening the tube leading down to the calorimeter caused a slight lowering of θ_1, and in order to re-establish the equality between θ_1 and θ_0 before commencing an experiment, it was necessary to switch on, for a second or two, the current from the exterior coil in the tank to the calorimeter coil. When the observer at the differential-thermometer galvanometer announced that $\theta_0 - \theta_1$ was small and steady, the time for commencing the experiment had arrived.

10. In our later experiments, when we had become more expert at the various operations, the time from the insertion of the dropper to the commencement of an experiment was from 3 to 5 minutes. In our earlier observations at 30° the time was, however, much longer—from about 8 to 15 minutes. During this interval there was generally apparent a slight lowering of θ_1, which made us fear that the evaporation through the capillary was appreciable, and it was not until our tenth experiment that a means of meeting this difficulty suggested itself. It was evidently necessary to wait until θ_1 became steady before commencing an experiment, and it was difficult to see how to shorten the time required to establish this condition.

From Experiment X. onwards the procedure was as follows. A glass rod whose upper end passed through a cork fitting the opening of the entrance-tube was lowered into the calorimeter. Round the lower three inches of this rod was strapped (by fine platinum wire) a thin roll of cotton-wool of which the upper two inches were saturated with benzene while the lower end was left dry, so that there was no danger of drops of benzene falling from it into the silver evaporating-flask.

The rod was withdrawn occasionally and more benzene added if the upper portion of the roll had become dry. Observation of the galvanometer showed when the cooling effect caused by the evaporation had ceased, and we were thus able to determine when the flask and connecting tubes were saturated. The rod was not finally withdrawn until the dropper was introduced: thus the space being saturated, no further evaporation took place; and we believe that from Experiment X. onwards any error arising from evaporation in the time preceding the experiment may be disregarded.

After we had completed our experiments, we made some observations with the object of ascertaining the probable magnitude of the error in Experiments I. to VII. due to evaporation when the flask had not been previously saturated. A dropper was placed within the evaporating flask for 20 minutes and kept at a temperature of 30° C. It was then withdrawn, and the loss determined. As might have been expected, it varied slightly according to the droppers used, probably owing to the different sizes of the capillary opening.

In 20 minutes, dropper II. lost 10 milligrams,
" " " III. " 12 "
" " " IV. " 13 "

After Experiment VII. we adopted the plan of noting the time from the insertion of the dropper to the commencement of the experiment, but unfortunately we had not previously done so. We consider that the average time in these experiments must have been from 10 to 15 minutes; the loss during this time would therefore appear to have been about 6 milligrams, or rather more than $\frac{M}{1000}$. Thus the values of L resulting from Experiments I. to VII. are probably too low by about 0·12. Fortunately we have six independent experiments at 30° C. (Nos. XXIX. to XXXIV.) in which this cause of error was absent.

We have entered into this matter fully, as it shows the importance of extreme attention to details in work of this kind, and also it was necessary to explain why we practically neglected Experiments I. to VII. when drawing our final conclusions. The same cause of error would slightly affect

Experiments VIII. and IX. at 40° C. At this time, however, we were attempting to minimize the evil by allowing as short a time as possible to elapse between the introduction of the dropper and the commencement of an experiment. We have also a note of the time, which in both cases was less than 4 minutes. The loss during this interval would probably not have affected the resulting values of L by more than ·05, and we therefore do not consider it necessary to reject these experiments.

At the close of an experiment, when observation of the galvanometer showed that all evaporation had ceased, the dropper was extracted by means of a bent wire, immediately placed in its case, and weighed again after standing some hours in the balance-case.

Let m_1 and m_2 be the weights of the case and dropper before and after an experiment; the temperature of the balance-case was usually about 20° C., the specific gravity of benzene at that temperature may be taken as 0·88, hence M (true mass corrected to vacuo)

$$= m_1 - m_2 + (m_1 - m_2) \times {\cdot}0012.$$

11. Before commencing the experiments it was necessary to ascertain if any alteration had taken place in the values of the various constants and variables since their determination in 1894.

Thermometers II. and A were re-standardized and it was found that a "zero-point rise" had taken place, as is customary with mercury thermometers.

Thermometer No. II. had risen 0·4 mm.$=0^{\circ}{\cdot}02$ C. since its standardization in August 1894.

Thermometer A had risen 1·4 mm.$=0^{\circ}{\cdot}06$ (nearly) since its standardization in July 1893, which is about the normal rise of thermometers of this description.

The platinum thermometer (AB) was removed from the calorimeter, strapped to its corresponding thermometer (CD), and placed in the tank whose temperature was raised from 18° C. to 40° C. Observations at different temperatures showed a rise of 0·45 mm. in the null-point whose position is now given by the formula $598{\cdot}8 + {\cdot}03\,\theta$ in place of $598{\cdot}35 +$

·03 θ. If, however, this change, which probably took place in the arms of the bridge rather than in the thermometers *, had not been detected the resulting error would have been negligible, for a difference of ·45 mm. in the setting of the null-point is equivalent (at 40°) to a temperature difference of about ·0041° C., and the total loss or gain by radiation, &c., corresponding to this difference between θ_1 and θ_0 would not exceed 0·2 thermal gram per half-hour, whereas the actual duration of the majority of these experiments was about 18 minutes.

The corrected formula was, however, used throughout these experiments for the adjustment of the contact-maker.

12. *Value of* R_1.

The values of R_1 were redetermined at temperatures 30°, 40°, and 50°. The corrected results showed falls of ·0012, ·0012, and ·0016 respectively from the values of 1894. These quantities have, therefore, to be subtracted from the values given in Paper W, table viii., but the correction is only 1 in 10,000 of R_1.

The following table gives the values of R_1 used during the reductions of the observations on benzene.

(The suffix to R denotes the potential-difference in terms of a Clark cell.)

TABLE III.

Temp.	R_{3e}.	R_{4e}.	R_{5e}.
20°	10·327	10·329	10·333
30	10·351	10·353	10·357
40	10·374	10·377	10·381
50	10·399	10·401	10·406

δR per 1° C. = ·0024.

The re-standardization of the Clark cells has already been referred to (section 6, p. 24).

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* [Note by E. H. G., August 12, 1895.—I have found by re-standardizing the bridge-arms, that the above supposition was correct.]

13. Q_s.

No alteration in the values of Q_s at 30° and 40° C. appears to have taken place.

The values of Q_s at 50° and 20° as given in Paper W, pp. 332, 333, appeared to have undergone alteration, and we therefore made a careful redetermination at those temperatures. The method adopted was that described on p. 292, Paper W, viz., the rate of rise in temperature at null-point was determined

(1) when the heat-supply was that due to the stirring only

$$=\left(\frac{d\theta_1}{dt}\right)_s, \text{ and}$$

(2) when the heat-supply was that due to the stirring and a potential-difference of 3 Clark cells

$$=\left(\frac{d\theta_1}{dt}\right)_{3es}.$$

Hence
$$\frac{\left(\frac{d\theta_1}{dt}\right)_s}{\left(\frac{d\theta_1}{dt}\right)_{3es}-\left(\frac{d\theta_1}{dt}\right)_s} \quad . \quad . \quad . \quad . \quad . \quad . \quad (A)$$

gives the ratio of the heat supplied by the stirring to the heat-supply when the potential-difference was that of 3 Clark cells; and as the latter can be calculated if R_1 is known, the value of Q_s can be obtained without any assumptions as to the thermometric scale, the capacity for heat of the calorimeter, &c.

The individual experiments were in close agreement with each other and give the following results:—

At 50°, $\left(\frac{d\theta_1}{dt}\right)_{3es}$ $=\cdot 14533$ expressed in mm. of bridge-wire where rate of stirring $=5\cdot 380$,

$\left(\frac{d\theta_1}{dt}\right)_s$ $=\cdot 001155$, at rate $5\cdot 380$.

Hence Q_s (at rate $5\cdot 380$) $=\cdot 003404$;

but (see Paper W, p. 330) $\frac{r^4}{Q_s}=$ a constant $=\frac{837\cdot 8}{\cdot 003404}$, hence $Q_s = \cdot 003206$ at rate 5·300.

A second determination gave

$Q_s = \cdot 003202$ at rate 5·300.

A third gave

$Q_s = \cdot 003205$ at rate 5·300.

We therefore assume

$$Q_s = \cdot 00321 + (r_1{}^4 - 789) \times \cdot 0000041,$$

as sufficiently accurate.

At 20°, $\left(\frac{d\theta_1}{dt}\right)_{3es} = \cdot 15941$ at rate 5·590,

$\left(\frac{d\theta_1}{dt}\right)_s = \cdot 00427$ at same rate.

Hence Q_s (at rate 5·590) $= \cdot 01174$;

but $\frac{r^4}{Q_s} =$ a constant $= \frac{976 \cdot 6}{\cdot 01174}$,

hence $Q_s = \cdot 00949$ at rate 5·300.

A second determination gave ·00951 at rate 5·300,
A third ,, ,, ·00991 ,, ,, 5·300.

In this last experiment, however, there is internal evidence of some error in the time over the second interval. If we omit this interval and calculate the value of Q_s from the remaining intervals of that experiment, we get ·00961 as the value.

It is evident that ·00950 is a sufficiently close approximation.

We therefore assume

$$Q_s = \cdot 00950 + (r^4 - 789) \times \cdot 0000120.$$

Our experiments at 30° and 40° show that the values of Q_s at those temperatures as given in Paper W are correct. The errors at 20° and 50° as given in that paper were due to an arithmetical mistake, a difference having been added, instead of subtracted, in each case.

The following Table gives the expressions by which the value of Q_s can be obtained at any of the temperatures or rates given in succeeding Tables :—

TABLE IV.

Temp.	Value of Q_s (in thermal grms.).
50°	$\cdot00321+(r_1^4-789)\times\cdot0000041$
40	$\cdot00466+(r_1^4-789)\times\cdot0000059$
30	$\cdot00665+(r_1^4-789)\times\cdot0000084$
20	$\cdot00950+(r_1^4-789)\times\cdot0000120$

These changes in Q_s probably indicate the changes in the viscosity of the oil.

14. *Alterations in the Apparatus.*

No alterations have been made except in exterior portions of the apparatus.

When working with water in 1894 there were, in the tubes leading to the air-pumps, one or two rubber joints which are now replaced by glass ones.

To prevent condensation, the benzene vapour after issuing from the tank passed over a row of small gas-jets and then down into a small Wolff's bottle, connected with the manometer and containing pumice-stone and sulphuric acid. It then passed through another tap* into a large globe (capacity about 35 litres) also containing pumice-stone and sulphuric acid. By means of a water-pump the pressure in this globe was reduced to that of the aqueous vapour at the temperature of the tap-water. Its capacity was so great that the pressure in the Wolff's bottle could, at any time, be brought below that required, at experiments above 20°, by simply opening the tap communicating with the globe. Thus the water-pump had not to be used during an experiment, and, consequently, the two motors worked with greater regularity. The Wolff's bottle was also in direct communication with a Geissler's mercury-pump, by means of which, when working at the lower temperatures, the pressure was greatly reduced near the close of an experiment in order to secure the boiling-off of the last drop of benzene. We found that the mercury-pump had to be kept in constant use during the experiments at 20°. With the exception of the above alterations and additions, the apparatus is the same in every respect as that figured in Plates 4, 5, and 6 of Paper W.

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* The grease on the core of taps traversed by benzene was replaced by phosphoric acid.

15. *On the Purity of the Benzene.*

The benzene, which was a sample of that used by Professor Ramsay and Miss Marshall for their comparative experiments, was supplied by Messrs. Kahlbaum of Berlin and guaranteed free from thiophene. It was redistilled twice from phosphoric anhydride, until it showed a perfectly constant boiling-point.

16. *Description of an Experiment.*

The dropper was placed in position (as described in section 9) and the contact-maker was then set, by means of a magnifying-glass, to the null-point corresponding to the tank temperature θ_0.

When the observer (Observer II.) at the thermometer-galvanometer (G_2) announced that θ_1 had become steady, three observations of the galvanometer-swing were taken, and the chronograph-key, being pressed at the second of those swings, recorded the time from which t_s was estimated. At the same moment Observer I. recorded the reading of thermometer II., which when reduced to the nitrogen scale gave the value of θ_0',—the initial tank temperature.

As a rule the initial swings were somewhere between 0 and $+50$ (a swing of $+50$ would correspond to about $+\cdot006°$ C.), and, for reasons which will appear later, we preferred to have this initial swing (which gave the value of d') positive.

The Wolff's bottle (condenser A) had previously been exhausted down to the pressure of the large globe (condenser B) already referred to, but the tap between the two condensers had been closed. A tap (immersed in the tank-water) between the silver flask and condenser A was then opened, and the air expanding into A caused a fall in the manometer attached to it. The tap connecting condensers A and B was then gradually opened, and the pressure in the flask fell until some benzene was expelled from the dropper. The instant this occurred the calorimeter temperature θ_1 commenced to fall, and Observer I. was acquainted with this fact by Observer II. who, throughout the whole experiment, was engaged in calling aloud the galvanometer-swings which resulted from the inversion of the battery connexions in the differential-thermometer circuit by means of a swinging key. The storage-cell current, which had been running for some time through a platinum-silver coil immersed in oil in the outer tank, was then switched on to the calorimeter-coil, the action recording itself on the chronograph-tape, and the potential balance then adjusted by means of the apparatus described in Paper J, p. 283. This balance had previously been approximately obtained while the current was running through the tank-coil; thus only a small additional adjustment was required and, as the temperature of the calorimeter-wire remained constant, the electrical balance required little attention, for the potential-difference rarely altered by as much as $\frac{E}{100,000}$ throughout an experiment. As the benzene vapour passed into condenser A the pressure in the flask increased, and thus the loss of heat by evaporation diminished. By a rapid movement of the tap between condensers A and B, the pressure could again be diminished and the cooling effect increased.

It was found impossible to control the rate of evaporation with the same perfection when using benzene as was the case in the experiments with water. The taps had to be constantly manipulated, and a moment's inattention on the part of Observer I. was immediately followed by a sudden rise or fall in θ_1. This was more especially the case during the experiments at low temperatures. From about experiment V. or VI. onwards, however, the swings rarely amounted to as much as 200 or so, except during the first minute when the thermal balance was being obtained, at which time a swing of ± 400 or 500 was generally experienced. Throughout the whole of an experiment care was taken that any positive swing should be succeeded by a corresponding negative one. Although the announcement of swings of "$+200$" appeared alarming at the time, the extreme attention devoted to the keeping down of these oscillations was really unnecessary. A swing of $+200$ indicated that θ_0 was lower than θ_1 (at 40°) by about 0°·02 C., and the radiation &c. coefficient of the calorimeter being about ·00009 (in degrees per second per difference of 1°, Paper W, p. 289), this swing, even if maintained throughout the *whole time* that evaporation was proceeding (on an average less than about 7 minutes), would only have resulted in a *total* loss of about $\cdot 00009 \times 318 \times \cdot 0216 \times 7 \times 60 = \cdot 25$ thermal gram *. In no case, however, was a swing of such magnitude allowed to remain unaltered for more than a few seconds.

The chief difficulty was experienced near the close of an experiment. When working with water, there were always some indications that the end was near, for the pressure had to be diminished in order to maintain the thermal balance if only a drop or so remained. In the case of benzene, however, there were rarely any such indications; for the galvanometer-swing might be announced as $+8$ or $+10$, and before the next announcement could be made it would be found that it had shot up to 300 or 400. After the preliminary experiment, it was of course possible to roughly estimate (knowing the weight of the dropper when filled) the probable duration of an experiment and, by adding it to the observed time of establishing the current, to predict approximately the time when evaporation would cease. Owing, however, to differences in the size of the air-bubbles necessarily left in the droppers, to the different rates of stirring, &c., and to errors in calculations made while all the attention of the observers was needed elsewhere, the current was very often switched off too late, thus increasing the value of the correction Σq unnecessarily. If any mistake was made, the final value of θ_1 was in consequence higher than the initial one, for in those cases where the current was switched off too soon, it was always possible to bring θ_1 up to its initial value by re-establishing the current for a second or two; if, however, θ_1 was too high there was no means of diminishing it, all the benzene having been exhausted. Observer I. endeavoured, if possible, to arrange so that the swing at the close of an experiment should be about −150, for the following reason. Evaporation having ceased and the current being switched off, the tap between the evaporating-flask and condenser A was closed, and a tap (also immersed in the tank-water) was slowly opened so that air (dried by having passed through sulphuric acid and phosphoric anhydride) was gradually admitted into the evaporating-flask through a 30 ft. copper coil immersed in the tank. Thus the air was at the temperature of the calorimeter. The heat liberated by its compression caused, however, a rise in θ_1 equivalent to a swing of about 120 ; thus no heat was, on the whole, gained by the calorimeter, for a corresponding loss had been experienced during the exhaustion. On this account we preferred to commence with a small positive swing, as it was not then necessary to reduce θ_1 so far at the close of an experiment to allow for this final increase. When Observer II. found that θ_1 had again become steady three final swings were taken, the chronograph-key being pressed at the middle one, thus giving the termination of the time t_s during which the stirring supply had to be estimated. Thermometer II. was read at the same time, and gave θ''_0, the final tank temperature. Throughout the experiment, every thousand revolutions of the stirrer had been automatically recorded on the chronograph-tape.

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* About $\frac{1}{2000}$ of the total "heat-supply."

The method of removing and weighing the empty dropper has already been indicated.

17. *Remarks on the Experiments.*

We give particulars of all our experiments with two exceptions. In No. XXI. *a.* the chronograph ceased to work during a critical portion of the experiment, and we were thus unable to determine t_E. In XXIV. *a.* Observer I. omitted to close the entrance tap to the evaporating-flask before connecting with the exhaust, thus the attempt to diminish the vacuum was a failure and the experiment was relinquished after a minute or so. In these two cases we at once performed other experiments to replace the failures, but retained the numbers for convenience of reference.

We have, however, rejected, when drawing our final conclusions, several of the experiments whose details are given. Such experiments are marked by a † in the Tables. We have, in no case, rejected any experiment *except as a consequence of some note made during that experiment*; that is, before reducing the observations we had already decided as to those which should be regarded as of little value. Thus we have in no way been guided by the results. For example, XXV. differs from the mean of its group by a greater quantity than either of the rejected experiments at that temperature: however, as we have no note against it, we are compelled to give it equal importance with any of the others. A simpler plan would have been to reject entirely the † experiments, but we think that a fairer idea of the general accuracy of the work is given by including all those that we completed.

The reasons for the rejection of the † experiments are given (as they appeared in our original notes) at the end of each group. We have, however, in each case given the mean of all in a footnote, and it will be seen that (except at 30°) our results would not have been appreciably affected by the inclusion of those rejected ones.

As a rule, we found that the thermal balance was most easily maintained when the potential-difference was that of 4 cells, but this supply of heat would have been somewhat too great at 20°, when we worked with 3 cells only. At all temperatures above 20° we performed experiments with 3, 4, and 5 cells, and we regard the agreement amongst the results as very satisfactory, and as establishing the validity of the various corrections. In thermal investigations it is as a rule difficult to alter all the conditions in so complete a manner as that caused by the changes above referred to. For example, if any "priming" had taken place (the usual cause of inaccuracy in determinations of the Latent Heat of Evaporation), its effect must have been greatly increased when the rate of evaporation was nearly trebled, which was the case when the potential-difference was altered from 3 to 5 cells. Again, the importance of the different terms undergoes such changes that any constant error in the determinations of $Q_s t_s$ and Σq would cause the values of L when $n=4$ to lie between the values when $n=3$ and $n=5$. An inspection of the Tables will show that no such effect is visible.

We also varied the "stirring supply" very considerably. The Tables show that (at the *same* temperature) the values of Q_s have been changed from ·00276 (Expt. XVII.) to ·00427 (Expt. XXIV.).

[*Note by* E. H. GRIFFITHS.—My experience with apparatus of a similar nature to that used in these experiments has convinced me of the severity of the test above referred to. During my attempts at a determination of the mechanical equivalent during the years 1887–1891 the results invariably broke down when thus tested. The following quotation, referring to the experiments anterior to 1892, is from p. 364, Paper J :—

"The agreement amongst individual experiments taken under the *same conditions*, was, if anything, at times more marked than in the experiments of 1892 ; nevertheless, when the final reduction took place, fatal discrepancies invariably showed themselves. . . . Experiments conducted with a high electromotive force invariably gave too great a value for the time as compared with that obtained when a lower electromotive force was used."

It appears to me that it is this power of altering all the conditions which renders electrical methods of such great value when applied to thermal determinations.]

18. *Explanation of the Tables.*

Tables lettered A give the experimental *data* ; the deductions from those data are given in Tables lettered B.

(We have arranged the experiments in order of temperature rather than historical order, as consecutive experiments were not always at the same temperatures.)

TABLES A.

Col. I. *a* and *b* give the number and date of the experiment.

" II. *a* and *b* give the number of Clark cells in series, and the number of the dropper used.

Col. III. gives the mass of benzene evaporated, after correction to *vacuo* (M).

" IV. The time during which the current was maintained (t_E).

" V. The time from commencement to end of experiment (t_s), *i. e.*, the time during which the "stirring supply" has to be estimated.

" VI. The number of revolutions per second (r_1) of the stirrer.

" VII. The difference between the initial (θ_0') and the final temperature (θ_0'') of the surrounding walls.

" VIII. Let θ_1' (initial calorimeter temperature) exceed θ_0' by d', and let θ_1'' (final calorimeter temperature) exceed θ_0'' by d''; then this column gives the value of $d'-d''$. Hence

$$\{(\theta_0'-\theta_0'')-(d'-d'')\}$$

gives the difference between the initial and final temperature of the calorimeter.

" IX. The capacity for heat of the calorimeter and its contents at the mean temperature of the tank (C_{θ_1}).

" X. The temperature of the Clark cells.

" XI. The value of R_{ne}, *i. e.*, the resistance of the calorimeter-coil at temperature θ_1, when the potential-difference of its end is *ne*. (From Table III. p. 34.)

19. Tables B.

(The numbers of the columns are similar to those in the corresponding tables of Paper W, where full particulars of the reductions are given.)

Col. XV. The value of $Q_E t_E = \frac{(ne)^2 \times t_E}{R_1 \times J}$, where e is the potential-difference of 1 Clark cell, at the temperature given in Col. X. (*supra*). The values of R_1 will be found in Table III. p. 34, and, as previously explained, the value of $J = 4{\cdot}199$ (see section 6, p. 24).

Col. XVI. The value of Q_s deduced from Col. VI. by means of Table IV. p. 37.

„ XVII. The value of $Q_s t_s$, the "stirring supply," from Cols. V. and XIV.

„ XVIII. The term $\Sigma q = C_{\theta_1}\{(\theta_0' - \theta_0'') - (d' - d'')\}$ from Cols. VII., VIII., and IX.

„ XIX. The sum of Cols. XV., XVII., and XVIII., that is, the total thermal grams (Σ) required for the evaporation of M grms. of benzene.

„ XX. The mean tank temperature (θ_0) of the experiment, expressed in the nitrogen scale.

„ XXI. The value of $L = \frac{\Sigma}{M}$.

Remarks on Experiments at 50°.

Exp. XIX. *a.* "Mistake in switching off current, did not do so till θ_1 had risen so far that d'' could not be obtained by means of galvanometer-swing. Had to readjust contact on bridge at close of experiment. Decided to regard experiment as a failure and repeat it."

The experiment is included in the table to show that even the large resulting value of the term $\Sigma q(-18{\cdot}64)$ has but a small effect on the value of L, which differs from the mean at this temperature by less than 1 in 400. This value for L(98·90) should, however, certainly not influence our final conclusions.

Exp. XXIII.—"Mistake in connecting with vacuum, reduced pressure far below right amount, hence could not obtain proper thermal balance throughout experiment."

Conclusion.

When $\theta_0 = 50{\cdot}014$, $L = 99{\cdot}14$.

TABLE V.A.

Experiments at 50° C.

I.		II.		III.	IV.	V.	VI.	VII.	VIII.	IX.	X.	XI.
a. Number.	*b.* Date. July	*a.* No. of Cells.	*b.* No. of Case.	M.	t_E.	t_S.	r_1.	$\theta_0'-\theta_0''$.	$d'-d''$.	C_{θ_1}.	Temp. of Cells.	R_{ne}.
XVII.	14	4	II.	5·0339	658·4	1319·2	5·107	0°	−·0010°	322	15·30°	10·402
XVIII.	14	4	III.	4·7071	622·4	1118·7	5·118	0	−·0154	,,	15·30	,,
†XIX. *a.*	14	4	IV.	4·9536	670·7	1204·3	5·126	−·0020	−·0560	,,	15·30	,,
XIX. *b.*	14	4	III.	4·7241	615·7	1093·2	5·241	+·0025	+·0046	,,	15·64	,,
XX.	16	4	V.	4·3654	569·1	997·0	5·171	+·0054	−·0030	,,	15·24	,,
XXI. *b.*	16	4	II.	4·9893	652·6	1314·9	5·219	+·0005	−·0030	,,	15·70	,,
XXII.	17	3	V.	4·3550	1010·1	1521·6	5·195	−·0024	−·0014	,,	15·58	10·399
†XXIII.	17	5	IV.	4·9726	416·2	1024·2	5·411	+·0024	−·0047	,,	15·77	10·406
XXIV. *b.*	18	5	IV.	5·0056	419·6	874·0	5·692	+·0017	−·0057	,,	15·50	10·406
XXV.	18	3	V.	4·3899	1011·4	1405·4	5·653	+·0054	−·0068	,,	15·50	10·399

TABLE V.B.

Reduction of Experiments at 50° C.

I. *a.*	XV.	XVI.	XVII.	XVIII.	XIX.	XX.	XXI.
Number.	$Q_E t_E$.	Q_S.	Q_{sts}.	Σq.	Σ.	θ_0.	L.
XVII.	496·00	·00276	3·64	− 0·32	499·32	$4\overset{\circ}{9}$·993	99·20
XVIII.	468·85	·00279	3·12	− 4·97	467·00	49·984	99·20
†XIX. *a.*	505·21	·00280	3·28	−18·64	489·95	49·983	98·90†
XIX. *b.*	463·55	·00307	3·35	+ 2·29	469·19	50·017	99·31
XX.	429·70	·00290	2·90	+ 0·78	433·37	50·026	99·27
XXI. *b.*	491·36	·00301	3·96	− 0·81	494·61	50·018	99·14
XXII.	427·93	·00296	4·50	− 1·22	431·21	50·027	99·02
†XXIII.	489·40	·00349	3·57	− 0·74	492·23	50·028	99·00†
XXIV. *b.*	493·52	·00427	3·74	− 1·29	495·97	50·021	99·08
XXV.	428·59	·00416	5·84	− 0·45	433·98	50·025	98·87
Mean (omitting those marked †)						50·014	99·14±·034
If we include all experiments at this temperature we get ...						50·013	99·10±·031

TABLE VI. A.

Experiments at 40° C.

I.		II.		III.	IV.	V.	VI.	VII.	VIII.	IX.	X.	XI.
a. Number.	*b.* Date. July.	*a.* No. of Cells.	*b.* No. of Case.	M.	t_E.	t_S.	r_1.	$\theta_0' - \theta_0''$.	$d' - d''$.	C_{θ_1}.	Temp. of Cells.	R_{ne}.
VIII.	8	4	III.	4·7228	628·2	1092·0	5·274	0°	−·0049°	318	16·27°	10·377
IX.	9	4	IV.	5·0424	676·8	1155·0	5·220	+·0010	−·0192	,,	15·90	,,
X.	9	4	II.	5·0212	665·4	1169·3	5·171	−·0120	+·0023	,,	15·70	,,
†XI.	9	4	III.	4·7228	635·5	1074·6	5·176	−·0015	−·0202	,,	15·71	,,
XII.	9	4	V.	4·3846	577·8	1105·3	5·118	−·0020	−·0016	,,	15·72	,,
XIII.	10	4	IV.	4·9994	670·2	1143·7	5·097	−·0145	−·0085	,,	15·45	,,
XIV.	10	4	II.	5·0258	662·0	1194·2	4·859	+·0030	+·0054	,,	15·45	,,
XV.	11	4	IV.	5·0051	668·6	1084·8	5·169	−·0051	−·0116	,,	15·40	,,
XVI.	11	4	II.	5·0286	678·3	1281·4	5·162	+·0051	−·0430	,,	15·50	,,
XXVI.	19	3	V.	4·4191	1035·2	1471·5	5·242	−·0030	+·0027	,,	15·32	10·374
XXVII.	19	5	III.	4·7469	406·0	779·8	5·237	+·0015	−·0118	,,	15·32	10·381
XXVIII.	19	4	II.	5·0292	664·6	1066·6	5·238	+·0070	−·0070	,,	15·34	10·377

TABLE VI. B.

Reduction of Experiments at 40° C.

I. *a*.	XV.	XVI.	XVII.	XVIII.	XIX.	XX.	XXI.
Number.	$Q_E t_E$.	Q_S.	$Q_S t_S$.	Σq.	Σ.	θ_0.	L.
VIII.	473·86	·00457	4·99	− 1·56	477·29	39°·965	101·04
IX.	510·68	·00439	5·07	− 5·79	509·96	40·090	101·12
X.	502·22	·00423	4·95	− 3·08	504·09	40·068	100·39
†XI.	479·49	·00424	4·56	− 7·12	476·93	40·078	100·99†
XII.	436·08	·00405	4·48	− 1·14	439·42	40·071	100·22
XIII.	505·96	·00399	4·16	− 7·31	502·81	40·055	100·57
XIV.	499·81	·00330	3·94	+ 2·67	506·42	40·065	100·76
XV.	504·79	·00422	4·58	− 5·31	504·06	40·058	100·70
XVI.	512·05	·00419	5·37	−12·05	505·37	40·063	100·50
XXVI.	439·76	·00446	6·56	− 0·10	446·22	40·021	100·97
XXVII.	478·73	·00445	3·47	− 3·28	478·92	40·025	100·90
XXVIII.	501·77	·00445	4·75	0	506·52	40·025	100·70
Mean (omitting that marked †)						40·045	100·71 ± ·057
If we include all experiments, we get						40·049	100·74 ± ·054

Table VII.A.

Experiments at 30° C.

I.		II.		III.	IV.	V.	VI.	VII.	VIII.	IX.	X.	XI.
a.	*b.*	*a.*	*b.*									
Number.	Date. July	No. of Cells.	No. of Case.	M.	t_E.	t_B.	r_1.	$\theta_0'-\theta_0''$.	$d'-d''$.	C_{θ_1}.	Temp. of Cells.	R_{ne}.
† Prelim.	4	3	I.	3·5975	878·8	1617·0	5·691	0	−·0640	312	15·63	10·350
I.	5	4	III.	4·9107	661·6	1341·4	5·515	−·0108	−·0178	"	15·44	10·353
† II.	5	4	IV.	5·0174	676·6	1210·9	5·177	−·0082	−·0212	"	15·59	"
III.	7	4	II.	5·1313	696·0	1102·3	5·473	0	−·0302	"	15·57	"
IV.	7	4	IV.	5·0071	674·7	1224·9	5·577	−·0088	−·0187	"	15·74	"
V.	7	4	V.	4·5336	607·9	1113·2	5·582	−·0096	−·0121	"	15·70	"
VI.	7	4	III.	4·8039	636·6	1144·5	5·120	+·0103	−·0063	"	15·77	"
VII.	7	4	II.	5·2472	692·4	1251·5	5·091	+·0061	+·0088	"	15·89	"
XXIX.	22	3	V.	4·4083	1045·0	1438·8	5·238	−·0088	−·0085	"	14·85	10·351
XXX.	22	5	IV.	4·9873	431·1	939·5	5·252	+·0031	−·0178	"	14·85	10·357
XXXI.	23	5	III.	4·7495	406·7	788·5	5·227	−·0077	+·0037	"	14·82	10·357
XXXII.	23	3	V.	4·4459	1050·7	1434·8	5·156	0	−·0010	"	14·90	10·351
XXXIII.	23	5	II.	5·0477	437·8	840·9	5·227	−·0108	−·0068	"	14·78	10·357
XXXIV.	23	3	IV.	5·0226	1185·7	1645·3	5·241	0	−·0022	"	14·85	10·351

TABLE VII. B.

Reduction of Experiments at 30° C.

I. *a*.	XV.	XVI.	XVII.	XVIII.	XIX.	XX.	XXI.
Number.	$Q_E t_E$.	Q_S.	$Q_S t_S$.	Σq.	Σ.	θ_0.	L.
† Preliminary.	373·98	·00883	14·3	−19·96	368·32	29°·820	102·36 †
I.	500·60	·00779	10·45	− 8·92	502·13	29·990	102·25
† II.	511·72	·00607	7·35	− 9·17	509·88	29·995	101·61 †
III.	526·68	·00756	8·33	− 9·42	525·59	29·962	102·44
IV.	510·41	·00815	9·98	− 8·58	511·81	30·000	102·24
V.	459·90	·00818	9·11	− 6·77	462·24	29·971	101·97
VI.	481·59	·00580	6·64	+ 1·25	489·48	29·969	101·90
VII.	523·62	·00567	7·10	+ 4·65	535·37	29·977	102·02
XXIX.	445·19	·00635	9·14	− 5·40	448·93	30·104	101·85
XXX.	509·70	·00641	6·02	− 4·59	511·13	30·117	102·46
XXXI.	480·95	·00629	4·96	− 1·25	484·66	30·034	102·05
XXXII.	447·50	·00596	8·55	− 0·31	455·74	30·047	102·50
XXXIII.	517·80	·00629	5·29	− 5·49	517·60	30·042	102·55
XXXIV.	504·26	·00636	10·46	− 0·69	514·03	30·052	102·34
Mean of Experiments XXIX. to XXXIV.						30·066	102·30 ± ·076
†Mean of all (omitting Preliminary and II.)						30·006	102·22

Remarks on Experiments at 40° C.

Exp. XI.—"Reduced pressure far too much at commencement, was unable to obtain thermal balance until near end of experiment."

Exp. VIII. & IX.—As pointed out (section 10, p. 33), the values of L are here probably in error by about 0·05. This, however, does not seem sufficient reason for rejecting them.

Conclusion.

When $\theta_0 = 40{\cdot}045$, $L = 100{\cdot}71$.

Remarks on Experiments at 30° C.

Preliminary Exp.—The oscillations in θ_1 were very violent, as this was our first attempt at balancing when evaporating benzene; also, as we had no idea when to finish, the value of Σq is very great. We have, however, inserted it to show how little such matters affect the result.

Exp. II.—"After inserting the dropper, had to withdraw it again owing to its sticking in the tube; dropper exposed for some time." There is little doubt that, owing to the cooling when withdrawn, some air was sucked back, and hence there must have been some expulsion on re-introduction. We regret that we continued the experiment: the result is useless.

20. It will be noticed that the irregularities in this Table are greater than in preceding ones. These are due to two causes:—(1) The want of practice in the observers; (2) The value of the galvanometer-swings were not determined with sufficient accuracy until after Experiment VII., and, as the values of Σq are large, any error might affect L, but probably not by more than $\pm 0{\cdot}1$.

In Section 10, p. 32, we have given in full our reasons for considering that all these experiments (I. to VII.) are too low by about 0·12. The mean value of L given by them (omitting Preliminary and II.) is 102·14: thus it is probable that, had the flask been previously saturated with vapour, we should have obtained about 102·26, which is in fair agreement with the mean of the last six (102·30), where this cause of error

TABLE VIII. A.

Experiments at 20° C.

I.		II.		III.	IV.	V.	VI.	VII.	VIII.	IX.	X.	XI.
a.	*b.*	*a.*	*b.*									
Number.	Date. July	No. of Cells.	No. of Case.	M.	t_E.	t_S.	r_1.	$\theta_0'-\theta_0''$.	$d'-d''$.	C_{θ_1}.	Temp. of Cells.	R_{ne}.
† XXXV.	24	3	III.	4·7593	1129·6	1575·8	5·562	$+$·0041	$-$·0194	305	15·30	10·327
XXXVI.	24	3	IV.	5·0083	1182·1	1833·2	5·586	$-$·0052	$-$·0143	,,	15·40	10·327

TABLE VIII. B.

Reduction of Experiments at 20° C.

I. *a.*	XV.	XVI.	XVII.	XVIII.	XIX.	XX.	XXI.
Number.	$Q_E t_E$.	Q_S.	$Q_S t_S$.	Σq.	Σ.	θ_0.	L.
† XXXV.	482·02	·01151	18·14	$-$4·67	495·49	19·947	104·10 †
XXXVI.	504·42	·01172	21·48	$-$5·95	519·95	19·947	103·82

was non-existent. In any case, taking *all* the values as they stand in Col. XXI. (rejecting the † experiments), we get 102·22. The number 102·30, however, is the most probable value.

Conclusion.

When $\theta_0 = 30·066$, $L = 102·30$.

Remarks on Experiments at 20° C.

Exp. XXXV.—This should be regarded as a preliminary experiment only. We had no idea of the pressure required to produce equilibrium, and had great difficulties in sufficiently reducing the pressure. "θ_1 was extremely irregular; the galvanometer-swing was often off the scale for a considerable period."

Exp. XXXVI.—"This was a very fair experiment; the thermal balance well under control."

We regret that we were unable to perform more experiments at 20° C. It was found necessary, however, to work the mercury-pump almost continuously, and this could not be done without further assistance[*]. As such assistance was but rarely available, and as our time was drawing to a close, we were compelled to content ourselves with the above experiments.

Conclusion.

When $\theta_0 = 19·947$, $L = 103·82$.

21. *Discussion of the Results.*

The values of L over the range 20° to 50° appear to be practically a linear function of the temperature.

If we assume

$$L = 107·05 - 0·1581\,\theta, \quad . \quad . \quad . \quad . \quad (\alpha)$$

we obtain a very close approximation to the experimental values, as shown by the following Table :—

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[*] We return our sincere thanks to Mr. C. T. Heycock, F.R.S., for his help during these experiments at 20° C.

Table IX.

Temperature. Nitrogen-scale.	L (expressed in terms of a thermal unit "at 15° C.").	
	By formula (α).	Experimental results.
50·014	99·14	99·14 ± ·034
40·045	100·72	100·71 ± ·057
30·066	102·30	102·30 ± ·076
19·947	103·90	103·82

It is evident, therefore, that the curvature over the above temperature-range must be very slight.

The value of L cannot continue to be a linear function of θ at very high temperatures, either in the case of benzene or water; for the critical point of benzene would fall at about 677° C., and that of water at about 990° C. It has, however, been shown that in the case of water the curvature from 0° to 100° (that is, over a pressure-range of about 4 mm. to 760 mm.) may be neglected (Paper W, pp. 316–321); and the above Table would indicate that we cannot be far wrong in making a similar assumption with regard to benzene, especially when we consider that both the temperature-range (0° to 80°·2 C.) and the pressure-range (26 mm. to 760 mm.) are in the latter case diminished.

We originally intended to perform a group of experiments at 60°, which is about the highest temperature to which it is advisable to expose the apparatus; but before doing so it would have been necessary to standardize a third mercury-thermometer—a work which would have occupied at least a week of the short time at our disposal.

Assuming formula (α) to hold to 760 mm., and taking the boiling-point at that pressure as 80°·20 C. (Ramsay and Young, Phil. Mag. 1887), we get

When $\theta = 80·2$, $L = 94·37$ (" thermal units at 15° C.").

The results of the experimental work described in this paper may be summarized as follows:—

The Latent Heat of Evaporation of Benzene over the temperature-range 20° to 50° C. (nitrogen-scale) is represented by the equation

$$L = 107{\cdot}05 - 0{\cdot}158\,\theta,$$

where L is expressed in terms of a "thermal unit at 15° C."

Assuming this expression to hold over the range 50° to 80°, the resulting value of L at 80°·2 C. (760 mm. pressure) is **94·37.**

22. *Historical.*

We have been unable to find records of any determinations in addition to the following:—

REGNAULT. *Mémoires de l'Académie*, 1862, vol. xxvi. p. 761: $L = 109{\cdot}0$.

R. SCHIFF. Liebig's *Annalen*, 1886, vol. ccxxxiv. p. 338: $L = 93{\cdot}4$ at 80°·35.

K. WIRTZ. Wiedemann's *Annalen*, 1890, vol. xl. p. 438: $L = 92{\cdot}9$ at 80°·1.

JAHN. *Zeitschrift für Physikalische Chemie*, 1893, vol. xi. p. 79: $L = 107{\cdot}6$.

IV. *A Method of Comparing directly the Heats of Evaporation of different Liquids at their Boiling-points. By* Miss Dorothy Marshall, *B.Sc., and* Prof. W. Ramsay, *Ph.D., F.R.S., University College, London*[*].

1. The heat of evaporation of a liquid is usually determined by measuring with the aid of a calorimeter the amount of heat liberated during the condensation of a known mass of the vapour; or conversely, by measuring the heat absorbed during the vaporization of a known mass of the liquid. The former method is the one more frequently adopted, *e. g.* by Berthelot and by Schiff.

Apart from the direct experimental difficulties of the method (and these are by no means inconsiderable), it is open to the objection that it measures, not simply the heat set free during the passage of a given mass of the substance from the gaseous to the liquid condition, but the sum of this quantity of heat with that liberated during the cooling of the liquid from the temperature of its boiling-point to the final temperature of the calorimeter. It is therefore necessary to make a separate determination of the thermal capacity of the liquid between these temperatures.

Whatever the cause may be, the fact remains that the data regarding heats of vaporization are very scanty. The subject has been worked at by few observers, and the results obtained show very considerable discrepancies. Taking, for instance, the case of ethyl alcohol, the heat of vaporization of a gramme is given as 208 calories by Despretz, 227 by Brix, 202·4 by Andrews, and 208·9 by Favre and Silbermann. It is clear that no method can be really satisfactory that leads to such discordant results.

2. Professor Ramsay suggested some time ago that it should be possible to compare directly the heats of evaporation of two liquids, by raising the temperature of each to its boiling-point (surrounding each vessel with a jacket of its own vapour), and then determining the loss of weight sustained by each vessel when a current of electricity was passed

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[*] Read November 8, 1895.

through a carbon filament immersed in the liquid. All other conditions being made the same for the two vessels, the ratio of their losses of weight should give the inverse ratio of the heats of evaporation of the liquids. This method of comparison we have since been engaged in working out; and having once overcome the initial difficulties of apparatus &c., we have been able to obtain some very satisfactory results, and intend to apply the method to a number of other liquids that we have not yet had time to investigate.

In such an arrangement as this there is no direct heat measurement, and therefore a fruitful source of error is eliminated. As the current passes, heat is developed in each carbon filament; and this heat is entirely expended in converting a portion of the liquid into vapour. The thermal capacity of the liquid no longer needs to be known, since the experiment is conducted entirely at the temperature of its boiling-point.

Each vessel is weighed at the temperature of the atmosphere at the beginning and end of the experiment; and the loss of weight of the vessel during the experiment gives the amount of liquid vaporized. If the same current be passed for the same time through two filaments of equal resistance, equal amounts of heat will be developed: hence the ratio of the loss of weight of the first vessel to the loss of weight of the second is the inverse ratio of the heats of vaporization of the liquids employed.

If the filaments have not equal resistances, the amounts of heat developed will be directly as the resistances. If the heat of evaporation of any one liquid be known, the absolute value of the heat of evaporation of any other liquid can be calculated from the ratio directly found.

3. [*Note, July* 1895, *by D. Marshall.*—At the time when this work was begun, the only liquid for which L was at all certainly known was water; but it was not considered advisable to compare directly with water in all the experiments: (i.) because the heat of evaporation of water is so very much greater than that of any other liquid; (ii.) because it is so exceedingly difficult to get it sufficiently pure; and (iii.)

because the continued use of water seemed to make the apparatus more liable to break.

Ethyl alcohol was in every way more suitable for a standard, and I therefore tried to make a particularly careful comparison between alcohol and water. But both in those first attempts, and again in several other attempts that I made later on, I had the greatest difficulty in getting even moderately concordant results; and I have never, when working with water, been able to approach the degree of accuracy that I know from my other results I am justified in expecting.

Thus I was finally obliged to conclude that water is not a liquid to use in these experiments at all, much less is it fitted to form our ultimate standard. It became necessary therefore to find some other liquid for which L was accurately known; and as we did not feel sufficient confidence in any of the published values with which we were acquainted, Mr. E. H. Griffiths, F.R.S., kindly consented to make a determination of the Heat of Evaporation of Benzene by the same method that he used with such conspicuous success in the case of water.

His experiments (in which I also took part) measured the heat of evaporation over the range 30° to 50° C.; and gave by extrapolation the value at 80°·2, the boiling-point. This number is therefore the one finally adopted by us in the calculation of our results; and we take this opportunity of recording our sincere thanks to Mr. Griffiths for furnishing us with such a trustworthy standard.

It should be observed that this number finally obtained for the heat of evaporation of benzene at its boiling-point does not in any way affect our work regarded as a method of comparison; but it does enter as the most important feature into the calculations by which we pass from our ratios to the absolute values of the heats of evaporation, and bring our results into line with those of other observers.]

4. *Description of the Apparatus.*

In the first experiments vessels were used closely resembling the ordinary incandescence-lamps, with carbon filaments; but a good deal of trouble was caused by the extreme difficulty of making satisfactory connexions. After the preliminary experiments, these lamps were therefore rejected in favour of others which had a spiral of fine platinum wire in place of the carbon filament: the ends of the spiral were attached to stout platinum terminals sealed into the glass; the terminals were gilt and amalgamated, and thus a safe and good contact was secured. The upper part of each lamp was drawn out into a rather narrow open tube, through which the liquid could be introduced, and which could be securely closed to prevent loss during weighing. The lamp was set up in an ordinary vapour-jacketing arrangement provided with side-bulb and condenser. The jacket was closed at the bottom by an indiarubber cork, through which passed two U-tubes containing mercury; the terminals of the lamp rested on the inner ends, into the outer ends dipped the wires carrying the current. The cork was protected by a layer of mercury so that it could not come into contact with the liquid. Each lamp was jacketed with the vapour of its own liquid, so that the temperature of its contents would be raised to the boiling-point without the possibility of ebullition taking place until the current was started; the liquid then lost by boiling from the lamp was condensed in the jacket, and there was no loss of material.

The two lamps were connected in series.

In the earlier experiments the current used was that supplied at 110 volts to run two incandescence-lamps of 32-candle power (in parallel): it was further cut down to a convenient strength by the introduction of an appropriate resistance. At first this resistance consisted of a solution of copper sulphate —therefore both indefinite and variable: in the later experiments a coil of wire of about 20-ohms resistance was substituted for the copper-sulphate solution.

In performing an experiment, the liquid in the side-bulb of the jacket was first caused to boil; and the current was not started until the whole contents of the lamp were judged to have reached the temperature of the condensing vapour: if this were so, the liquid would pass into tranquil ebullition the moment the circuit was completed. It was generally found advisable to drop into the lamp a little glass capillary tube to provide a starting-point for boiling, as most of the liquids showed a great tendency to become superheated and bump. A number of experiments were rejected for this reason.

5. *Preliminary Experiments.* February 1894.

In the experiments with carbon lamps, the resistance of each filament was determined by the Wheatstone-bridge method, using a post-office box of coils for bridge, at five different temperatures; and the results were plotted on squared paper. The observations for each lamp lie approximately on a straight line, the resistance falling off uniformly as the temperature rises.

The resistance of each filament at 0° and 180°, the decrease per degree, and the coefficient of change of resistance between 0° and 180° were found from the curves.

TABLE I.

	I.	II.	III.
Resistance at 0°	42·5 ohms.	43·17	42·60
Resistance at 180°	36·69 „	36·58	36·10
Decrease per degree	0·0323	0·0366	0·0361
Coefficient of change of resistance	0·00076	0·00085	0·00085

These experiments gave no satisfactory results.

6. *First Series of Experiments.* March 1894.

Two lamps were now made with spirals of fine platinum wire in place of carbon filaments. These platinums were each 18 inches long, and coiled into spirals with all possible care to prevent neighbouring turns from touching and short-circuiting. The resistances of the platinums were determined by the Wheatstone-bridge method. The coefficient of increase of resistance of platinum with rise of temperature being (approximately) known, the resistance of each spiral for any given temperature could be calculated. It was assumed that the temperature of the spiral was the same as that of the liquid boiling round it.

A comparison of chlorobenzene and alcohol may serve for an example :—

Date.	B.	Weight evaporated.		R_1.	R_2.	Corrected Weights.		Ratio $\frac{L_1}{L_2}$.	L_1.
		C_6H_5Cl.	C_2H_5OH.			C_6H_5Cl.	C_2H_5OH.		
March 15.	746·3	14 5830	4·7975	4·89	4·27	14·5830	5·5252	0·364	76 1
April 20.	763·0	16·6645	5·4195	4·87	4·31	16·6645	6·1236	0·367	

TABLE II.

Substance.	Date.	B.	Ratio.	L.
Alcohol referred to water*............	May 1 „ 4	767 755	0·387 0·388	208·1
CS_2	Mar. 11	741·9	0·405	85·0
Acetic Acid	May 10	754·4	0·475	98·6
„ „	„ 15	756·8	0·472	
„ „	„ 16	757·7	0·472	
Ether......................	„ 17	752·7	0·396	82·5
„	„ 30	752·3	0·397	
„	June 4	753·9	0·396	
Ethyl Acetate	„ 5	754·4	0·409	85·1
„ „	„ 6	753·8	0·409	
Methyl Formate	„ 7	755·2	0·514	106·7
„ „	„ 11	754·1	0·511	
Methyl Butyrate	„ 12	758·3	0·389	81·0

* The other substances are referred to Alcohol.

7. *Second Series of Experiments.* June 1894.

Up to this time the resistance of the platinum spiral during the passage of the current had never been directly measured, but was calculated from the cold resistance and the coefficient of increase with rise of temperature, on the assumption that the temperature of the spiral was the same as that of the surrounding liquid. The truth of this assumption was now called in question, and it was found to be unjustifiable. The resistance of the spiral was directly measured, first when surrounded by a liquid at its boiling-point, then when the current was passed so that the liquid boiled.

The resistance was always found to be greater when the current was passing; that is to say, the temperature of the spiral is always higher than that of the liquid boiling round it*. One does not exactly know what conditions govern the difference of temperature between spiral and liquid. The probability is that the ratio of the resistances when two liquids are compared will be altered, as well as the absolute values of the resistances themselves; and this would tend to alter all the results hitherto obtained.

It should also be observed that the coefficient of increase of resistance of platinum with rise of temperature is that given by Callendar (1·338; Phil. Trans. clxxxii. A, p. 136) for a particular sample of wire employed by him; but as this coefficient is found to be different for different samples of wire, the value adopted may very probably not hold for the particular wire here used.

It was therefore necessary to determine directly the resistance of the platinum spirals in each experiment. The resistance of the wire during the passage of the current was now determined by Poggendorff's method:—viz., comparison with the resistance of a standard wire carrying the same current. [For diagram of arrangements *vide infra*, p. 65.]

The standard resistance was a piece of uninsulated german-silver wire, immersed in cold water to keep its temperature constant.

Table III. shows the results obtained.

TABLE III.

Substance.	Date.	Ratio $\frac{L_1}{L_2}$.	L_1.
Water	July 2.	0·391	209·74
,, ...	,, 6.	0·390	
,, ...	,, 11.	0·391	
Chlorobenzene	,, 18.	0·355	75·2
,, ...	,, 19.	0·363	
,, ...	,, 19.	0·358	
Ether	,, 12.	0·398	83·5
,, ...	,, 13.	0·398	
Ethyl Acetate	,, 20.	0·392	82·2
,, ,, ...	,, 23.	0·392	
Acetic Acid	,, 24.	0·471	100·2
,, ,, ...	,, 25.	0·484	

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* Griffiths, Phil. Trans. clxxxiv. A (1893), pp. 400–407.

8. *Third Series of Experiments.* February 1895.

In the early part of this year the apparatus was further modified with a view to increasing the accuracy of the resistance measurements.

The readings of resistance had been found very troublesome and very uncertain. In many experiments it was almost impossible to obtain anything like a point of balance on the bridge-wire: it was useless to attempt to read to within 1 centim. length of the wire; and during any one experiment the points of balance did not remain steady at all, but seemed to shift indefinitely. This might be the consequence either of variations in the strength of the current in the main circuit, or of change in the resistance of the bridge-wire from unequal heating. [The bridge consisted of a coil of uncovered platinoid wire immersed in cold water and connected in series with a platinum wire stretched against a scale.] Probably both causes were working together.

Two important changes were made:—

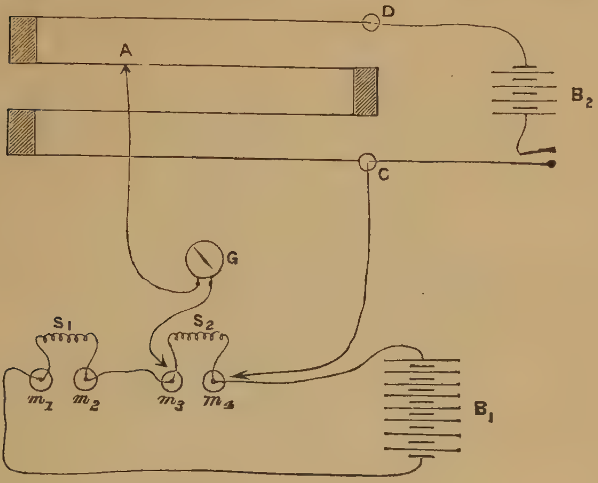
(i.) The current in the primary or lamp-circuit was taken from 8 secondary cells, instead of being taken from the mains through a (probably) variable resistance.

(ii.) A new bridge was made, consisting of four parallel german-silver wires of approximately equal resistance stretched against four metre-scales.

The whole of the resistance in the secondary circuit now falls into the bridge itself; whatever heating-effect there may be is distributed all along the bridge, and its effect is much lessened if not altogether removed. The resistance of the bridge-wire does not need to be known, provided it is tolerably uniform; because we are only concerned with comparative measurements, which are simply proportional to the lengths measured.

For the same reason the standard wire of known resistance is really unnecessary, and was omitted in all the remaining experiments.

It became evident at once—working in these improved conditions—that the whole method had been made very much more sensitive and accurate than before; hence it was of



S_1 S_2 are the platinum spirals of the lamps.
$m_1 \ldots\ldots m_4$ are mercury-cups by which connexion is made with S_1 and S_2.
B_1, B_2 are the batteries supplying the current in the primary and secondary circuits respectively.
C, D are the terminals of the fourfold potentiometer wire.
A is a sliding contact, and has to be moved until the fall of potential on the bridge between C and A is equal to the fall of potential between m_4 and m_3, or m_2 and m_1.
G is a simple reflecting-galvanometer.

the utmost importance to repeat the fundamental comparison between alcohol and water from which all the other results were calculated. These experiments are entered in Table IV.

It was at this point that we gave up the idea of taking water for our standard, and looked round for some other liquid to use in its place. It is quite clear, from the numbers obtained, that there is something in the water itself that interferes with the experiments. Probably there is some electrolysis during the passage of the current along the uninsulated wire; it appears impossible by any ordinary means to get water so pure that its conductivity may be disregarded.

In the later tables the numbers given for the heats of

TABLE IV.

No.	Date.	B.	Weights evaporated.		V_1.	V_2.	Ratio $\frac{V_1}{V_2}$.	Corrected weights.		Ratio $\frac{L_1}{L_2}$.
			Alcohol.	Water.						
1	March 1.	754·1	15·2470	5·4645	274·7	260	1·057	15·2470	5·7740	0·379
2	„ 2.	754·7	14·7365	5·2370	326·2	308·8	1·060	14·7365	5·5510	0·377
3	„ 5.	757·8	14·1615	5·1310	227·2	210·3	1·057	14·1615	5·4215	0·383
4	„ 6.	756·0	13·5130	4·7255	208·0 248·5	195·7 233·1	1·063 1·065	13·5130	5·028	0·372
5	„ 7.	755·6	15·3565	5·5735	207·4 247·4	194·9 233·1	1·064 1·061	15·3565	5·9255	0·385
6	April 9.	759·5	14·0570	5·6460	327·2 275·1	307·8 258·4	1·063 1·064	14·0570	6·0015	0·427
7	„ 9.	759·5	13·7710	5·5770	327·9 275·7	307·4 258·5	1·066 1·066	13·7710	5·9450	0·432
8	„ 10.	761·3	13·7425	5·5420	328·3 276·2	307·6 258·7	1·067 1·067	13·7425	5·9130	0·430

Note.—Experiments 1 to 5 were made with the ordinary distilled water of the laboratory; experiments 6 to 8 were made with some water that had been specially purified for use in water-analysis.

evaporation are calculated from the absolute value for benzene, viz.:—L = 94·4 at 80°·2.

Table V. records the results obtained by us for benzene, toluene, metaxylene, acetic acid, and ten of the lower esters prepared by Prof. Sydney Young. Each liquid is compared with alcohol.

In Table VI. we give for each liquid:—

(A) the ratio compared with benzene;
(B) the value of L in thermal units at 15°;
(C) the value of L obtained from the thermodynamic equation (see p. 71, *infra*);
(D) values obtained by other observers.

In Table VII. we give for each,

L, the heat of evaporation;
t, the temperature of the boiling-point;
M, the molecular weight;
$\frac{ML}{T}$, the quotient of the molecular heat of vaporization by the absolute temperature; according to Trouton's Law this should be a constant.

9. TABLE V.

No.	Date.	B.	Weights evaporated.		Ratio $\frac{V_1}{V_2}$.	Corrected weights.		Ratio $\frac{L_1}{L_2}$.	Mean.
				Alcohol					
Benzene.									
1 ...	May 21.	752·8	16·976	8·193	0·907	16·976	7·431	0·438	
2 ...	„ 22.	755·7	18·662	8·950	0·906	18·662	8·1085	0·434	
3 ...	„ 23.	757·9	19·010	9·150	0·901	19·010	8·244	0·434	
4 ...	„ 24.	756·7	17·098	8·286	0·905	17·098	7·499	0·438	0·436
Toluene.									
5 ...	May 28.	...	17·359	7·136	0·972	17·359	6·936	0·400	
6 ...	„ 29.	763·2	17·046	7·040	0·972	17·046	6·843	0·401	0·401
Metaxylene.									
7 ...	May 30.	755·7	16·309	6·063	1·031	16·309	6·251	0·386	
8 ...	June 6.	767·6	16·755	6·150	1·036	16·755	6·3715	0·380	0·383
Acetic Acid.									
9 ..	June 11.	758·6	19·517	8·956	0·980	19·517	8·777	0·450	
10 ...	„ 12.	762·2	17·033	7·754	0·984	17·033	7·630	0·448	
11 ...	„ 19.	752·4	19·330	8·613	0·984	19·330	8·475	0·447	0·448

Table V. (*continued*).

No.	Date.	B.	Weights evaporated.		Ratio $\frac{V_1}{V_2}$.	Corrected weights.		Ratio $\frac{L_1}{L_2}$.	Mean.
Methyl Formate.									
12 ...	March 25.	740·8	15·896	10·1455	0·800	15·896	8·1165	0·511	
13 ...	„ 26.	742·3	14·5725	9·289	0·795	14·5725	7·385	0·507	
14 ...	„ 27.	736·8	10·1955	6·5265	0·794	10·1955	5·181	0·508	0·509
Ethyl Formate									
15 ...	April 3.	758·8	17·208	8·883	0·843	17·208	7·4885	0·435	
16 ...	„ 3.	758·8	17·8375	9·2205	0·846	17·8375	7·8105	0·437	
17 ...	„ 5.	754·3	12·9885	6·712	0·844	12·9885	5·665	0·437	0·436
Methyl Acetate.									
18 ...	March 27.	735·7	17·158	9·0365	0·851	17·158	7·690	0·448	
19 ...	„ 28.	731·6	18·1105	9·547	0·853	18·1105	8·1435	0·449	
20 ...	„ 28.	731·6	16·220	8·5165	0·853	16·220	7·2645	0·448	0·448
Propyl Formate.									
21 ...	Dec. 20.	...	16·896	7·7695	0·916	16·896	7·117	0·421	
22 ...	Feb. 6.	754·5	17·6585	8·108	0·908	17·6585	7·362	0·417	
23 ...	„ 20.	768·1	17 675	8·2215	0·910	17·675	7·4815	0·423	0·420
Ethyl Acetate (referred to Benzene).									
24 ...	Oct. 24.	748·5	16·137	15·451	1·121	16·137	17·321	0·932	
25 ...	„ 25.	748·2	15·001	14·275	1·129	15·001	16·110	0·931	
26 ...	„ 26.	748·9	15·279	14·449	1·131	15·279	16·342	0·935	0·933
Methyl Propionate.									
27 ...	March 19.	760·5	17·6085	8·012	0·903	17·6085	7·235	0·411	
28 ...	„ 22.	758·8	16·204	7·369	0·904	16·204	6·6615	0·411	
29 ...	„ 23.	758·0	16·158	7·3375	0·903	16·158	6·626	0·410	0·411
Propyl Acetate.									
30 ...	March 18.	764·9	17·8735	7·206	0·951	17·8735	6·855	0·384	
31 ...	„ 28.	731·6	17·508	7·0885	0·950	17·508	6·734	0·385	
32 ...	„ 29.	736·8	14·545	5·9035	0·948	14·545	5·5965	0·385	0·385
Ethyl Propionate.									
33 ...	April 4.	762·9	18·5595	7·3745	0 942	18·5595	6·947	0·374	
34 ...	„ 4.	762·9	14·111	5·676	0·943	14·111	5·3625	0·380	
35 ...	„ 5.	754·3	16·2815	6·5735	0·944	16·2815	6·2055	0·381	0·378
Methyl Butyrate.									
36 ...	March 29.	736·8	14·836	5·738	0·952	14 836	5·4625	0·368	
37 ...	April 2.	755·7	15·207	5·8455	0·950	15·207	5·553	0·365	
38 ...	„ 2.	755·7	15 064	5·853	0·951	15·064	5·466	0·372	0·368
Methyl Isobutyrate.									
39 ...	April 8.	759·9	17·0765	6·4145	0·926	17·0765	5·940	0·348	
40 ...	„ 8.	759·9	15·7095	5·814	0·926	15·7095	5·384	0·343	0·346

TABLE VI.

	A.	B.	C.	D.
Benzene	1·000	94·4	......	109·0 Regnault II. 93·4 Schiff X. 92·9 Wirtz XI. 107·6 Jahn XII.
Toluene	0·920	86·8	......	83·6 Schiff X.
Metaxylene	0·877	82·8	......	78·3 Schiff X.
Alcohol	2·293	216·5	......	202·4 Andrews I. 227 Brix VI. 208 Despretz VII. 208·9 Favre & Silbermann VIII. 206·4 Schall IX. 205·0 Wirtz XI. 201·4 Longuinine XIII.
Acetic Acid	1·028	97·0	92·7	120·8 Berthelot IV. 84·9 Berthelot & Ogier V. 101·9 Favre & Silbermann VIII.
Methyl Formate	1·167	110·1	119·1	117·1 Andrews I. 115·2 Berthelot III.
Ethyl Formate	1·000	94·4	93·4	105·3 Andrews I. 100·4 Berthelot III. 92·2 Schiff X. 113·25 Jahn XII.
Methyl Acetate	1·028	97·0	96·8	110·2 Andrews I. 94·0 Schiff X. 113·86 Jahn XII.
Propyl Formate	0·956	90·2	87·3	85·3 Schiff X.
Ethyl Acetate	0·933	88·1	84·3	92·7 Andrews I. 83·1 Schiff X. 84·3 Wirtz XI. 102·14 Jahn XII.
Methyl Propionate	0·943	89·0	87·7	84·2 Schiff X.
Propyl Acetate	0·881	83·2	79·8	77·3 „
Ethyl Propionate	0·867	81·8	79·2	77·1 „
Methyl Butyrate	0·844	79·7	72·5	77·3 „
Methyl Isobutyrate	0·794	75·0	76·3	75·5 „

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TABLE VII.

	Ratio to benzene.	L.	*t*.	M.*	$\frac{M.L}{T}$.
Benzene	1·000	94·4	80°·2	77·40	20·65
Toluene	0·920	86·8	110·8	91·30	20·61
Metaxylene	0·877	82·8	138·5	105·20	21·03
Water	......	536·6	100°	17·86	25·64
Alcohol	2·293	216 5	78·2	45·66	28·09
Acetic Acid	1·028	97·0	118·5	59·52	14·72
Methyl Formate	1·167	110·1	31·8	59·52	21·45
Ethyl Formate	1·000	94·4	54·3	73·42	21·13
Methyl Acetate	1·028	97·0	57·1	73 42	21·53
Propyl Formate	0·956	90·2	80·9	87·32	22·38
Ethyl Acetate	0·933	88·1	77·15	87·32	21 93
Methyl Propionate ...	0·943	89·0	79·7	87·32	21·99
Propyl Acetate	0·881	83·2	101·25	101·22	22·45
Ethyl Propionate	0 867	81·8	99·2	101·22	22·22
Methyl Butyrate	0· 44	79·7	102·7	101·22	21·43
Methyl Isobutyrate ...	0·794	75 0	92·3	101·22	20·74

* The molecular weights are calculated by taking

$$\left\{\begin{array}{l} H=1. \\ O=15{\cdot}86 \text{ (Rayleigh \& Scott).} \\ C=11{\cdot}90 \text{ (Stas :—}C=12{\cdot}005 \text{ when } O=16). \end{array}\right.$$

The absolute zero is taken as −273°·7.

10. The results of these experiments may, we think, be regarded as satisfactory so far as they go.

A good many difficulties have been encountered, but most of them have been overcome; and with a few more improvements in matters of detail we believe that the method will become almost as simple in practice as in theory, and will give results as good as can be required for ordinary purposes.

We do not profess or desire to aim at such accuracy as Mr. Griffiths has reached in his work on water; our method does not and cannot lend itself to such refinements.

We rather hope to obtain for a large number of liquids values that are certainly within one per cent. of the truth, and often much nearer.

Up to the present our experiments have mostly been limited to Dr. Young's esters, which were ready to hand; but we propose to extend them now, as far as possible, to any other liquids that we can obtain pure, and whose boiling-points fall between 30° and 150°.

We desire, in conclusion, to express our thanks to Prof. Carey Foster for the use of his laboratory in the later part of the work, and to Mr. A. W. Porter for his kind assistance and advice in the matter of the resistance measurements.

Appendix.

I. Calculation of L from the thermodynamic equation; worked out for the case of *Methyl Acetate* from experimental data given by Prof. Young (J. C. S. lxiii. p. 1191):—

$$L=(S'-S)\,.\,\frac{dp}{dt}\,.\,\frac{T}{J}.$$

(i.) To find $\frac{dp}{dt}$, by means of Biot's formula

$$\log p=a+b\alpha^t.$$

t.	P.
50°	588·2 millim.
60°	837·5 ,,
70°	1167·0 ,,

hence $\log p=5{\cdot}2852200-2{\cdot}515695\times({\cdot}9937262)^{t-50}$

at 57°·0 . . . $p=755{\cdot}0043$ millim.

at 57°·2 . . . $p=760{\cdot}2895$,,

$$\therefore\ \frac{dp}{dt}=26{\cdot}4260 \text{ millim. per } 1^\circ \text{ at } 57^\circ{\cdot}1.$$

$$\frac{dp}{dt}=2{\cdot}64260\times 13{\cdot}596\times 980{\cdot}94 \text{ dynes per sq. cm. per } 1^\circ.$$

(ii.) $S'=$vol. of 1 gr. vapour at $57^\circ{\cdot}1=350{\cdot}6$ c.c.

$S=$vol. of 1 gr. liquid at $57^\circ{\cdot}1=1{\cdot}1$ c.c.

$\therefore\ (S'-S)=349{\cdot}5$ c.c.

$T=273{\cdot}7+57{\cdot}1=330{\cdot}8.$

*J $=4{\cdot}199\times 10^7$.

Hence **L = 96·81 thermal units.**

---

* The value of J given here is that determined by Griffiths (Phil. Trans. vol. clxxxiv. 1893 A), and used in working out the experiments on benzene; even if not correct, it is still the right value to use here, because it was determined by means of the same standards as those by which the quantity of heat developed in the benzene experiments was determined; so that any errors would eliminate.

This assumption as to the value of J is the only one that enters into the calculation; all the other data were obtained by direct experiment.

II. *Regarding the Purity of the Liquids employed.*—The benzene was bought from Messrs. Kahlbaum, of Berlin, and guaranteed free from thiophene. It was distilled twice from phosphoric anhydride until its boiling-point remained constant.

The toluene had been used in some of Prof. Ramsay's previous work; it was distilled once and showed a constant boiling-point.

The metaxylene was fractionated three times, and the portion distilling between 138·1 and 138·7 was used; it was probably not quite pure.

The alcohol was obtained from Messrs. Baird and Tatlock, and distilled from quicklime, and finally with a little sodium, until its boiling-point remained constant.

The acetic acid was bought from Messrs. Kahlbaum; it was fractionated by freezing, and the portion melting at 16°·2 was separated and used.

The esters were prepared by Prof. S. Young, F.R.S., and all details may be found in his paper (J. C. S. lxiii. p. 1191).

DISCUSSION.

Prof. RAMSAY drew special attention to the table giving the values of the quotient $\frac{ML}{T}$, where M is the molecular weight, T the absolute temperature, and L the latent heat. Very curious differences are noticeable in the case of water, alcohol, and acetic acid.

Prof. CAREY FOSTER expressed his admiration for the method, since it obviated the necessity for knowing the specific heat of the liquid or vapour.

Prof. SILVANUS THOMPSON said the difficulty experienced in the case of water, due to electrolysis, might be obviated by the employment of a spiral of lower resistance and a larger current, so that the difference of potential between the ends of the spiral should be less than 1·7 volts.

The CHAIRMAN said Capt. Abney had asked him to enquire to what extent the temperature of the liquid was affected by radiation.

Mr. J. W. RODGER asked if any direct experiment had been made to determine if the temperature of the liquid was not above its true boiling-point. In some cases differences of as much as 2° might exist between the temperature of the liquid and that of the vapour given off. The differences in the value of $\frac{ML}{T}$ in the case of water, alcohol, and acetic acid might be due to the fact that the vapours of alcohol and water were simple, while the vapour of acetic acid was complex.

Mr. R. Appleyard suggested that the differences obtained in the case of water might be due to the presence of dissolved air.

Mr. Griffiths said that the objection to the adoption of Prof. Thompson's suggestion was the fear that, with short wires, an excessive difference in temperature between the wire and the liquid might exist.

Mr. Rhodes asked if Mr. Griffiths could trust his determinations of temperature to $\frac{1}{8000}$th of a degree?

Mr. Griffiths, in reply, said that he thought there was no limit to the accuracy with which a difference of temperature could be measured; the absolute temperature, however, he only relied upon to $\frac{1}{1000}$th of a degree.

Prof. Ramsay said the fact of superheating existing would not affect the results, since near the temperatures at which they were working the latent heat did not vary appreciably with the temperature. In reply to Captain Abney, he said some previous experiments by Dr. Young and himself had shown that the vapour-jacket was quite impervious to radiant heat from without.

V. *A "Direct-reading" Platinum Thermometer.*

By ROLLO APPLEYARD*.

MESSRS. CALLENDAR and GRIFFITHS and their colleagues at Cambridge have proved that the platinum thermometer is capable of measuring temperature with an accuracy of *one thousandth* of a Centigrade degree. For general purposes, temperature is never required to be measured with a precision greater than *one tenth* of a degree. It is my object to consider how the reserve of precision may be utilized in facilitating the processes of testing. The results seem to indicate that it is possible, with a platinum thermometer, to determine ordinary temperatures by a single operation, of such a kind that the readings are "direct"—that is to say, no calculation requires to be applied to them.

In the course of some experiments upon the variation of the electrical resistance of dielectrics with temperature, an apparatus was needed which, by a quick and simple operation, would measure the temperature within the substance of the dielectric. As dielectrics, generally, are bad conductors of heat, it is desirable to take simultaneous readings of temperature at many points within their mass. A platinum thermometer may be regarded as having an *extensible bulb*, for the wire may be distributed to various points within the dielectric; the temperature deduced from the platinum thermometer, at any instant, is therefore equivalent to the mean of the simultaneous readings of a large number of distributed mercurial thermometers. Further, the dielectrics, in their protecting covers, are submerged in a tank of water; the stems of ordinary mercurial thermometers placed, vertically, beneath a water-surface would be difficult to observe; the platinum instrument is far more convenient for this work, for it has the advantage of a *portable stem and scale.*

The resistance of a dielectric[†] is by no means such a definite quantity as the resistance of a metallic conductor. The difference in quality between samples of the same name, the age of the substance, the nature of the contact between the dielectric and its electrodes, and the so-called "electrification," are such variable factors that the term "dielectric resistance" has only an empirical significance. To arrange for the measurement of temperature with extreme precision is therefore unnecessary.

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* Read November 22, 1895.

[†] See "Dielectrics," Proc. Physical Soc. xiii. p. 155, 1895; Phil. Mag. 1894, xxxviii. p. 396.

Range.

For certain practical reasons, the temperatures mentioned in this paper are referred to the Fahrenheit scale. The required range was 32°–120° F.; and, within the present limits of accuracy, it is safe to assume that the resistance, R_t, of the platinum coils, at any temperature t, within this range, is represented by the expression

$$R_t = R_{32}[1+\alpha(t-32)]. \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The Platinum Coils.

There were six platinum coils, forming six thermometers, each having a resistance of about 7 ohms. Each consisted of about 2 metres of pure platinum wire, of 8 mils diameter, wound in double spirals upon an ebonite rod. Thick copper leads were soldered to the ends of the coils. *To determine* α *and* R_{32} the six coils were separately tested in melting ice, and then at various temperatures in heated water, side by side with a mercurial thermometer. These measurements were made with the coils submerged in a large tank, the temperatures being controlled by a gas-regulator. The resistances were measured on an ordinary "dial" bridge having german-silver coils. This part of the work occupied much time: α was derived from the mean of the best of 30 determinations; it was somewhat tedious, but the values R_{32} and α proved, in the result, very accurate. The six coils of the six thermometers were afterwards connected in series, and regarded as one thermometer. The mean values were:—

$R_{32} = 40{\cdot}05$ ohms (six coils, in series).
$\alpha = {\cdot}002097$.

The six platinum coils, still connected in series, were then imbedded within their respective dielectrics. *Hollow metal tubes*, for mercurial thermometers, were inserted at the same time within the dielectrics, for check-readings. The dielectrics were then immersed together into one tank of water.

Measurement of R_t *by "Dial" bridge.*

The apparatus was now complete, and tentative tests were made for R_t at different temperatures. The results, when compared with the mean of the readings of the mercurial thermometers in the tubes, were rather discouraging. The temperatures could not be depended upon within about 2° Fahr. These errors were due either to the german-silver coils, or the plugs, of the dial-bridge. Slightly better results were obtained by reducing the battery to a single cell. It was impossible to determine the bridge-temperature, at any time, with accuracy; and, moreover, even if that temperature could have been found *, the temperature-coefficient for different samples of german-silver varies so widely with different samples, that no dependence whatever could have been placed upon values "corrected" by text-book coefficients. The "dial" bridge was therefore abandoned, and a slide-bridge of special form was designed for the purpose.

Measurement of R_t *by Slide-bridge.*

The slide-bridge, fig. 1, consists of a 12-ft. plank, P, with a groove G cut along its whole length. A metre-scale, S, can slide in the groove, flush with the upper surface of the plank. Datum-marks are made at successive metres along the plank; subdivisions being read upon the sliding metre-scale. The contact-piece, L, is attached to the metre-scale at its zero-point. This saves the cost of an elaborate graduation of the plank; and keeps the sliding-contact very accurately in position. When necessary, the metre-scale with its contact-piece is easily transferred to other such slide-bridges. There is no distortion of the slide-wire, and indentation is

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* A platinum coil interwound with one of the resistance-bobbins would probably be the best method under these circumstances.

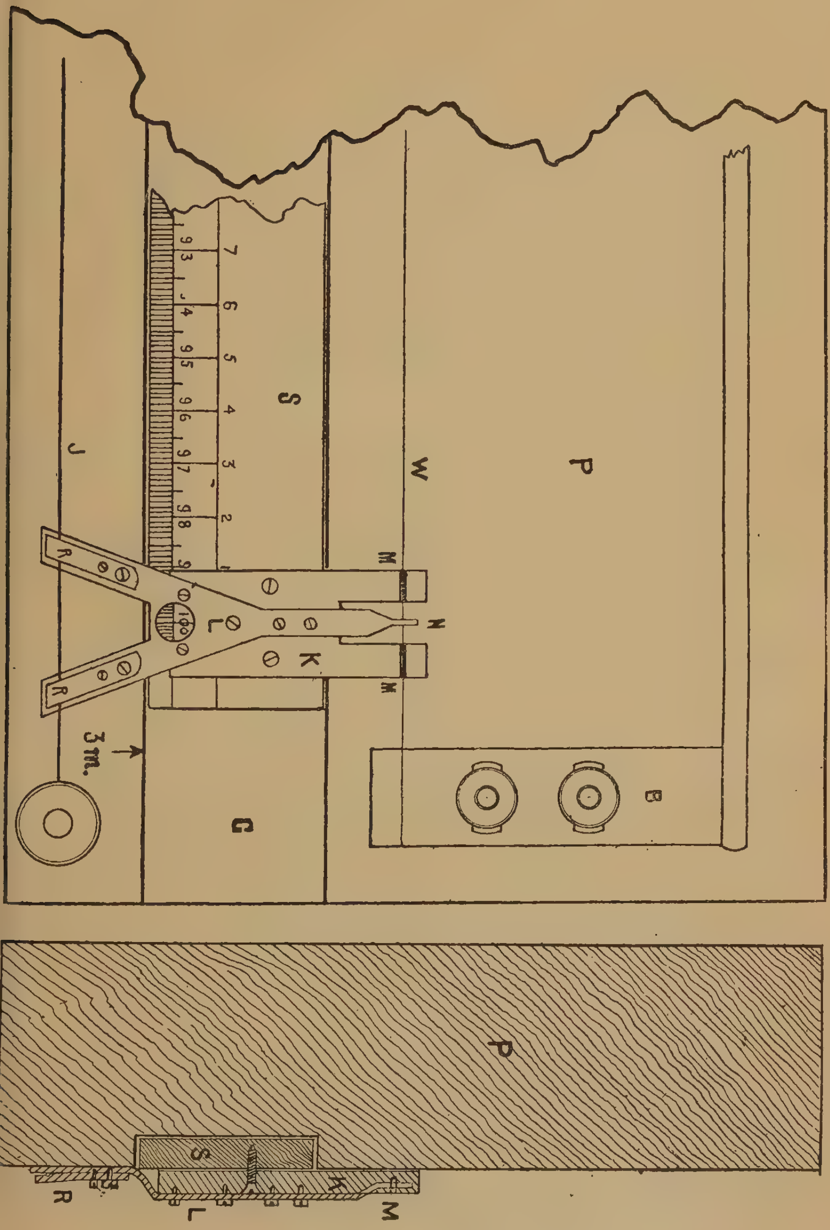

Fig. 1.

almost impossible*. The slide-wire, W, is of platinoid † stretched between two adjustable blocks of copper, B. Its ends are soldered into saw-cuts made in the copper blocks. On the other side of the scale-groove there is a copper wire, J, for the galvanometer-contact. The contact-piece consists of an ebonite block K, with a gap at one end. The platinoid wire slides through a saw-cut M transversely to this gap in the ebonite. A Y-shaped piece of spring-brass, L, is screwed to the ebonite block. One end of the Y, projecting over the gap, is provided at N with a platinum knife-edge. The other two arms of the Y project beyond the other end of the ebonite block, and carry clips, R, through which the wire for the galvanometer-contact slides.

The bridge-coils, fig. 2, consist of five silk-covered platinoid resistance-bobbins, immersed in a double vessel of copper, the inner vessel containing $1\frac{1}{2}$ gallons of paraffin-oil.

A glow-lamp, submerged in the oil, is used for maintaining the temperature constant at 80° F.; this temperature was chosen as being just above ordinary air-temperature.

Copper rods (a, b, c, e, f, g, k) connect the coils; their ends pass upward, through the wooden cover of the calorimeter, to the switch.

Fig. 2 shows the arrangement of these coils. The two ends of the slide-wire go to f and g respectively; to which points the 100-ohm coils (f, e) and (g, a) are connected. By switching p to l, the fixed arm (45 ohms) of the bridge, connected between a and b, is balanced against the 45 ohms between b and e; the slider has then a position at the middle of the slide-wire. When p is switched to n, the platinum thermometer is substituted for the (b, e) coil.

Calibration.

To calibrate the bridge, the switch (fig. 2) is set with p, q horizontal, and s screwed firmly to m. This puts $45+10=55$ ohms between b and e, and the slider has to be moved to a

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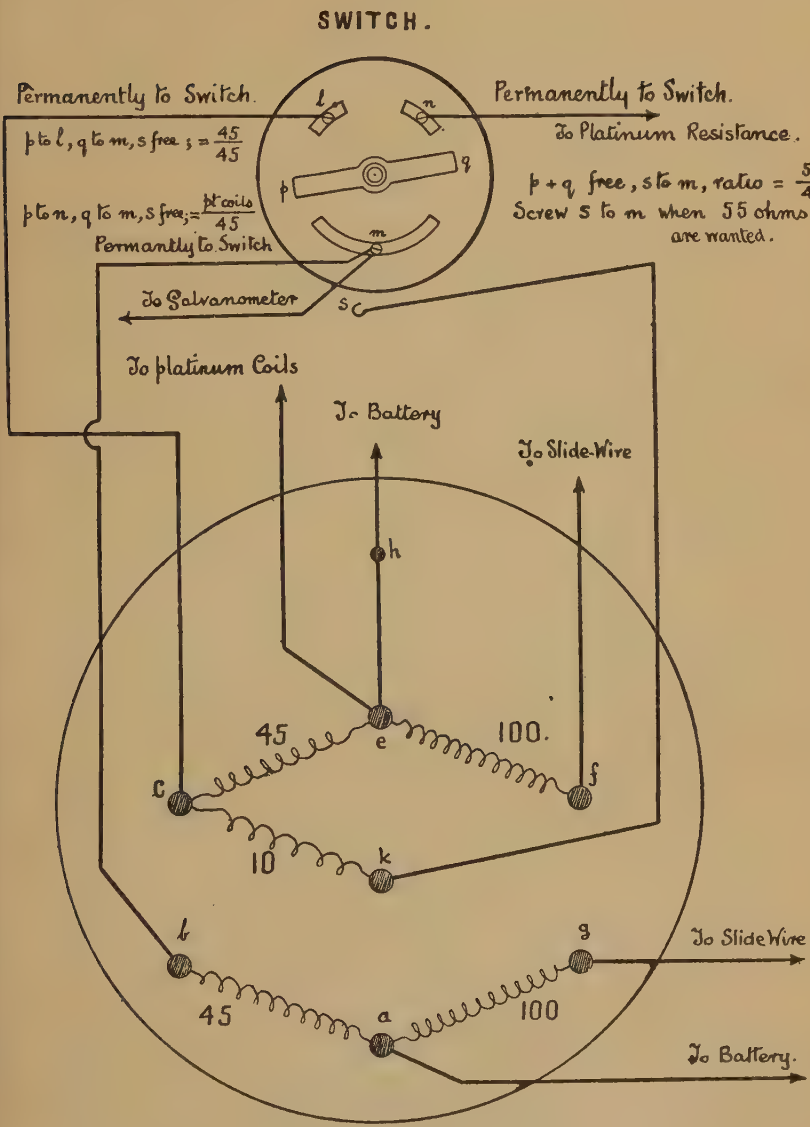
* A very convenient *optical bench* could be made by such a grooved plank.

† Platinoid is not good for slide-bridge wires. After about two months' exposure this wire has become blackened, and I am replacing it by a wire of platinum-silver.

Fig. 2.


SWITCH.
Permanently to Switch.
p to l, q to m, s free ; = 45/45
p to n, q to m, s free ; = pt coils/45
Permantly to Switch
To Galvanometer
l
n
p
q
m
s
Permanently to Switch.
To Platinum Resistance.
p + q free, s to m, ratio = 55/45
Screw s to m when 55 ohms are wanted.
To platinum Coils
To Battery
To Slide-Wire
h
45
e
100.
c
f
10
k
b
g
45
a
100
To Slide Wire
To Battery.

new position along the plank. A simple calculation enables us to find an expression for the temperature, t, of the platinum coils in terms of n, the distance in cms. of the sliding-contact from the extreme left of the plank-scale. The actual figures for this particular bridge are here given.

Corrected values of the bridge-coils, at 80° F.

Between a and b = 45·138 ohms.
" c " e = 45·150 "
" k " c = 10·032 "

The coils 100, 100 require no correction; they were adjusted at 64°·5 F., and we are only concerned with their *equality* at the slightly higher temperature.

Value of r, the resistance of 1 cm. of the slide-wire.—Balancing (45·15 + 10·032) ohms against 45·138 and noting the corresponding value* of n, we have

$$45{\cdot}138[100+241{\cdot}95\,r]=55{\cdot}182[100+58{\cdot}05\,r].$$

Whence $$r={\cdot}1301 \text{ ohm.} \quad \ldots\ldots \quad (2)$$

R_t *in terms of* t.—Putting the values for R_{32} and α in (1), the equation becomes

$$R_t=40{\cdot}05[1+{\cdot}002097(t-32)], \quad \ldots \quad (3)$$

that is, within the required range of temperature.

To this may be added, without sensible error, the resistance of the thick copper leads going to the platinum coils; this was ·025 ohm, so that

$$R_t=37{\cdot}39+0{\cdot}084\,t. \quad \ldots\ldots \quad (4)$$

R_t *in terms of* n.—The general equation of the bridge will be

$$R_t[100+(300-n)r]=45{\cdot}138(100+nr).$$

Or $$R_t=\frac{4513{\cdot}8+5{\cdot}8745\,n}{139{\cdot}04-{\cdot}1301\,n}. \quad \ldots\ldots \quad (5)$$

t in terms of n.—From (3) and (5),

$$t=\frac{10{\cdot}7406\,n-684{\cdot}9}{11{\cdot}677-0{\cdot}0109\,n}. \quad \ldots\ldots \quad (6)$$

That is, $$n=\frac{11{\cdot}677\,t+684{\cdot}9}{10{\cdot}7406+0{\cdot}01093\,t}. \quad \ldots\ldots \quad (7)$$

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* The full length of the slide-wire is $n=300$ cms.

From (7) the plank may be calibrated "directly," in temperature degrees. It will, however, be more convenient to plot a curve co-ordinating n and t. Such a curve does not greatly differ from a straight line; it is therefore sufficiently accurate to calculate n for every successive five degrees within the range.

Accuracy attained.

Mr. E. H. Griffiths has been good enough to examine my figures, and he agrees that, if the original standardization of the platinum coils is carefully attended to, this apparatus may be relied upon to measure temperature with an accuracy of one tenth of a Fahrenheit degree. Unfortunately, the platinum coils were sealed hard and fast within their respective dielectrics before this slide-bridge method was adopted; so that it has not been possible to check the readings in a liquid, as against a mercurial thermometer. This is very much to be regretted. By taking the mean of the readings of mercurial thermometers in the tank, and the readings of the mercurial thermometers in the tubes within the dielectrics, and comparing this mean with the value of t derived from (6), it is possible to make a very rough comparison. It should be carefully noticed, however, that "tube" temperatures include all the errors which it is the very object of the present method to avoid. The following table must be regarded with that limitation.

TABLE I.

Date.	Tube-temperature.	n.	t, from (6).	Air-temperature.
Aug. 22	76°·0 F.	135·8	75°·9 F.	81° F.
" 23	72·5	132·9	72·6	77
" 28	68·3	128·9	68·1	73
" 29	80·5	139·75	80·4	76
" 30	90·6	147·7	89·6	76
Sept. 2	101·0	156·64	100·1	78
" 3	110·1	164·8	109·9	79

It may be remarked here that, with the same apparatus, by taking the mean of the two values of n, corresponding to reversal of the battery, a second approximation could have been made. Also the slide-wire is thinner than is required by the temperature range. A thicker wire would have given greater possibilities of accuracy, and it would have been less liable to mechanical injury and temperature fluctuations.

The Action of Sulphur Vapour upon Copper.

While standardizing platinum thermometers in the vapour of sulphur (444° C.) some mica plates, which formed part of the apparatus within the vapour, were bound together with copper wires which passed through holes in the mica. After five hours' exposure to the vapour the apparatus was dismantled, and, as had been expected, the copper wire was completely demetallized. An examination of the resulting wire, a brittle compound of cuprous or cupric sulphide, showed that it was, in some cases, perforated by a small axial hole; the wire had, in fact, become a tube of very small bore.

I am able to show this to you by projecting upon the screen one of the pieces of mica through which the original wire was threaded; the ends of the sulphide wire were broken off flush with each face of the mica, thus forming a section of the sulphide wire.

Prof. Ramsay tells me that, as a rule, it is possible to draw an inner metallic core* from a copper wire which has been treated with sulphur vapour. It therefore seemed probable that my specimen was a piece of sulphide wire out of which a core had fallen. This, however, does not appear to be the explanation, for I have since succeeded in threading an iron wire, of 8 mils diameter, completely through a sulphide wire, which was more than half an inch long and not quite straight. It is impossible that a core of 8 mils could have fallen out of this specimen.

An examination, with the microscope, of the original copper wire does not reveal the slightest perforation; although the end was carefully removed with nitric acid so as to avoid the closing of the hole, if one existed, by any mechanical cutting of the wire.

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* I have obtained lately several specimens which show this core.

It was suggested that an occasional hole might occur in the process of drawing the wire. Prof. Boys asked me to try a piece of copper cut from a block. I therefore had a piece, of square section, sawn from a large commutator-bar. After six hours' boiling in sulphur vapour, this copper rod became circular in section; it had a central perforation, as could be proved by passing a wire into it. The colour of the surface was a deep "beetle" blue.

In order to show the amount of expansion which the copper experiences during the change to sulphide, a second piece of copper, of square section, was sawn out of the commutator-bar, half the length being turned down to a circular section. This was put into sulphur vapour for six hours, with the result that the diameter of what had been the square part had increased from 215 to 386 mils; the circular part had increased from 120 to 232 mils; and a third piece, of ordinary drawn copper wire, increased from 31 to 77 mils. During the transformation from copper to sulphide the diameter in all cases is approximately doubled. The length is only slightly increased; all sharp edges disappear; and there is a general tendency towards the circular section. The fracture is crystalline, resembling that of an aerolite.

A piece of Delta metal submitted to the same test came out of the sulphur vapour without appreciable change. It was discoloured but otherwise unaltered, and I propose to adopt this metal in future for the electrodes of platinum thermometers.

Summary.

The foregoing results indicate that platinum-coil thermometry may be reduced to a simple operation, and that the simplified method is sufficiently precise for general work. The exact conditions which determine the numerical limits of accuracy, generally, cannot be very concisely stated. The possible causes of error are well known, and with a little care may be nearly eliminated. The first precaution is to maintain the bridge-coils at some constant temperature just above the maximum air-temperature. For this purpose the oil-bath and glow-lamp will be found satisfactory.

If the temperature of the slide-wire is likely to vary considerably, r may be redetermined; or a correction-factor may be applied. This will seldom be necessary; no such correction was used in the above tests.

There must be some easy arrangement for calibrating the slide-wire. The auxiliary-coil test is expeditious. The wire now supplied by good makers is sufficiently uniform; and, except where great precision is essential, it will not require to be corrected for differences of diameter; it should be of platinum-silver.

The resistance of the leads can generally be added as in equation (4). In other cases one of the "dummy-lead" methods of Siemens or Callendar must be adopted.

In conclusion I have to thank Mr. E. H. Griffiths, Mr. W. A. Price, and Prof. W. N. Stocker for their help in preparing this paper; and the Silvertown Telegraph Company, who have kindly allowed me to carry out the experiments and publish the results.

Discussion.

Mr. Trotter said he agreed with the author that the "reserve of precision" at our disposal, on account of the delicacy of some of our modern instruments, ought to be made use of to facilitate the rapid performance of many measurements where the utmost accuracy is not necessary. He had the impression that platinum silver was not now considered the best material for use as the bridge-wire.

Mr. H. F. Burstall explained the differences between temperature as measured on the mercury, air, and platinum thermometers. At a temperature of about 40° the platinum thermometer read about 0°·4, and the mercury thermometer about 0°·1 below the air thermometer. Prof. Callendar had obtained measurements of temperature correct to within 0°·1 by using a Weston voltmeter and an ordinary Wheatstone bridge; the variations of resistance, and hence the temperature, being read directly from the deflexions on the voltmeter.

Mr. Rhodes thought that, except where extreme accuracy was necessary, the mercury thermometer was very much more convenient than the platinum thermometer.

Mr. **Burstall** said the great convenience of the platinum thermometer lay in the fact that the scale could be read at a distance of many yards from the point where the temperature was being measured, and hence could be used in many places where it would be impossible to read a mercury thermometer.

Mr. **Blakesley** considered that the author was somewhat bold when he stated that for general purposes it was never necessary to measure temperature to nearer than one-tenth of a degree.

VI. *On the Existence of Vertical Earth-Air Electric Currents in the United Kingdom.* *By* A. W. Rücker, *M.A., F.R.S.*[*]

In a paper by Dr. Adolph Schmidt, read before Section A of the British Association at Oxford (Report Brit. Assoc. 1894, p. 570), the author stated that he had expanded the components of the earth's magnetic force in series, and had deduced expressions, two of which give the magnetic potential on the surface of the earth in so far as it depends on (1) internal, and (2) external forces. "The third series represents that part of the magnetic forces which cannot be expressed in terms of a potential, but must be due to electric currents traversing the earth's surface." The author concludes that such currents amount on the average to about 0·1 ampere per square kilometre.

It appeared therefore desirable that this conclusion, drawn from the magnetic state of the earth as a whole, should be tested by means of those portions which have been most fully studied.

The test to be applied is, whether the line-integral of the magnetic force taken round a re-entrant circuit on the surface of the earth is or is not a vanishing quantity.

The irregular form of the United Kingdom makes the application of this test more difficult than it would otherwise be; but as two detailed Surveys of Great Britain and Ireland have been carried out by Dr. Thorpe and myself for the epochs 1886 and 1891 respectively, the data at our disposal are so numerous that I thought it worth while to undertake the inquiry.

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[*] Read December 13, 1895.

The actual work of calculation has been carried out almost entirely by two of my students, Messrs. Kay and Whalley. My best thanks are due to them for the care and skill they have displayed.

The facts on which the investigation is based are as follows.

The first survey (1886) included 205 stations, at all of which observations were made by Dr. Thorpe or myself.

The *true*, and therefore irregular, isomagnetic curves were drawn for the epoch January 1, 1886, and the *terrestrial curves*, from which the local disturbances were eliminated, were also calculated for the same date (Phil Trans. vol. clxxxi. A, 1890).

The second survey included observations at 677 stations. These were made by ourselves, or, under our superintendence, by Messrs. Briscoe, Gray, and Watson. The results are about to be published by the Royal Society. The terrestrial isomagnetic curves were drawn for the epoch Jan. 1, 1891. The secular change having been carefully determined by special observations and methods, the values of the elements and the terrestrial curves obtained for the earlier date were reduced to Jan. 1, 1891. Thus the whole of the 882 stations were available for drawing the true isomagnetics for the latter date. The two sets of terrestrial curves obtained from the second survey and from the first survey reduced to the second epoch did not agree exactly, and the lines bisecting the intervals between them were taken as our final result for the terrestrial curves in 1891.

The following sets of curves will be considered in this paper:—

(1) The terrestrial isomagnetics obtained in the first survey for Jan. 1, 1886. These will be referred to as *the* 1886 *curves.*

(2) The same curves reduced by the secular change to Jan. 1, 1891. These will be called *the first survey* 1891 *curves.*

(3) The terrestrial curves for 1891 deduced from the second survey. These will be called *the second survey* 1891 *curves.*

(4) The mean terrestrial curves for Jan. 1, 1891, deduced from (2) and (3). These will be called *the mean* 1891 *curves.*

(5) Lastly, the true isomagnetic curves deduced from the results at all the 882 stations for the epoch Jan. 1, 1891. These will be called *the true* 1891 *curves.*

(1) *The* 1886 *Curves.*

The advantage of using the calculated terrestrial curves is that they can be carried across the sea from England to Ireland, or extended a few miles from the coast by extrapolation. The area included can therefore be made as large as possible. On the other hand, the method of obtaining these curves is such that the errors in their positions will probably be greatest near the boundaries of the land area over which the survey was carried. In order therefore that such errors might affect different calculations as differently as possible, it was determined to take two circuits, which should have their greatest extensions N. and S., and E. and W. respectively. They will be called the α and β circuits.

The α circuit was bounded by long. 2° W., lat. 58° N., long. 7° W. and lat. 52° N.

The β current was bounded by long. 1° W., lat. 55° N., long. 9° W. and lat. 52° N.

In the published account of the 1886 survey (*loc. cit.* p. 322), the values of the declination (δ) and horizontal force (H) are given for all points within the United Kingdom defined by the intersection of whole degrees of latitude and longitude. From these the northerly components of the force ($H \cos \delta$) were calculated for all such points on the lines of latitude, and the westerly components ($H \sin \delta$) for all such points on the lines of longitude which bounded the circuits.

The method of calculating the line-integral of the force may best be shown by an example, for which we may select long. 2° W. between lat. 52° and 58°.

Let C be the number of cm. in a degree of latitude, and N the northerly component of the force. Assume that $N = N_{52} + al' + x$, where a is a constant, $l' = l - 52$, and x is a small variable.

Let W be the work done as the unit pole moves due North from lat. 52° to lat. 58°.

Then

$$W = C\int_{52}^{58} N dl = C\{N_{52} \times 6 + 18a + \int_0^6 x dl'\}.$$

The value of a was found from the values of the northerly components at the points on latitudes 52° and 58° respectively. The integral was calculated by quadrature, graphic methods being employed.

To give an idea of the relative magnitudes of the terms, I append the following data :—

$$6N_{52} + 18a + \int_0^6 x dl' = 1{\cdot}00896 - 0{\cdot}06864 - 0{\cdot}00152$$
$$= 0{\cdot}93880.$$

The constant C was taken = 11119320 cm., so that the work done in this part of the circuit is $1{\cdot}04387 \times 10^7$ ergs.

Treating the other parts of the circuit in the same way, the four quantities, the algebraical sum of which is the work done in completing the circuit, are :—

$$(1{\cdot}04388 + 0{\cdot}17082 - 1{\cdot}00637 - 0{\cdot}20902) \times 10^7 = -6{\cdot}9 \times 10^3 \text{ ergs.}$$

Dividing by 4π, we find that the total current within the circuit is -550 C.G.S. units, and, since the area is $2{\cdot}13 \times 10^5$ square kilometres, this amounts to $-0{\cdot}026$ ampere per square kilometre. The negative sign indicates that the current flows downwards.

A similar calculation carried out with respect to the β circuit, of which the greatest extension is East and West, and the area is $1{\cdot}77 \times 10^5$ square kilometres, indicates a current of only $-0{\cdot}004$ ampere per square kilometre.

(2) *The First Survey* 1891 *Curves.*

When the 1886 curves are reduced to the Epoch Jan. 1, 1891, by methods which are fully described in the account of the later survey, the results obtained from the α and β circuits are $-0{\cdot}045$ and $-0{\cdot}030$ ampere per square kilometre respectively. It would at first sight appear as though the fact that these values are larger than those calculated for

Jan. 1, 1886, might be due to errors introduced by the assumed values of the secular change; but, as will immediately be seen, they are not larger than those obtained by another method, which this cause of error does not affect.

(3) *The Second Survey* 1891 *Curves.*

Treated in exactly the same way as the last, these give values of about the same magnitude but of opposite signs; viz. for the α circuit $+0{\cdot}016$, and for the β circuit $+0{\cdot}020$ ampere per square kilometre. Thus two different methods of calculating the same quantity lead to very different results, which point to the conclusion that the apparent effects of the hypothetical currents are due to small errors in the determination of the exact positions of the lines.

In the final calculation of the results of our survey, we have taken the means of the positions of these two sets of lines as the isomagnetic lines for 1891, hence the mean values of the currents deduced from them by the

(4) *Mean* 1891 *Curves*

are $+0{\cdot}001$ and $-0{\cdot}005$ ampere per square kilometre for the α and β circuits respectively.

(5) *The True* 1891 *Curves.*

We have further checked the above results by means of the true curves, taking two circuits—one (γ) in England and Scotland, and the other (δ) in Ireland.

The first of these was as follows:—

Long. 1° E. from lat. 51° to lat 53°.
Lat. 53° from long. 1° E. to 1° W.
Long. 1° W. from lat. 53° to 55°.
Lat. 55° from long. 1° W. to 3° W.
Long. 3° W. from lat. 55° to 53°.
Lat. 53° from long. 3° W. to 4° W.
Long. 4° W. from lat. 53° to 51°.
Lat. 51° from long. 4° W. to 1° E.

The area is $1{\cdot}054 \times 10^5$ square kilometres. The values of the horizontal force and declination for every $10'$ of latitude or longitude were read off from the maps on which the values at the different stations were entered, and the true isomagnetics drawn. This operation was performed by Messrs. Kay and Whalley and checked by myself. The northerly or westerly component of the force was then calculated for each of these points, and the average value for each short section was assumed to be equal to the mean of the values at its initial and final points.

No difficulty arose except at a point in Wales, where the curves are closed, and where it was therefore necessary to assume an average value for a section of the line on which a maximum occurred.

The result of the calculation gave a current of −0·008 ampere per square kilometre.

The second circuit was taken in Ireland. It traversed the district of Antrim, in which there are violent local disturbances, and is interesting chiefly as showing to what extent the result may be affected by such causes.

The circuit was as follows:—

Long. 6° 30′ W. from lat. 52° to lat. 55°.
Lat. 55° from long. 6° 30′ to long. 8°.
Long. 8° from lat. 55° to 54°.
Lat. 54° from long. 8° to 9°.
Long. 9° from lat. 54° to lat. 52°.
Lat. 52° from long 9° to long. 6° 30′ W.

The area is 48×10^3 square kilometres.

The current is −0·046 ampere per square kilometre.

The fact that these different circuits, including areas of very different magnitudes and situated in different parts of the United Kingdom, all give very small values for the hypothetical currents, is strong evidence that the smallness of the calculated numbers is not due to the fact that large positive and negative values mutually cancel each other. It is, for instance, conceivable that the directions of the current-flow might be opposed on what may be called the oceanic and continental sides of the kingdom.

If this were so, it is probable that circuits γ and δ would have given results of opposite signs. By way of testing the matter further, the current was calculated both from the 1886 and the 1891 lines for the relatively small area in the West of Ireland bounded by latitudes 52° and 53°, and longitudes 9° and 10°.

The results were :—

for 1886, $-0{\cdot}04$ ampere per square kilometre,
„ 1891, $+0{\cdot}11$ „ „ „

Hence the difference of direction which characterized the currents deduced from the two surveys when applied to large areas, also distinguishes, and in an exaggerated degree, the results obtained from a small border district. It is therefore evident that either the distribution of the vertical currents has entirely altered in five years, or they are too small to be detected by the method employed.

The former of these hypotheses is negatived by the fact that different calculations, based on the first and second survey 1891 curves respectively, lead to discordant results for the same date, and we are therefore compelled to fall back upon the second alternative.

Effect of the Ellipticity of the Earth.

The question may fairly be raised whether, in dealing with such minute quantities, the ellipticity of the earth ought not to be taken into account.

In answer to this, it may be stated that the work done when the unit pole traverses the α circuit was also calculated, using the data as to the form of the earth given by Captain Clarke and quoted by Prof. Everett ('C.G.S. System of Units,' ed. 1891, p. 71). The numerical values thus obtained differed from those given above, but the differences between them were of the same order. Both methods of calculation lead to opposite conclusions as to the directions of the hypothetical currents according as the 1886 or 1891 curves are used ; thus proving that the small outstanding uncertainties as to the magnetic state of the United Kingdom are the cause of the discrepancies, which are not reduced by using a closer approximation to the form of the earth.

Conclusion.

I can only conclude from these various figures that the local magnetic surveys furnish no evidence of vertical electrical currents in the United Kingdom. The largest number obtained from the larger circuits is less than half that which Dr. Schmidt assigns as the mean value for the whole earth. Different calculations lead to results of different signs for the same quantity ; and the data which would *a priori* be accepted as the best give the smallest values.

As far as the terrestrial curves are concerned, the final results for the two surveys are embodied in the 1886 and the mean 1891 curves respectively. The local irregularities in the North of Ireland are so great that the calculations based on the true isomagnetics in that country may be neglected as compared with the probably much better results obtained in Great Britain.

Selecting, then, only the most trustworthy values, the results may be summed up as follows in terms of amperes per square kilometre.

	Circuit.		
	α.	β.	γ.
1886.	−0·026	−0·004	
1891.	+0·001	−0·005	
1891.	......	......	−0·008

From these figures we may conclude that there is not in the United Kingdom a vertical current amounting on the average to 0·1 ampere per square kilometre. They are not inconsistent with the existence of a current of about a tenth or a twentieth of that amount ; but on account both of the smallness of the results and of the discrepancy between the values obtained for 1891 by two methods, we cannot assert that such a current actually exists. The calculations taken by themselves do not disprove the hypothesis that electric currents traverse the earth's surface, as we cannot argue from the condition of a small portion of the globe to that of the whole. The most that can be said is that no evidence in favour of the existence of vertical currents can be drawn from one district, which has been very minutely surveyed.

P.S.—No reference to Dr. Schmidt's original paper was given in the short notice published in the Report of the British Association. I had therefore supposed that the latter was a preliminary note. Professor Schuster has, however, recently shown to me Dr. Schmidt's complete investigation, and he has kindly calculated the current-density at latitudes 50° and 55° on the prime meridian from formulæ given by Dr. Schmidt. The result is *upward* currents and 0·20 to 0·15 ampere per square kilometre at lat. 50° and 55° respectively. The mean of these two numbers, viz. 0·175, is nearly equal to Schmidt's mean for the whole earth (0·17). It is opposite in direction to and very much greater in magnitude than any vertical current the existence of which is compatible with the results of the Magnetic Survey.

Discussion.

Mr. Watson said a few words on the difficulty experienced in determining the line-integral in South Wales, due to the presence of closed curves.

VII. *On Resultant Tones.*

By Professor J. D. Everett, *F.R.S.*[*]

1. The received theory of the generation of resultant tones in the ear may be summed up with rough accuracy as follows[†]:—

The drumskin being pulled inwards by the end of the handle of the "hammer," which is attached to its centre, offers unsymmetrical resistance to displacement in the inward and outward directions, so that the equation for the movement of its centre in free vibration would be

$$-\ddot{x}=\omega^2x+\alpha x^2,$$

or to a closer approximation

$$-\ddot{x}=\omega^2x+\alpha x^2+\beta x^3,$$

ω, α, β being constants.

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[*] Read January 24, 1896.

[†] See *Tonempfindungen*, Appendix xii.; Rayleigh on Sound, art. 68; Bosanquet, Proc. Phys. Soc. vol. iv. p. 240, arts. 57–69.

The value of ω when the second is the unit of time is less than 60, hence the frequency of free vibration, being $\omega/2\pi$, is less than 10.

When two harmonic forces of frequencies m and n act upon the drumskin, they produce, in addition to their own tones, certain "resultant tones," the one of largest amplitude being the "first difference-tone," of frequency $n-m$. The next largest amplitude is about $\left(\frac{n-m}{n+m}\right)^2$ of this, and belongs to the "first summation-tone," of frequency $n+m$. Neither of these tones will be audible unless the excursion x is so large that αx^2 is sensible in comparison with $\omega^2 x$. There will also be difference-tones of frequencies $2m-n$ and $2n-m$, but neither of these will be audible unless βx^3 is sensible compared with $\omega^2 x$.

2. This theory does not appear sufficient to account for the loud resultant tones which are sometimes heard. When a Helmholtz siren is driven rapidly, with the rows of holes 9, 12, 15, and 18 open, the resultant tone of frequency 3 on the same scale is the most prominent tone in the whole volume of sound.

3. The view which I desire to put forward is closely connected with the well-known theorem of Fourier, that every periodic variation can be resolved in one definite way into harmonic constituents, whose periods must be included in the list 1, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, &c., where 1 denotes the period of the given variation itself. The corresponding frequencies will be as 1, 2, 3, 4, &c.

In the majority of cases, when this analysis is carried out, the fundamental constituent, represented by 1 in the above lists, is the largest or among the largest; but in the case of a variation compounded of two simple tones with frequencies in the ratio of two integers, neither being a multiple of the other, the fundamental will be absent, and the Fourier series will consist of only two terms, which in the language of acoustics are harmonics of the fundamental.

4. Clearness of thought is facilitated in these matters by supposing a curve to be drawn, in which horizontal distance represents time and vertical distance represents the quantity whose variation is in question. Since the variation is periodic,

the curve will consist of repetitions of one and the same form, in other words it will consist of a number of equal and similar waves, and the wave-length stands for the complete period of the variation.

The point on which I wish to insist is, that if such a curve representing the superposition of two harmonics of the fundamental is in the first instance very accurately drawn, and is then inaccurately copied in such a way that all successive waves are treated alike, the inaccuracy is morally certain to introduce the fundamental.

Let y denote any ordinate, and θ the time (or abscissa) expressed in such a unit that 2π is the numerical value of the wave-length or period ; then the amplitude of the fundamental is the square root of A^2+B^2, where

$$A=\frac{1}{\pi}\int_0^{2\pi} y\cos\theta d\theta, \qquad B=\frac{1}{\pi}\int_0^{2\pi} y\sin\theta d\theta.$$

In the original curve both A and B vanish. Let y' be the altered value of y in the new and inaccurate curve, and let z denote $y'-y$; then we have, between the above limits,

$$\int y'\cos\theta d\theta=\int y\cos\theta d\theta+\int z\cos\theta d\theta=\int z\cos\theta d\theta,$$

since $\int y\cos\theta d\theta$ is zero.

But z may be regarded as a random magnitude, hence it is infinitely improbable that its different values exactly fulfil the condition $\int z\cos\theta d\theta=0$. Therefore the new A is finite ; and similar reasoning shows that the new B is finite.

If all the ordinates were changed in one uniform ratio, A and B would remain zero, and no new constituent would be introduced ; but any other change, unless specially planned to avoid introducing A and B, is practically certain to give A^2+B^2 a finite value.

5. I maintain that such a change is effected in the form of sonorous waves during their transmission from the external air to the sensory fibres by which we distinguish pitch. The waves are transmitted first from the air to the drumskin, then through two successive levers, the hammer and anvil, to the head of the stirrup, while the foot of the stirrup sits upon the membrane of the oval window, and passes on the vibrations

through the membrane to the liquid on the other side in which the sensory fibres are immersed. The levers turn upon ligamentous fulcra, and have rubbing contact with each other. The wave-form cannot run the gauntlet of all these transmissions without being to some extent knocked out of shape. It is much as if a very accurately drawn curve, representing the original wave-form, were copied and recopied, five times in succession, by five different pantagraphs not very firm in their connexions. The final copy so obtained would be sure to exhibit sensible departures from the original.

6. It appears likely that the chief seat of the disturbing actions in the ear is the junction of the hammer and anvil. "When the drumskin with the hammer is driven outwards, the anvil is not obliged to follow it. The interlocking teeth of the surfaces of the joint then separate, and the surfaces glide over each other with very little friction"*. Such action is likely to introduce derangement, increasing generally with the excursions of the drumskin, but not expressible as a definite function of the ordinates of the wave-curve. For a given pressure on the drumskin, the pressure communicated to the liquid in the cochlea will vary according to the relative position and relative motion of the two portions of this joint.

7. The principal resultant tone due to these actions is likely to be that which corresponds to the complete period of the actions, in other words the highest common fundamental of the two primaries, or what old writers called the "grave harmonic." This will not be the same as the "first difference-tone" unless the ratio of the two primaries is of the form $m : m+1$; and I have satisfied myself, both by my own trials and by a study of Kœnig's experimental results, that when the difference-tone and the common fundamental are not identical, the common fundamental is usually the predominant, and often the only audible resultant tone. (See Appendix.)

8. The common fundamental is, however, not the only resultant tone that can be thus accounted for. Similar reasoning to that employed in reference to A and B suffices to explain the introduction of any or all of the harmonics of

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* Ellis's 'Helmholtz,' p. 133, 2nd edition.

the fundamental; but it is to be expected, from the analogy of ordinary experience in harmonic analysis, that the successive constituents will usually be smaller and smaller as we advance in the series. The octave is likely to be the largest of them; and Kœnig found, in several experiments with primaries in the ratio of 3 : 5, that both the fundamental 1 and its octave 2 were distinctly heard as resultant tones.

9. The following investigation bears on the relation between beats and resultant tones. The expression

$$a \cos m\theta + b \cos n\theta$$

can be reduced to the form

$$A \cos\left(\frac{m+n}{2}\theta - \epsilon\right),$$

where A and ϵ are given by

$$A^2 = a^2 + b^2 + 2ab \cos (n-m)\theta,$$

$$\tan \epsilon = \frac{a-b}{a+b} \tan \frac{n-m}{2}\theta,$$

and the beating together of two tones not differing much in pitch is explained by the fact, definitely expressed in these formulæ, that the whole effect may be regarded as a succession of waves with gradually varying amplitude. The frequency of the beats is the frequency of the maxima of A^2, and is the difference of m and n.

We have ascribed resultant tones to alterations made in the wave-form by the action of the ear, such alterations being in general largest at those points at which the excursions of the drumskin are largest. These excursions are measured by $\pm A$, and the above investigation shows that their maxima have a frequency corresponding to the difference-tone. This is true whether m and n are commensurable or incommensurable. If they are commensurable, their greatest common measure will be the frequency of the complete cycle of change. This cycle will not be conspicuous in the curve if the ratio of $n-m$ to $\frac{1}{2}(n+m)$ is very small, but will assert itself more and more as this ratio increases; and these remarks will apply to the comparison of the fundamental with the first difference-tone.

10. If the ear is able so to alter the form of waves impinging upon it as to generate resultant tones, it is natural to seek for some instance of a similar action in external bodies. A violin is very susceptible, like the ear, to vibrations of all frequencies between wide limits, and the sound-post serves, like the ossicles of the ear, to transmit vibrations from one portion to another. It is easy to produce resultant tones by bowing two strings of a violin together. For example, in the ordinary process of tuning, when the fourth and third strings with frequencies as 2 : 3 are combined, the resultant tone 1 is very observable if attention be directed towards it. But more striking effects are obtained when the resultant is at a larger interval from the primaries. The major sixth 3 : 5, the major second 8 : 9, and the minor seventh 5 : 9, are suitable intervals for calling out the fundamental 1, the strings employed being either the first and second or the second and third. The deep resultant tone thus obtained can not only be heard by the ear but felt as a tremor by the hand which holds the instrument. This is clear evidence of its objective existence, and I have succeeded in confirming the fact by means of a Helmholtz resonance-globe, the largest of the ordinary set, responding to C of 128 vibrations. When held with the edge of its mouth resting against the side of the violin, it responds to the combination C of 256 and G of 384 on the 4th and 3rd strings, or to the combination C of 512 and E of 640 on the 2nd and 1st; or, still better, to the 3rd and 2nd open strings each flattened one note, so as to be C of 256 and G of 384. Here, then, we have distinct evidence that the violin possesses the power which I have ascribed to the ear—the power of manufacturing the fundamental when the two primaries are supplied.

11. Sir John Herschel, in his treatise on Sound (*Enc. Met.* arts. 238, 239), mentions the fact that the common fundamental can be called out by sounding two or more of its harmonics on very accurately tuned strings or pipes, and says that the effect cannot be obtained from a pianoforte tuned in the ordinary way, because the intervals are tempered. I find, however, on trying the experiment with an upright Broadwood of date about 1860, that C of 64 is easily called out by simultaneously striking eight or ten of its harmonics;

and the effect is greatly enhanced if the key of C 64 is held down. In the latter case its note continues to be heard for a long time after the keys which were struck are released. From these experiments it appears probable that the sounding-board of a piano possesses the same property which we have proved to exist in the violin.

12. I now come to the explanation of the experiments of Professor Rücker and Mr. Edser (Proc. Phys. Soc. vol. xiii. p. 412; Phil. Mag. 1895, xxxix. p. 341). They were made with a Helmholtz siren, and in each instance the two primaries were produced in the same box, sometimes the upper and sometimes the lower box. The following explanation is a development of suggestions contained in Appendix xvi. of the *Tonempfindungen*.

The rate of escape of air from the box containing the two rows of holes which are employed may as a first approximation be assumed to be jointly proportional, at each instant, to the aperture for escape and the differential pressure which produces the escape. Again, this differential pressure may be regarded as the algebraic sum of two terms, one of them constant, and representing its average value, while the other represents the difference from the average due to the varying amount of the aperture from instant to instant. As a first approximation, equal increments of aperture must be regarded as producing equal decrements of pressure, so that the variable term will be proportional (with reversed sign) to the excess of the aperture above its mean value. This excess (defect being counted negative) will be a periodic function of the time, and if the ratio of the two primaries in lowest terms be $m:n$, the frequency for the complete period will be represented on the same scale by 1. In other words it will be the period of their common fundamental.

Let the aperture at time t be expressed in a Fourier series, θ being put for $2\pi t/\mathrm{T}$, where T is the complete period; and let the variable part of the expression be denoted by $f(\theta)$, while a_0 denotes the mean aperture, so that the aperture at time t is $a_0+f(\theta)$. We shall have

$$f(\theta)=\mathrm{A}\sin\theta+\ldots+a_1\sin(m\theta+\epsilon_1)+\ldots+b_1\sin(n\theta+\epsilon_2)+\ldots$$

The largest amplitudes will be a_1 and b_1 corresponding to the two primaries; but A, which corresponds to the fundamental, is likely to be sensible.

The pressure at time t is proportional to

$$C-f(\theta),$$

C being a constant; and the aperture is

$$a_0+f(\theta).$$

Hence the rate of escape is proportional to

$$Ca_0+(C-a_0)f(\theta)-\{f(\theta)\}^2.$$

a_0 is comparable with the maximum value of $f(\theta)$, and C is much greater; hence a_0 may be neglected in comparison with C.

Developing $\{(f(\theta)\}^2$, we shall obtain a term

$$2a_1b_1 \sin(m\theta+\epsilon_1)\sin(n\theta+\epsilon_2)$$
$$=a_1b_1[\cos\{(n-m)\theta+\epsilon_2-\epsilon_1\}-\cos\{(n+m)\theta+\epsilon_2+\epsilon_1\}],$$

representing a difference-tone and a summation-tone. From $(C-a_0)f(\theta)$ we have the common fundamental

$$(C-a_0)A\sin\theta, \text{ or } CA\sin\theta,$$

and the two primaries

$$Ca_1\sin(m\theta+\epsilon_1),\quad Cb_1\sin(n\theta+\epsilon_2).$$

Suppose for simplicity that $a_1=b_1$, then, taking the amplitude of each of the primaries as 1, the amplitude of the common fundamental will be A/a_1, and the amplitudes of the summation-tone and difference-tone will each be a_1/C.

When $n-m=1$, the difference-tone coincides with the fundamental, and their joint amplitude may be taken as the square root of the sum of the squares of a_1/C and A/a_1.

13. Professor Rücker and Mr. Edser in experiments i. and ii. obtained the difference-tone 64 from five distinct combinations of primaries,

256 & 320, 192 & 256, 320 & 384, 51·2 & 115·2, 96 & 160,

their ratios being

4 : 5, 3 : 4, 5 : 6, 4 : 9, 3 : 5.

The second combination appears to have given a stronger effect than either the first or the third; whence it would appear that low frequencies are favourable to strong effects.

Nevertheless the fourth combination is mentioned as giving a rather feebler effect than any one of the first three. This confirms our conclusion that the difference-tone is weaker when it is distinct from the common fundamental than when it coincides with it.

Experiment iii. was directed to testing for the presence of the resultant 64 when the primaries were 256 and 576, which are as 4 : 9. Their common fundamental is 64, and it could not be detected. This may have been because the pitch 576 was too high to give a good effect. Or the failure may be an indication that A/a_1 is decidedly smaller than a_1/C. It would be interesting to repeat the experiment, employing 192 and 320 as the primaries.

14. Near the end of chapter vii. of the *Tonempfindungen* Helmholtz makes prominent mention of the slipping of the hammer on the anvil as an important cause of resultant tones, and appears to regard it as exemplifying his mathematical formula for the restoring force as a function of the displacement. But it is clear that if the hammer, which holds the drumskin, is liable to shift in its supports, the restoring force cannot be a mere function of the displacement, but must also depend on the relative position and relative velocity of the hammer and anvil at the moment considered. I accept all the consequences which Helmholtz deduces in the passage in question from this slipping, including its application to explain first difference-tones ; but I regard these consequences as lying outside the range of his general mathematical formulæ as given in Appendix xii.

15. To sum up my objections to the received mathematical theory of resultant tones :—

First. It assumes that the reaction of the drumskin against the air is a definite function of the displacement of the drumskin from a certain fixed position, whereas this reaction depends also on the position and motion of the further end of the hammer at the time.

Secondly. Even if the vibrations of the drumskin were in accordance with the received formulæ, there is plenty of scope for the introduction of additional constituents on the road from the drumskin to the liquid in the cochlea. The auditory ossicles, with their ligamentous supports and attachments, probably serve to protect the oval window of the cochlea against shocks and jars, and to smooth down asperities in the wave-form, thus mitigating the harshness of sounds and rendering them more musical. The changes thus introduced are very unlikely to fulfil the special conditions required for the vanishing of the common fundamental.

Thirdly. The received theory makes the common fundamental, when not coincident with the first difference-tone, depend on a term involving the cube or some higher power of the displacement. When the primaries are as 3 : 5, the fundamental 1 comes in as $2m-n$, and depends on the cube of the displacement. When they are as 4 : 11, the tone 1, which Kœnig found to be the loudest resultant, is $3m-n$, and depends on the fourth power. When they are in the ratio 4 : 15, as in Kœnig's experiment with the simple tones ut_5 and si_6, the common fundamental ut_3, which was the only resultant tone heard, is $4m-n$, and depends on the fifth power of the displacement; the first difference-tone, which depends on the second power and should in theory be the loudest, being inaudible. This is surely a *reductio ad absurdum* of the received theory.

I do not wish to be understood as denying that the theory has any basis of truth. My contention is that the actions to which it is truly applicable play only a subordinate part in the production of resultant tones.

APPENDIX.

Examples selected from Kœnig's *Expériences d'Acoustique*, pp. 103 and 104, illustrating the production of the common fundamental. The "single vibrations" of the original are here reduced to double or complete vibrations :—

ut_5 and si_5, which are as 8 : 15, gave only ut_2.

ut_5 and 2816, which are as 4 : 11, gave ut_3 corresponding to 1 louder than any other tone.

ut_5 and si_6, which are as 4 : 15, gave no audible tone but ut_3.

ut_5 and 3968, which are as 8 : 31, gave no audible tone but ut_2.

ut$_6$ and 3584, which are as 4 : 7, gave *ut*$_4$ more distinct than the difference-tone *sol*$_5$.

ut$_6$ and *si*$_6$, which are as 8 : 15, gave *ut*$_3$ distinct, the difference-tone 7 being inaudible.

ut$_6$ and 3968, which are 16 : 31, gave *ut*$_2$ only.

ut$_6$ and 4032, which are 32 : 63, gave *ut*$_1$ only.

Discussion.

Dr. C. V. Burton, after explaining several portions of Prof. Everett's paper, said that he (Dr. Burton) considered that the author's view in many ways seemed to fit in with the observed facts better than the accepted theory, but still did not appear itself quite free from objection. Prof. Everett supposes that the first term in a Fourier series is always the most important, and although in most cases which occur in practice this may be so, it hardly seems legitimate to take this as a characteristic of a Fourier series.

VIII. *The Magnetic Field of any Cylindrical Coil.*

By W. H. Everett, *B.A.**

[Abstract.]

Applying Ampère's formula for the magnetic force at any point due to an element of current, the force perpendicular to any plane circuit, carrying a current *i*, is found to be, at any point P,

$$Z' = i\int \frac{r^2 d\theta}{(r^2+h^2)^{\frac{3}{2}}};$$

h being the distance of P from the plane of the circuit, and r, θ the polar co-ordinates of any point of the circuit referred to the projection of P as origin.

The longitudinal force at any point due to a current in a cylindrical coil, or solenoid, is given by a second integration. It is the sum or difference of two terms, each of the form

$$Z = inh\int \frac{d\theta}{\sqrt{r^2+h^2}};$$

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* Read November 8, 1895.

where h denotes the distance of the point from an end plane of the solenoid, and n the number of turns per unit length of h. The depth of the coil, normal to the cylindrical surface, is assumed to be inconsiderable. The limits of integration are 0 and 2π for any point whose projection, taken parallel to the axis, falls within the solenoid.

Similarly, the transverse force at any point, due to a solenoid, is found to be

$$\mathrm{R}=in\Sigma\left(\frac{1}{\sqrt{r^2+h_1^2}}-\frac{1}{\sqrt{r^2+h_2^2}}\right)ds,$$

the summation being vectorial.

The latter two formulæ can be readily applied, for approximate calculation, to a cylindrical coil of any cross-section, including coils of circular and rectangular sections. But in the case of rectangular coils the formulæ become integrable.

Let p be the perpendicular distance of any point P from one of the faces of the rectangular coil, and a one of the two parts into which the corresponding side of a cross-section is divided by the perpendicular from P. Then the longitudinal force at P is given by the algebraic sum of sixteen terms, each of the form

$$\mathrm{Z}=inhp\int_0^a\frac{dx}{(x^2+p^2)\sqrt{x^2+p^2+h^2}},$$

$$=in\sin^{-1}\frac{ah}{\sqrt{(a^2+p^2)(h^2+p^2)}}.$$

And the transverse force at P is the resultant of eight terms, each of the form

$$\mathrm{R}=in\int_0^a\left(\frac{1}{\sqrt{x^2+p^2+h_1^2}}-\frac{1}{\sqrt{x^2+p^2+h_2^2}}\right)dx,$$

$$=\frac{in}{2}\log\left(\frac{p^2+h_2^2}{p^2+h_1^2}\cdot\frac{a+\sqrt{a^2+p^2+h_1^2}}{a+\sqrt{a^2+p^2+h_2^2}}\right).$$

The first formula in the paper can be used to find an expression for the force due to a circular current, at any point P in the plane bounded by the circle. Draw any chord through P, and call its segments r, r'. Write c for the distance of P from the centre, b for half the minimum chord through P, and a for the radius. Then for the force at P the formula reduces to

$$\begin{aligned} F &= i\int_0^{2\pi}\frac{d\theta}{r}, \\ &= i\int_0^{\pi}\left(\frac{1}{r}+\frac{1}{r'}\right)d\theta, \\ &= \frac{2i}{b^2}\int_0^{\pi}\sqrt{a^2-c^2\sin^2\theta}\,.\,d\theta. \end{aligned}$$

This can be written

$$F = i\frac{s}{b^2},$$

s being the perimeter of the ellipse with a and b as semi-axes, and having some value between $2\pi a$ and $4a$, according to the position of the point considered.

IX. *On the Determination of High Temperatures with the Meldometer.* By WILLIAM RAMSAY, *Ph.D., F.R.S., Professor of Chemistry, University College, London, and* N. EUMORFOPOULOS, *B.Sc., Demonstrator of Physics, University College, London**.

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THE meldometer, an instrument devised by Dr. Joly, has been sufficiently described by him (Proc. Roy. Irish Acad. 3rd series, ii. p. 38, or Chem. News, vol. lxv.), and we need therefore only give a very brief account of it here. The essential part of the instrument is a length (about 10 centim.) of thin uniform platinum ribbon, about 1 millim. wide. This is heated by a current of adjustable strength, and the increase in length of the ribbon is measured by a delicate micrometer-screw, the ribbon being kept gently stretched by a small spring. The temperature of the ribbon is, of course, lower where the two forceps hold it; but if it is suitably cut at each end nearly to a point, a length of, say, 6 centim. in the middle may be made of a very uniform temperature, as can be proved experimentally by taking the reading of the melting-point of the same substance at different points along the ribbon.

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* Read February 14, 1896.

An infinitesimal quantity (scarcely visible with the naked eye) of the substance to be melted is placed on the ribbon and viewed with a low-power microscope. The small quantity of the substance required enables it to be purified very completely. The current is then put on and increased rapidly until the substance melts, and thus an approximate reading is obtained. This is repeated more cautiously, to obtain an exact reading. Several readings can be taken by remelting the same, and also by using fresh substance, the latter method being usually adopted.

To translate the readings into temperatures, it is necessary to standardize the instrument by taking the readings with substances of known melting-points. Of these, unfortunately, there are none known with certainty beyond about 350° C. One reading is obtained by taking the temperature of the air, another is the melting-point of potassium nitrate (339°), and for a third one the melting-point of potassium sulphate (1052°) was adopted, for reasons that will appear. The melting-point of silver is irregular, apparently because of absorption of oxygen, and consequent spitting. Gold can be used, and also palladium. As, however, the expansion of the ribbon is almost a linear function of its temperature, and as the observations hitherto taken did not extend beyond about 1050°, it was considered unnecessary to take readings with palladium, the general character of the expansion of the ribbon being already known from Dr. Joly's observations. The question next arises, what is the melting-point of gold? There have been two or three determinations of value, which unfortunately differ from one another.

M. Violle (*C. R.* 1879) determined it by a calorimetric method, and obtains as a result 1045° (on the air-thermometer).

Messrs. Holborn and Wien (Wied. *Ann.* xlvii. and lvi., 1892 and 1895), who give 1072°, compared a thermo-element with an air-thermometer, and then used the former for determining the melting-points of silver, gold, and copper. This was done by inserting in a porcelain crucible the thermo-element, and also two platinum wires connected by a wire of the substance whose melting-point was to be taken. The platinum wires formed part of a circuit containing a battery and a galvanometer. When the wire melted, the circuit was broken, and the temperature read at the same moment with the thermo-element. The melting of the substance must in general lag a little behind the thermo-element; and as no mention is made of the rate at which the temperature was raised, it is difficult to know how far the results can be trusted. They obtain, however, very concordant readings.

Besides these, there are two determinations with the platinum pyrometer: one by Professor Callendar (Phil. Mag. Feb. 1892), and the other by Messrs. Heycock and Neville (J. C. S. Trans. 1895, p. 160). By a somewhat violent extrapolation * from a formula which, as far as we know, has been compared with the air-thermometer only to about 625°, they obtain :—

	Callendar.	Heycock and Neville.
Freezing- or melting-point of silver ...	982°	961°
„ „ gold	1091°	1062° †

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* In Holborn and Wien's last paper (*loc. cit.*) the resistance of pure platinum is determined at different temperatures with their thermo-element, and their results cannot be expressed quite satisfactorily ("*nur ungenügend*") by means of Callendar's formula. The resistance of two of their pure platinum wires began to differ beyond about 900°, while agreeing below this temperature. They therefore consider this property unsuited to extrapolation.

† In a recent paper (J. C. S. Trans. 1895, p. 1025) Heycock and Neville state that "Callendar did not rigidly follow the method of calibration, which was afterwards developed by Griffiths and himself, and which we have always adhered to. That method requires that the resistance of the pyrometer should be determined at three standard temperatures," viz., at 0°, 100°, and the boiling-point of sulphur, 444°·5. And lower down they say, "If Callendar had standardized his thermometer on the boiling-point of sulphur....;" hence they infer that Callendar did not use the boiling-point of sulphur for this determination. But the determination of this boiling-point is given in Phil. Trans. 1891, *i. e.*, it was published before the paper referred to, and the latter is also later than another paper (Phil. Mag. July 1891, p. 109), in which the boiling-point of sulphur is directly referred to. We do not quite see how to reconcile the various statements.

Callendar then, taking the melting-point of silver as 945°, makes the melting-point of gold 1037°. Now Violle's determination for silver is 954°, which, using Callendar's formula, would give for gold 1049°. Holborn and Wien's value for silver is 968°. In view of the variance between the numbers, it was determined to take Violle's value, though the correct value may be a few degrees higher (compare Le Chatelier, *C. R.* cxxi. p. 323, 1895). In any case, allowance can easily be made, when further researches have determined the true melting-point.

The gold used by us was a very pure specimen obtained from Messrs. Johnson and Matthey. The salts used, with the exception of some of the iodides, were pure specimens, prepared specially by ourselves. In a few cases the salts so prepared were recrystallized, and melting-points were taken both of the recrystallized salt and also that obtained from the mother liquor. No difference could be detected, and hence no further mention is made of these determinations.

Gold on melting alloys with the platinum, and hence must destroy to a certain extent the uniformity of the ribbon. A remedy for this is to dust the ribbon lightly with finely powdered talc. This, however, is not very satisfactory, and interferes somewhat with the observations; but with care, fairly good results can be obtained, as the following numbers show:—

	With talc.	Without talc.
NaCl.	799°	792°
K^2CO^3	883	880
KCl	752	762
$Ba(NO^3)^2$ . . .	583	575

It was then found that the melting-point of potassium sulphate is very little different from that of gold, viz., 7 degrees higher or 1052°; and this salt was afterwards used instead of the gold; thus there is no need to use talc.

Some melting-points are sharply marked, others are not. In these cases the lowest point was taken at which spreading over the ribbon could be detected.

For purposes of comparison determinations by other observers are given; some determined the melting-points (*e.g.*, Carnelley, and Meyer, Riddle and Lamb), others the freezing-points (Carnelley, Le Chatelier, Heycock and Neville, and McCrae). The data are taken from the following references:—

Carnelley (calorimetric method): J. C. S. Trans. 1876, p. 489; 1877, p. 365; 1878, p. 273.

Le Chatelier (thermo-electric method, assuming melting-point of gold as 1045°): *Bull. Soc. Chim.* t. xlvii. p. 301; *C. R.* t. cxviii. pp. 350, 711, and 802.

V. Meyer, Riddle and Lamb: *Ber.* xxvii. (1894) p. 3129. —In this method the salt has been previously fused in a platinum tube with a wire down its centre, and to this wire is attached a weight passing over a pulley. When the salt melts, it is pulled out by the weight, and the temperature is determined at the same moment by an air-thermometer.

J. McCrae (Wied. *Ann.* lv. p. 95), relying on Holborn and Wien's results, standardized his thermo-element with boiling diphenylamine (304°) and boiling sulphur (444°·5). For his thermo-element he used platinum against an alloy of platinum rhodium, and also against an alloy of platinum and iridium. The numbers in brackets given below refer to the latter, and the others to the former. It will be noticed that they do not agree perfectly. This may, of course, be due to his metals not being of the same purity as those of Holborn and Wien. The latter also from their observations find that the iridium alloy is not as well suited as the rhodium alloy.

Salts of Lithium.

The lithium carbonate bought as pure gave a conspicuous sodium coloration to the Bunsen flame. It was purified by successive precipitation and washing, until it gave a brilliant carmine coloration to a Bunsen flame, free from yellow fringe, showing an absence of sodium. The melting-points are not very well marked as a rule.

	Ramsay & Eumorfopoulos.	Carnelley.	Le Chatelier.
$Li^{2}SO^{4}$	853	818	830
$Li^{2}CO^{3}$	618	695	710
LiCl	491	598	
LiBr	442	547	
LiI	below 330	446	

It will be noticed that there is here no agreement between the results of different observers. Our results are as a rule about 100 degrees lower than Carnelley's, except that of the sulphate. Our resistances did not allow us to take the melting-point of lithium iodide, as we had not arranged for temperatures below 330°, but its melting-point is below that of potassium nitrate (339°). Lithium iodide is very hygroscopic, so that on placing it on the ribbon it quickly liquefies; then on putting on the current, it becomes solid and then liquid again at the temperature given above, *i. e.*, it has melted, and remains so indefinitely.

Salts of Sodium.

The salts of sodium and potassium were prepared from the bicarbonate, precipitated by carbonic acid from a solution of the pure carbonate. The melting-points are well marked, though all are not equally so, *e. g.*, the iodides.

	Ramsay & Eumorfopoulos.	Carnelley.	Le Chatelier.	Meyer, Riddle & Lamb.	Heycock & Neville.	McCrae.
Na^2SO^4 ...	884	861	860	863	883	883
Na^2CO^3 ...	851	814	820	849	852	861 (854)
NaCl	792	772	778	815	...	813
NaBr	733	708	...	758	...	761
NaI	603	628	...	661	...	695 (668)

The agreement between Messrs. Heycock and Neville's results and our own is only apparent, as they find 1062° as the melting-point of gold, while we assume 1045°. The fact that they are determining freezing- and not melting-points may introduce some difference.

Salts of Potassium.

	Ramsay & Eumorfopoulos.	Carnelley.	Le Chatelier.	Meyer, Riddle & Lamb.	Heycock & Neville.	McCrae.
K^2SO^4 ...	1052	...	1045	1078	1066·5	1059 (1166)
K^2CO^3 ...	880	834	860	879	...	893 (885)
KCl	762	734	740	800	...	800
KBr ...	733	699	...	722	...	746 (709)
KI	614	634	640	685	...	723 (677)

M. Le Chatelier gives an earlier value 1015° for potassium sulphate, and also 885° for potassium carbonate. Our value for potassium sulphate is 7 degrees above that of gold.

Messrs. Heycock and Neville's is 4·5 above their value for gold, but they mention that there may be an error of 2 degrees, due to the alkalinity of their potassium sulphate.

Salts of Calcium, Strontium, and Barium.

The salts of calcium, strontium, and barium were prepared from their carbonates precipitated from the purified nitrates. Their melting-points are not well marked, especially those of calcium; and here again the iodides are less well marked than the chlorides.

	Ramsay & Eumorfopoulos.	Carnelley.	Le Chatelier.	Meyer, Riddle & Lamb.	McCrae.
$Ca(NO^3)^2$	499	561			
$CaCl^2$	710	719	755	806	802
$CaBr^2$	485	676			
CaI^2	575 (?)	631			
$Sr(NO^3)^2$	570	645			
$SrCl^2$	796	825	840	832	854
$SrBr^2$	498	630			
SrI^2	402	507			
$Ba(NO^3)^2$	575	593	592		
$BaCl^2$	844	860	847	922	916 (941)
$BaBr^2$	728	812			
BaI^2	539				

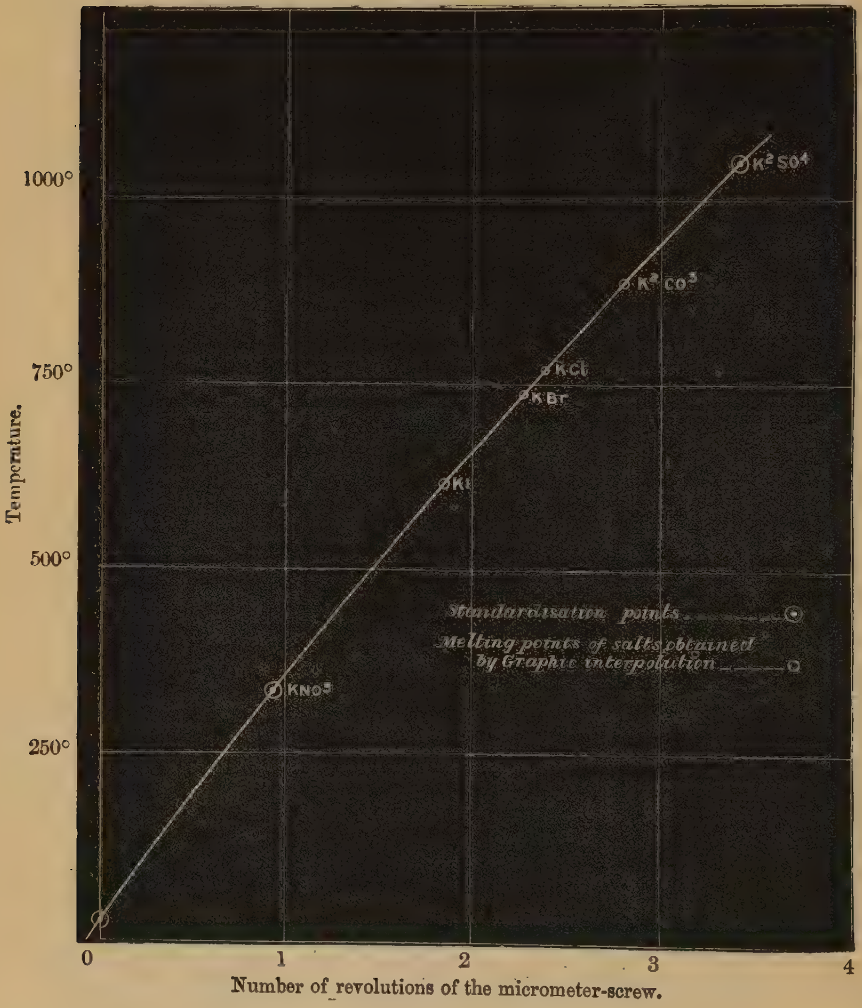
Calcium chloride is very difficult to observe, as it slowly softens. We found it practically impossible to take the melting-point of calcium iodide, as it is exceedingly hygroscopic, and, on heating, it is almost immediately oxidized. We do not think it can be above the value given, though it may be below it.

Salts of Silver and Lead.

These were prepared by precipitation. Their melting-points are well marked.

	Ramsay & Eumorfopoulos.	Carnelley.
Ag^2SO^4	676	654
$AgCl$	460	451
$AgBr$	426	427
AgI	556	527
$PbSO^4$	937	
$PbCl^2$	447	498
$PbBr^2$	363	499
PbI^2	373	383

A curve is appended with the melting-points of the salts of potassium marked on it. In drawing this curve there must



of necessity be a small uncertainty, as there is no datum-point between 339° and 1052°; but this cannot amount to more than a very few degrees.

In conclusion, we may point out that plotting our meldometer-readings against other observers' melting-points does not give a smooth curve.

University College, London.

DISCUSSION.

Prof. RAMSAY added that the chief advantage of the meldometer was that only a very minute fragment of the substance was required for the measurement, so that extreme purity of the sample could be secured. There was the disadvantage, however, that many substances undergo some change when heated in air. In reply to a question from Mr. Blakesley, Prof. Ramsay said that the property of the platinum which was used to measure the temperature was its expansion.

Mr. CAMPBELL asked whether the zero of the instrument was found to be constant. In Cardew voltmeters it often took several hours for the needle to come back to zero after heating.

Mr. EUMORFOPOULOS, in reply, said that the zero was constant to within a quarter of a degree.

Prof. RAMSAY also exhibited a small direct-vision spectroscope, in which the eye-piece is moved in a plane perpendicular to the axis of the instrument by means of a micrometer screw. This form of spectroscope is found to be of great utility in verifying the position of lines in the spectrum.

X. *On some Geometrical Instruments invented by* Mr. E. Scott *and* Signor Monticolo.

[Abstract.]

Mr. E. Scott, at the meeting on January 24, 1896, showed some Geometrical Instruments invented by himself and Signor Monticolo.

The instrument designed by Signor Monticolo is intended for drawing arcs of circles of such large radius that compasses cannot be employed. It can be used to trace arcs of circles of which the radii vary from 50 cm. to infinity.

The second instrument exhibited was a modified form of hatchet planimeter, which Mr. Scott has devised with a view of avoiding some of the defects of the ordinary form of instrument. Thus, to avoid the cutting of the paper, which occurs when the knife-edge is sharp, and the side-slip which occurs when the knife-edge is blunt, the author uses a wheel with a sharp edge. To avoid the inclination of the instrument to one side, which may easily occur with the ordinary form, a flat celluloid plate with a dot at the centre is used as the tracing point; this plate being kept pressed flat on the surface of the paper. A small wheel with a recording disk is attached, and may be used to measure the distance between the first and last position of the knife-edge.

Mr. Scott also described a form of planimeter which he had invented, and in which the disk and cylinder movement is used to perform the integration

Discussion.

Mr. C. V. Boys said that an instrument designed by Mr. Clarkson had been exhibited before the Royal Society which was capable of drawing arcs of circles of large radius. This instrument only drew an approximation to a circle, but the approximation was so close that it nowhere was more than the thickness of a thin ink line away from the truth. It would be interesting to hear from the author whether Signor Monticolo's instrument drew a rigorously exact circle or not. The upright position of the hatchet planimeter might be secured by using two wheels in place of one. The planimeter described was really a modified form of one he (Mr. Boys) had described before the Society in 1881.

Mr. Blakesley gave a short geometrical proof showing that the curve traced by Signor Monticolo's instrument was rigorously an arc of a circle. Mr. Blakesley also drew attention to the fact that the instrument in its present form cannot be used to trace the arc on both sides of the starting point.

Dr. C. V. Burton described an idea for an instrument for drawing circular arcs which had occurred to him, depending on the use of two wheels of different radii connected by an axle conveying a tracing point.

XI. *An Analytical Study of the Alternating Current Arc. By* J. A. Fleming, *M.A., D.Sc., F.R.S., Professor of Electrical Engineering in University College, London, and* J. E. Petavel*.

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Although the physical phenomena of the Electric Arc have received of late years very considerable attention, there are many points in connexion with the study of the alternating current arc which seem to us to have been insufficiently explored. The investigation which is here described is a contribution to the further examination of the mode in which the variation of the luminous and electric effects takes place in the alternating current arc.

The first portion of the research is concerned with an analytical study of the variation of the light thrown out from different radiating regions in the arc, and the delineation of the periodic value of this illuminating power by graphical methods. The object of the first set of experiments was to record and represent by an appropriate graphical method the periodic value of the light thrown out from the carbons and from the true electric arc region when the arc is supplied with electric power of known constant amount and varying magnitude, and at the same time to record the periodic variation of current through the arc, and potential-difference of the carbons. The instrumental appliances involved in the first place the employment of means for keeping constant and measuring the mean value of the power supplied to the arc. This was accomplished by the use of a suitable non-inductive resistance and a bifilar reflecting wattmeter, the series coil of which was in the circuit of the arc, and the shunt coil of which was connected across the carbons of the arc. The following is a description of this wattmeter:—

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* Read February 28, 1896.

The series coil was wound with 10 turns of thick copper wire (No. 10 S.W.G.) and had a resistance of ·01 ohm. The shunt coil was wound with 23 turns of thin wire, and had a resistance of 3·76 ohms, and was placed in series with a non-inductive platinoid resistance of 1200 ohms. The shunt-coil was suspended by two fine silver wires, which also served to conduct the current in and out of the coil. The controlling and deflecting forces so balanced each other at small displacements as to make the angular displacement of the movable coil very nearly proportional to the power passing through the wattmeter. The movable coil was provided with a mirror by means of which the image of a wire illuminated by an auxiliary arc lamp was reflected on to a fixed scale. This scale was carefully graduated, so as to read directly in watts. It was found that the earth's magnetic field caused a small deflexion of the movable coil when using continuous currents and when the shunt current was passing through it, when at the same time no current was flowing in the series coil. This terrestrial field was neutralized by magnets, or else the scale was shifted so that the part of the deflexion of the shunt coil due to the terrestrial field was eliminated. The best way would have been to have turned the wattmeter round through a certain angle, but as it had to be screwed up against the wall in a fixed position for steadiness, it was found that the above method was the simplest plan for obviating this source of error. The vibrations of the movable coil were damped by means of a mica vane dipping into a dash-pot filled with oil. When all the adjustments were made, it was found that this wattmeter produced a deflexion of the spot of light on the scale almost exactly proportional to the power passing through the instrument. This wattmeter was then connected up to the arc lamp, so that the whole current actuating the arc lamp passed through the series coil, the terminals of the shunt coil being connected to the carbons of the arc.

A series of preliminary experiments were then made for the purpose of enabling us to eliminate from the wattmeter-readings, firstly, the power taken up in the wattmeter itself; secondly, the power taken up in the shunt coil of the arc-lamp and other shunt resistances; and, thirdly, the power taken up in the carbons themselves, right up to the point where the arc is being formed. The observations, when reduced by applying these corrections, gave us at once the true mean power being taken up in the arc. The wattmeter was constructed with all the precautions necessary in making a wattmeter for measuring alternating-current power, so that it served to measure the power taken up either in alternating or continuous current arcs. The current supplied to the arc passed through a series of non-inductive resistances, consisting of carbon plates, in such a manner that the power given to the arc could be regulated with the greatest exactness. In addition to the wattmeter, an ammeter was placed in series with the arc lamp so as to measure the current passing through the arc, and a voltmeter connected to the carbons so as to measure the potential-difference of the carbons, these instruments being suitable both for continuous and alternating currents. In all the experiments the mean power was kept constant in the arc, and this was done by adjusting the current so that the wattmeter took a certain deflexion corresponding to the power desired, and the wattmeter was kept at a constant deflexion by regulating the carbon resistances in series with the arc. In addition to the instruments described above, a lens was fixed so as to enable the length of the arc to be measured in the usual manner.

In the course of the experiments three arc lamps were employed—a hand-regulated arc lamp in which the distance of the carbons was adjustable by a screw with great accuracy; a continuous-current arc lamp (the Waterhouse Arc Lamp); and an alternating-current arc lamp (the Helios Arc Lamp);

these last two being selected as excellent arc lamps of their respective types, our object being to select an arc lamp, the mechanism of which enabled it to be worked with electric powers varying over wide limits, and yet to yield a perfectly steady arc. In addition to these instruments there was set up the apparatus for delineating the curves of current of electromotive force, which have been described by one of us in the 'Electrician,' vol. xxxiv. p. 460, and which consists of a synchronizing alternating-current motor, having its fields separately excited and its armature circuit traversed by a shunt current from the circuit operating the arc lamp. This alternating-current motor was set up on the photometer bench, and it was used to drive an aluminium disk pierced by four openings, in such a manner that the disk revolved synchronously with the alternating current operating the arc. In addition to this duty the alternating-current motor carried a contact-maker on its shaft for the purpose of enabling the curves of current of electromotive force to be delineated. A brief description of the motor is as follows :—

It consists of two sets of field magnets M, M (see fig. 1), having eight poles on each side which are secured to two cast-iron disks. Between these field magnets revolves a small armature A, the iron core of which is formed of a very thin strip of transformer iron wound up into a ring, the armature coils being wound on this ring. The armature coils are joined up in series with one another, so as to give a series of alternating magnetic poles round the ring when a current flows through the armature circuit. The diameter of this armature is about six inches. The field magnets have eight poles, and the armature eight coils. The field magnetic coils are bobbins about 2 inches long and $1\frac{1}{2}$ inch in diameter, and when joined up in series in the proper manner the field magnets take a current of about eight amperes to give them a proper amount of saturation. The armature is carried upon a hard wooden boss fixed to a steel shaft ; and the steel shaft is carried through small ball bearings made like bicycle bearings. In order to prevent any side shake of the armature, there are at opposite ends of the base cast-iron pillars with a gun-metal screw through each, against which the rounded end of the shaft bears ; the position of the shaft can thus be adjusted with great nicety, and runs with great freedom from friction. The ends of the armature circuit are brought to two small insulated collars fixed on the shaft, against which press two light brass brushes marked B, B kept against the shaft by means of an expanding steel wire W. The armature

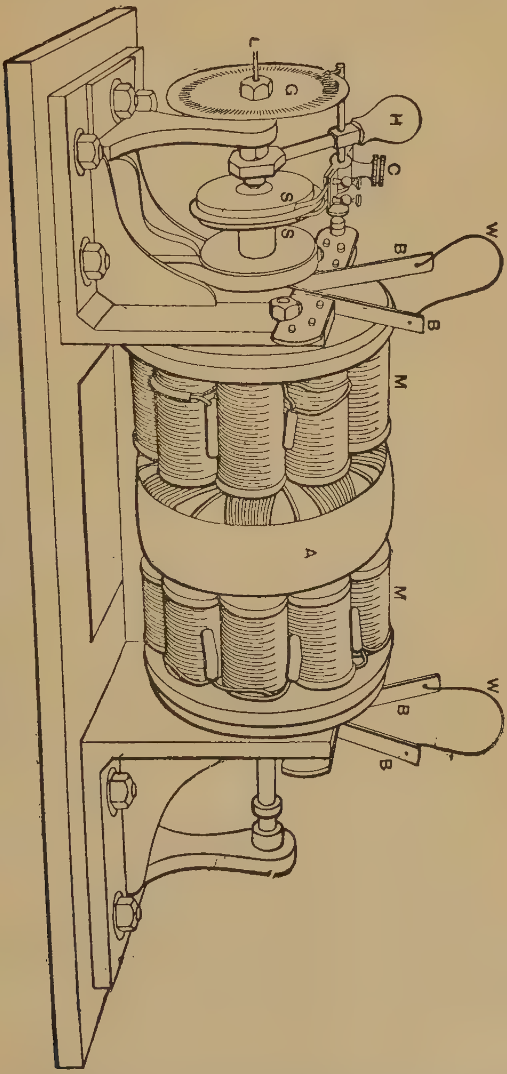


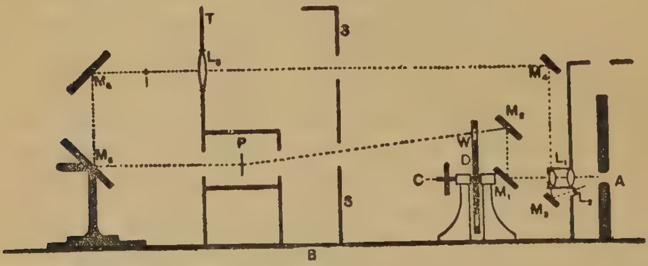
Fig. 1.—The Alternating Current Curve Tracer.
[From the 'Electrician,' vol. xxxiv. p. 460, 1895.]

shaft carries on one side an ebonite disk with a steel slip let into it. Two insulated springs S, S are carried on a rocking arm H; this rocking arm can be traversed through half a circumference, and is centred upon the gun-metal screw which prevents side shake in the shaft, and a pointer and graduated scale enables the exact position of the contact-springs to be determined. One of the stop-screws keeping the shaft from side shake is pierced with a longitudinal hole, and through this hole passes a stiff steel wire ; this serves to drive an aluminium disk 27 centims. in diameter and 4 millimetres thick. This disk is carried on a shaft which runs in a cast-iron bearing, and the disk is therefore driven synchronously by the motor. This aluminium disk has four slits in it separated by angular intervals of 90° ; the slits are 0·5 centimetre wide and 4·5 centimetres long. If the field-magnets of the motor are excited by a continuous current of about eight amperes, and if an alternating current of about two amperes is passed through the armature of the motor, then, on turning the motor rapidly round by hand, which can best be done by passing a strap round the shaft and pulling at the strap so as to spin the motor like a top, the motor will, if sufficient speed be gathered, drop into step with the alternating current driving it. Since the motor has eight magnetic poles, it makes one complete revolution in four complete periodic times, so that if the motor is being driven from an alternating-current circuit having a frequency of 100, then the motor has to run at 1500 revolutions per minute before it will drop into step, but at that speed it will fall into step with the current passing through its armature, and will be driven as a synchronous motor. Under these circumstances, if a ray of light is passed transversely to the disk in such a manner as to pass through the slits of the aluminium disk during the progress of rotation of the disk, the beam of light will be interrupted, but will obtain passage four times during each revolution through the slits in the disk as it goes round.

If the motor is being driven by the same alternating current circuit which supplies the alternating current to an arc lamp, it is evident that, on looking through the slits in the revolving disk at the alternating current arc, it will be seen in one constant condition during its periodic variation, such instant being determined by the position of the slits with reference to the phase of the current. Without entering into a longer description, it will be evident that this synchronizing motor driving a disk and a contact-breaker enabled two things to be done—first, to delineate all the current and electromotive-force curves of the arc taken in the usual way; and, secondly, a ray to be taken from the arc selected at one particular instant during the complete period through which the variation of illumination passes. These arrangements were completed by the construction of a photometer of a particular kind. Owing to the slow variation of position of the electric discharge in the alternating current arc, it would have been useless to photometer the instantaneous value of the light coming from the alternating current arc against any fixed standard of light; but it was found possible to make a very exact comparison between the intensity of the light coming from any part of the arc, and selected at any one constant instant during the complete phase, with the mean value of the light coming from that same part of the arc during the complete period; in other words, it was found possible to photometer the arc against itself, and so eliminate to a large extent the difficulties arising from slow periodic variations of the light sent out from the arc in any one direction. It is well known that the light of an alternating current arc, taken in any one direction, undergoes a slow periodic variation quite independently of the variation of current during the phase, neither is it dependent upon any variation of the mean square value of the current, because it takes place even when that current is perfectly constant. It appears to be due to slow changes of position of the points on the carbons between which the discharge takes place. The discharge is as it were seeking out new points between which to take place, and it continually changes these positions as the arc burns. The photometric arrangements finally adopted were as follows, and are shown in outline in fig. 2.

A represents an alternating current arc. This arc was enclosed in a metal lantern in which were three openings. The light from this arc passed through a lens L_1 in a horizontal direction and fell upon a mirror M_1 placed at an angle of 45°, capable of being rotated at this constant inclination round a horizontal line co-linear with the axis of the revolving disk, and the motor placed in front of the lens L_1. The ray was then reflected upwards into another mirror M_2, and by this mirror reflected at an angle very nearly equal to 45° in such a manner as to pass through the slits in the revolving disk D, when any one of the slits was in a position to allow

Fig. 2.



Arrangement of the Photometer and Revolving Disk.

the ray to pass. The two mirrors M_1 and M_2 were rigidly connected to a rocking-arm so centered that the line M_1 M_2 could be rotated round into any required position, always moving parallel to a radial line of the disk D. The disk D was the disk carried on the shaft of the synchronizing motor above described; the motor, together with its associated disk, was placed on the photometer bench in the required position opposite to the arc-light lantern. In fig. 2 the motor itself is not shown, but its position is indicated by the letter C. The angular position which the rocking-arm carrying the two mirrors M_1 and M_2 occupied with respect to the vertical line passing through the centre of the revolving disk could be observed on a graduated scale. It will be clear, then, that if the disk driven by the motor was in synchronism with the alternating current producing the alternating arc at A, an observer, looking through holes in the rapidly revolving disk, would see by reflexion in the mirrors M_1 and M_2 the alternating current arc at A; but he would see it, not as it is seen when looked at directly, but in some constant condition taken at one definite instant during the phase, which instant would depend upon the position of the line $M_1 M_2$ with regard to the vertical line through the centre of the disk. Thus by rocking over the arm $M_1 M_2$ in various positions the observer could see, through the window W as the disk revolved, the arc at A, either in the condition when the electric discharge is taking place, or when the true arc is extinguished, according to the position in which the arm $M_1 M_2$ was set. If, instead of observing with the eye, a disk of paper with a photometric grease spot upon it was placed at P, the lens L could be so adjusted as to throw an enlarged and well-defined image of the arc upon the disk at P; and by rocking over the arm carrying the mirror into successive positions, the observer would see the image of the arc pass slowly through all those successive phases which in the arc itself actually take place during one periodic time. As the image of the arc is much larger than the photometer disk, it was possible, by slightly shifting one of the mirrors, to bring any desired part of the image of the true arc or of the craters of either of the carbons to cover the grease spot. In addition to this interrupted ray, another ray was gathered from the same part of the arc by a lens L_2, placed on the same level with the lens L_1 but slightly to one side of it. This lens gathered a beam which was reflected by a mirror M_3 placed at an angle of 45°, and which reflected the ray upwards to another mirror M_4. In fig. 2, for the sake of clearness the lens L_2 is shown beneath the lens L_1, but it must be understood that in the real apparatus the lenses L_1 and L_2 were on the same level and placed side by side. The ray reflected from the mirrors M_3 and M_4 was set horizontally, so as to be received on a lens L_3, and by this lens L_3 was gathered to a focus at a point I. The lens L_2 was so adjusted as to form a large image of the arc on the screen which carried the lens L_3, and by slightly moving the mirror M_4 any part of this image could be made to cover the lens L_3. It will be seen, therefore, that the light gathered together at a focus at the point I could be made to be light coming from any assigned area in the arc or from the craters. A movable stand carried two other mirrors M_5 and M_6 fixed at angles of 45° in such a way as to reflect the ray coming from focus at 1, and reverse its direction so as to bring it round and make it fall on the left-hand side of the photometer-disk P. It will thus be seen that, by moving the mirrors M_5 and M_6, the light falling on the left-hand side of the photometer-disk P could be made to have any desired intensity within certain limits, and could be gathered from any desired part of the arc or craters; and, moreover, this illumination was the mean illumination, or proportional to the mean illuminative power of any part of the arc selected for examination. It will thus be clear that the arrangement enabled us to project on to the right-hand side of the photometer disk P the rapidly intermittent ray taken from any part of the arc, and always gathered at one constant phase condition during the complete period; whilst on the left-hand side of the photometer disk we could project a ray gathered from the same part of the arc, but not interrupted. We could therefore compare the mean value of the light proceeding from any part of the arc with the instantaneous value of the light taken from the same part of the arc and selected at any assigned instant during the period. Thus the arc itself became its own standard, and difficulties due to slow fluctuation of the mean light of the arc disappeared. At the same time the contact-maker on the motor enabled us to delineate in the usual manner the curves of current and potential difference of the arc, and thus to record the variation in the arc of the arc current, the carbon potential difference, the power expended in the arc, the resistance of the arc, and the luminous intensity of any part of the arc. A long series of experiments was then made with alternating current arcs of different lengths and powers, the periodic electric quantities being delineated and the light being taken, either from the centre of the true arc halfway between the carbons or from one of the craters of the carbon terminals,—generally the bottom carbon. The process of taking measurements was as follows:—

After setting the lenses and the mirrors so that the lens L_1 and the mirrors M_1 and M_2 gave a sharp image of the arc on the right-hand side of the photometer disk with the selected area of the image covering the grease spot, the mirrors M_5 and M_6 were moved backwards and forwards until the balance was obtained between the illumination falling on the right- and on the left-hand side of the photometer disk. The right-hand side of the photometer disk being illuminated by an intermittent stream of light always selected in the same phase, when considered as belonging to a periodically varying illuminating beam, whilst the light falling on the left-hand side of the photometer disk is a uniformly illuminating beam of the same quality and colour coming from the same part of the arc but not interrupted, and representing therefore the mean value of the light emitted from that selected area of the arc. Observers were deputed to measure all the various quantities by the different instruments, and a power of constant definite amount was supplied to the electric arc from an alternating-current machine driven by a continuous-current motor. By means of the carbon resistance and the reflecting wattmeter this power was kept constant at a selected value for a sufficient time to enable all the various periodic quantities to be observed at sufficiently frequent intervals during the phase. In all cases the arc was allowed to burn quietly for half an hour to get the carbons into a constant position before any observations were taken. It is hardly necessary to go into the details of delineating the current and electromotive-force curves, as the process of doing this is now well understood. The Kelvin vertical multicellular voltmeter, having a half microfarad condenser placed across its terminals, was employed for the measurement of the potential difference of the carbons in the following manner :—

The voltmeter, with its associated condenser, was connected, through the contact-maker driven by the shaft of the alternating motor, to the carbons of the arc ; the contact-maker thus closed the circuit at a certain instant during the phase, and the voltmeter gave the instantaneous value of the potential-difference of the carbons. In series with the arc was placed a non-inductive resistance of suitable magnitude. A switch was arranged so that the voltmeter with the condenser in parallel with it, both being in series with the revolving contact-maker, could be put across either the terminals of this non-inductive resistance, or else between the carbons of the arc. By rocking over the arm carrying the spring-brushes of the contact-breaker, the voltmeter circuit was closed at a particular instant during the phase, and the voltmeter reading gave therefore, when corrected, the instantaneous value either of the potential-difference of the carbons or the instantaneous value of the current through the arc. As the Kelvin multicellular voltmeter used by us only begins to read at 60 volts, in order to get readings for lower values than 60 volts, it is necessary to add a known electromotive force to the voltmeter circuit in order to block up the needle of the voltmeter to a false zero. This was done by connecting a known number of small Lithanode secondary batteries, the potential of which was determined by the same voltmeter, in series with the voltmeter. In this way the series of observations were successively taken of the following quantities:—First, the instantaneous value of the potential difference of the carbons taken at equidistant intervals throughout a complete period; secondly, the instantaneous values of the current through the arc taken throughout the complete period; and, thirdly, the instantaneous values of the luminous intensity of a certain selected portion of the arc taken at intervals throughout the complete period and expressed in terms of the true mean luminous intensity of the same portion of the arc at that time. These quantities having been obtained, it was then possible to plot them down in a series of curves, and to deduce therefrom curves representing the periodic variation of power through the arc, and the periodic variation of the resistance of the arc. Five sets of experiments were made, taken for different frequencies and different lengths of arc, each set comprising an observation taken with the light proceeding from the centre of the true arc, and also an observation taken with the light proceeding from the centre of the crater of the lower carbon. The frequencies employed were 83, 50, and 26. In all cases the current was kept at 14 amperes (mean square value); the results of these observations are embodied in the following 10 tables arranged in 5 pairs. Table I. A, for instance, gives the results of the observations taken with an arc having a frequency of 83·3, the light being taken from the centre of the true arc. Table I. B gives similar results of observations for the same arc, the light being taken from the centre of the lower crater. By holding a magnet at the back of the arc, noticing which way the instantaneous image of the true arc was projected by the magnet, and noting the pole of the magnet presented to the arc, it was possible to determine when the lower carbon was positive and when it was negative; and the diagrams corresponding to the above 10 tables are marked so as to show the half of the wave when the lower carbon is positive and when it is negative, in all those diagrams which refer to the light coming from the crater. The results of the observations given in the 10 tables are delineated graphically in diagram in figs. 3 to 12; and on referring to these diagrams a periodic line will be seen in each, delineating the variation of the potential difference of the carbons, and another periodic line indicating the variation of the current through the arc, whilst a third line indicates the periodic variation of the luminous intensity of the selected portion of the arc. In the diagrams figs. 13 to 17 are given curves representing the periodic variation of the light from the crater in the various cases, and extending over several periods. It will be noticed that those diagrams representing the periodic variation of the light from the arc show that this light undergoes a regular fluctuation between a maximum and a minimum, the maxima having equal values. The light in the centre of the true arc never falls quite to zero. This seems to be due to a little luminosity which hangs in the interspace between the carbons, but at the present moment it is difficult to say whether this persistence is due to a very small admixture of stray light (although every effort was made to keep this out), or to a persistence of the illuminating-power of the incandescent vapour in which the arc has been formed. On examining the true arc during its complete periodic variation, it is found that the blue or purple strip of light forming the true arc undergoes a periodic variation in intensity. As far as the eye can judge, the blue or purple light completely vanishes at a certain instant during the phase; but there is, outside the true arc, a dim halo of golden light which is persistent; and it is therefore probably on account of this persistent aureole of faint light round the true arc that the ordinates of the curve representing the periodic variation of the luminous intensity of the arc never become zero, but always indicate the outstanding constant amount of light. On the other hand, in the diagrams which represent the periodic variation of light coming from the centre of the crater of the lower carbon, we find the luminous intensity of the crater varies between a minimum value and two maximum values of different magnitude. During the time when the crater is positive it reaches a higher maximum intensity of illuminating power than during the time when it is negative, and, moreover, the curve representing the periodic variation of light rises more steeply than it comes down, which indicates a slow cooling of the carbons after they have been heated; in other words, they heat more quickly than they cool. This is particularly noticeable in that part of curve corresponding to the crater being negative; and it is only what would be expected, because after the carbon has reached its negative maximum and is beginning to cool, the opposite carbon is cooling from a condition in which it has been positive, and as it has been heated to a higher temperature than the negative carbon, it must assist by its radiation in keeping up the temperature and retard the cooling of the negative carbon. These diagrams will also show many other interesting facts: they show, for instance, that in the case of the long arc the self-induction of the arc is more marked than when the short arc is employed, and in that case there is a very distinct lag of current behind the potential difference.

It may also be noted in comparing the diagrams V. B and IV. B in figs. 12 and 10, which represent the curves for arcs of the same frequency but of lengths 1·2 centim. and ·32 centim. respectively, that in the case of the long arc there is no lag of light behind power as far as regards the points of maximum when the carbon is positive, but in the case of the short arc there is a sensible lag. This might be expected to be the case, because for a long arc the influence of the opposite carbon in keeping up the temperature of its neighbour as this last is cooling is less felt than in the case of a short arc.

Broadly speaking, the facts may be summed up as follows:—The purple light of the true arc undergoes a periodic variation, and, as far as the eye can judge, is completely extinguished during a certain interval during the phase; it has equal maxima values during the period, at instants slightly lagging behind the instants of maximum power-expenditure in the arc. On the other hand, the illuminating-power of the carbon crater varies between a minimum value and two maxima of unequal values; the greatest maximum occurring when the carbon is positive and at an instant slightly lagging behind the instant of maximum power-expenditure in the arc. A series of curves are also given (see figs. 18, 19, and 20) in which the periodic variation of the current, potential difference, power, and apparent resistance of the arc for various powers and frequencies are represented; and it will be seen from these curves that the resistance of the arc, including in this any counter electromotive force which may exist, varies periodically, the resistance being a minimum when the current is a maximum, and *vice versa.*

TABLE I. A.

Observations on the Periodic Variation of the Intensity of the Light of an Alternating Current Arc.

Light from centre of Arc. Short Arc.

Frequency . . . . = 83·3 ~
Length of arc . . . . = 0·55 cm.
Potential-difference (P.D.) of carbons (mean square value) . . . . . = 39 volts.
Current (mean square value) . . . . . = 14 amperes.
Power expended in arc . . . . =546 watts.
Pressure at alternator terminals . . . =104·5 volts.

Angle of Phase.	Current through Arc (instantaneous value).	P.D. of Carbons (instantaneous value).	Angle of Phase.	Intensity of Light from Arc.
0			26	38
20			38	23
40	+ 5·6	+20·2	58	16
60	+ 0·7	+ 0·2	76	6
80	− 3·7	−24·2	80	16
100	− 9·7	−34·5	102	24
120	−16·2	−38·0	132	47
140	−20·3	−43·0	146	60
160	−21·0	−44·8	156	56
180	−18·4	−43·5	168	68
200	−13·0	−32·5	176	57
220	− 6·2	−19·2	196	48
240	− 0·4	+ 1·9	202	31
260	+ 4·2	+25·5	242	11
280	+10·2	+38·1	252	19
300	+16·3	+40·8	274	12
320	+20·0	+46·5	304	50
340	+20·4	+48·3	320	58
360	+18·4	+50·5	326	55
			340	65
			346	63
			356	68

TABLE I. B.

Observations on the Periodic Variation of the Intensity of the Light of an Alternating Current Arc.

Light from Crater. Short Arc.

Frequency . . . . = 83·3∼.
Length of arc . . . . = 0·42 cm.
Potential-difference (P.D.) of carbons (mean square value) . . . . . . = 36 volts.
Current (mean square value) . . . . . = 14 amperes.
Power expended in arc . . . . =504 watts.
Pressure at alternator terminals . . . = 98 volts.

Angle of Phase.	Current through Arc (instantaneous value).	P.D. of Carbons (instantaneous value).	Angle of Phase.	Intensity of Light from Crater of lower Carbon.
0	+16·5	+42	12	60
20	+10·2	+25·3	33	34·5
40	+ 3·1	+10·2	53	28
60	− 1·1	−14·2	74	31
80	− 5·4	−27·7	95	24
100	−13·4	−32·4	116	44
120	−18·7	−38·4	137	71
140	−20·4	−48·9	157	101
160	−19·8	−52·3	178	148
180	−15	−48·4	200	91
200	− 8·8	−31	205	93
220	− 3·3	−11	226	61
240	+ 1·5	+11·4	247	42
260	+ 6·3	+26·4	268	31·5
280	+13·0	+34·9	289	41
300	+18·7	+39·4	309	56
320	+21·5	+44·4	330	65
340	+21·2	+45·8	351	55
360	+16·5		119	163
160	−19·8	−49		
140	−20·3	−48		

TABLE II. A.

Observations on the Periodic Variation of the Intensity of the Light of an Alternating Current Arc.

Light from centre of Arc. Medium Arc.

Frequency . . . . = 50 ∼.
Length of arc . . . . = 0·63 cm.
Potential-difference (P.D.) of carbons (mean square value) . . . . . . = 39 volts.
Current (mean square value) . . . . . = 14 amperes.
Power expended in arc . . . . = 546 watts.
Pressure at alternator terminals . . . = 99 volts.

Angle of Phase.	Current through Arc (instantaneous value.)	P.D. of Carbons (instantaneous value).	Angle of Phase.	Intensity of Light from centre of Arc.	
0	+19·8	+44	10	44	
20	+14·1	+38·5	32	13	Arc out.
40	+ 6·3	+27	55	11	„ „
60	+ 2·7	+10·5	77	18	
80	− 2·2	−16	100	17	
100	− 6·3	−36	122	45	
120	−13·5	−37·5	144	74	
140	−18·3	−47	165	68	
160	−19·8	−54	187	52·5	
180	−19·5	−54	208	32	
200	−14·8	−50	230	8·5	
220	− 7·9	−34	252	9	
240	− 1·9	−11·2	273	11	Arc out.
260	+ 1·9	+13	294	26	„ „
280	+ 7·2	+31	315	(71)?	
300	+12·6	+36	336	68	
320	+19·5	+40	356	42·5	
340	+21·7	+47·3	348	68	
360	+19·8	+46·3	327	45	

N.B.—Owing to an accidental shift of the brushes, in this table the angle corresponding to the instantaneous light is not the same as that corresponding to the current. The position of the curve of instantaneous light with regard to the E.M.F., current, and power curves cannot therefore in this case be determined.

TABLE II. B.

Observations on the Periodic Variation of the Intensity of the Light of an Alternating Current Arc.

Light from Crater. Medium Arc.

Frequency . . . . = 50 ∼.
Length of arc . . . . = 0·63 cm.
Potential-difference of carbons (mean square value) . . . . = 38 volts.
Current (mean square value) . . . . = 14 amperes.
Power expended in arc . . . . =532 watts.
Pressure at alternator terminals . . . = 96 volts.

Angle of Phase.	Current through Arc (instantaneous value).	P.D. of Carbons (instantaneous value).	Angle of Phase.	Intensity of Light from Crater of lower Carbon.
			0	56
			20	43
30	+ 6·3	+17	40	45
50	− 0·5	− 2·5	60	32
70	− 3·9	−22	80	39
			100	52
			120	137
			140	277
			160	277
			180	227
190	−11·3		200	165
210	− 5·5	−22	220	63
230	+ 0·1	− 1	240	64
250	+ 4·2	+16	260	61
270	+11	+28·4	280	78
			300	85
			320	72
			340	87
			360	(37)?
			340	69
			320	75

TABLE III. A.

Observations on the Periodic Variation of the Intensity of the Light of an Alternating Current Arc.

Light from centre of Arc. Medium Arc.

Frequency . . . . = 26 ∼.
Length of arc . . . . = 0·63 cm.
Potential-difference (P.D.) of carbons (mean square value) . . . . = 37 volts.
Current (mean square value) . . . . . = 14 amperes.
Power expended in arc . . . . = 518 watts.
Pressure at alternator terminals . . . = 62 volts.

Angle of Phase.	Current through Arc (instantaneous value).	P.D. of Carbons (instantaneous value).	Angle of Phase.	Intensity of Light from Centre of Arc.	
0	+19	+48	7	45	
20	+12·5	+39·5	28	26	
40	+ 5·8	+21·5	49	6	Arc out.
60	+ 0·5	+ 2·5	71	12	,, ,,
80	− 2·2	−22·5	92	7	,, ,,
100	− 9·7	−24	114	14	Arc just starting.
120	−16·8	−34·5	136	67	
140	−19·5	−45·5	158	71	
160	−21·8	−47·5	180	66	
180	−19·3	−50	202	24	
200	−11·5	−42	224	19·5	Arc out.
220	− 4·8	−22	246	4	,, ,,
240	− 0·5	− 2	268	8	,, ,,
260	+ 2·0	+17	290	22	
280	+10·8	+26	312	40	
300	+15·8	+36	333	57	
320	+20·7	+47·5	354	75	
340	+21·3	+50	15	53	
360	+19	+48			
340	+21	+51	345	62	
320	+20	+48	323	70	

TABLE III. B.

Observations on the Periodic Variation of the Intensity of the Light of an Alternating Current Arc.

Light from Crater. Medium Arc.

Frequency . . . . = 26 ∼.
Length of arc . . . . = 0·63 cm.
Potential-difference (P.D.) of carbons (mean square value) . . . . . . } = 39 volts.
Current (mean square value) . . . = 14 amperes.
Power expended in arc . . . . . . = 546 watts.
Pressure at alternator terminals . . = 62 volts.

Angle of Phase.	Current through Arc (instantaneous value).	P.D. of Carbons (instantaneous value.)	Intensity of Light from Crater of lower Carbon.
0	+18	+52	64
20	+14·3	+55	77
40	+ 6·7	+24·5	46
60	+ 1	+ 4·5	42
80	− 1·2	−21	43
100	− 5	−27	52
120	−17·5	−37	64
140	−18·2	−47	145
160	−20·6	−52	285
180	−20	−54	310
200	−11	−47	260
220	− 5	−25·5	200
240	− 1	− 5·5	119
260	+ 2	+15	87
280	+ 8·5	+30	50
300	+14	+40	104
320	+19	+45	110
340	+22	+53	130
360	+21		120

TABLE IV. A.

Observations on the Periodic Variation of the Intensity of the Light of an Alternating Current Arc.

Light from centre of Arc. Short Arc.

Frequency . . . . = 50 ~
Length of arc . . . . = 0·32 cm.
Potential-difference (P.D.) of carbons (mean square value) . . . . } = 36 volts.
Current (mean square value) . . . = 14 amperes
Power expended in arc . . . . =504 watts.
Pressure at alternator terminals . . = 85 volts.

Angle of Phase.	Current through Arc (instantaneous value).	P.D. of Carbons (instantaneous value).	Angle of Phase.	Intensity of Light from Centre of Arc.	
0	+18·4	+41·4	12	28	
20	+13·8	+38·4	34	15	
40	+ 5·7	+21	56	24	Arc going out.
60	+ 0·6	− 8·6	78	13	Arc out.
80	− 6	−22	100	14	Arc out.
100	−13·1	−25	122	18	
120	−19	−43	144	28	
140	−19	−46·5	166	20	
160	−18·3	−49	188	26	
180	−14·4	−46	210	17	
200	−10·2	−44·6	232	15	Arc going out.
220	− 5·0	−20	253	6	Arc out.
240	+ 1·1	+ 6·8	274	4	Arc out.
260	+ 8·6	+22·6	296	16	
280	+14·5	+29	317	26	
300	+18·9	+37·6	338	23	
320	+22·1	+40·6	360	30	
340	+20·3	+44	350	29	
360	+18·1	+42·6	328	23	
340	+21·5	+45·4			
320	+21·1	+40·4			

Table IV. B.

Observations on the Periodic Variation of the Intensity of the Light of an Alternating Current Arc.

Light from Crater. Short Arc.

Frequency . . . . = 50 ~
Length of arc . . . . = 0·32 cm.
Potential-difference of carbons (mean square value) . . . . = 38 volts.
Current (mean square value) . . . . = 14 amperes.
Power expended in arc . . . . . . =532 watts.
Pressure at alternator terminals . . = 93 volts.

Angle of Phase.	Current through Arc (instantaneous value).	Angle of Phase.	Intensity of Light from Crater of lower Carbon.
0			
20	+13·1	20	61
40	+ 7·2	41	56
60	+ 0·8	63	45
80	− 5·0	84	58
100	....	105	51
120	....	126	101
140	....	148	220
160	....	170	227
180	....	191	216
200	....	212	111
220	− 6·3	232	64
240	− 0·4	253	73
260	+ 4·6	273	66
280	+12·6	294	63
300	+17·6	315	77
320	+19·0	335	60
340	....	359	63
360	....	338	78
		317	88
		36	79

TABLE V. A.

Observations on the Periodic Variation of the Intensity of the Light of an Alternating Current Arc.

Light from centre of Arc. Long Arc.

Frequency . . . . = 50 ~.
Length of arc . . . . = 1·2 cm.
Potential-difference (P.D.) of carbons (mean square value) . . . . . = 53 volts
Current (mean square value) . . . . = 14 amperes.
Power expended in arc . . . . . . = 742 watts.
Pressure at alternator terminals . . . = 88 volts.

Angle of Phase.	Current through Arc (instantaneous value.)	P.D. of Carbons (instantaneous value.)	Angle of Phase.	Intensity of Light from Centre of Arc.	
0	+20·4	+58·6	10	40	
20	+15·9	+48·6	31	44	
40	+ 8·4	+26·4	52	9	
60	+ 0·7	− 7·6	76	12	Arc out.
80	− 3·6	−46	98	19	" "
100	− 8·1	−53·6	120	24	
120	−15·1	−63·6	142	72	
140	−18·6	−67·6	164	50	
160	−19·6	−68	187	63	
180	−17·6	−67	209	44	
200	−12·6	−53·6	232	12	Arc out.
220	− 5·6	−21	253	7	" "
240	− 0·6	+24	275	7	" "
260	+ 4·4	+47	297	34	
280	+ 8·4	+53	320	62	
300	+17·4	+55	343	87	
320	+21·4	+59·6	348	63	
340	+23·4	+59·6	326	78	
360	+20·4	+55·6			
340	+22·4	+59·6			
320	+21·4	+58·6			

Table V. B.

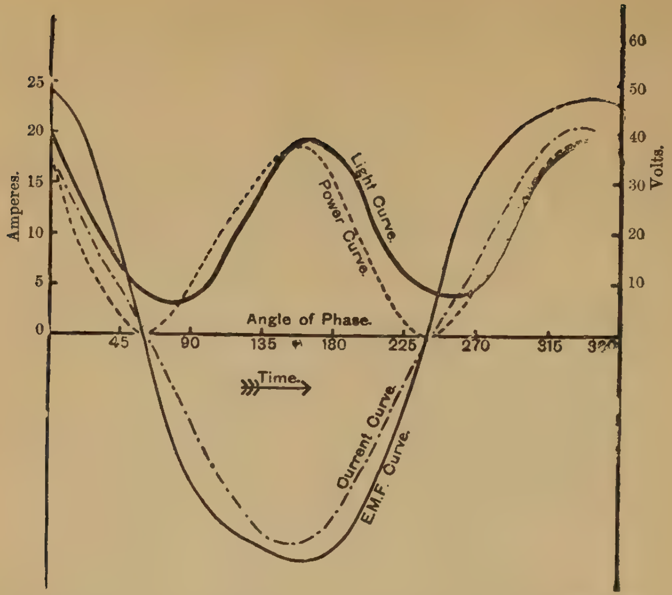
Observations on the Periodic Variation of the Intensity of the Light of an Alternating Current Arc.

Light from Crater. Long Arc.

Frequency . . . . = 50 ∼.
Length of arc . . . . = 1·2 cm.
Potential-difference (P.E.) of carbons (mean square value) . . . . . = 53 volts.
Current (mean square value) . . . . = 14 amperes.
Power expended in arc . . . . . . = 742 watts.
Pressure at alternator terminals . . = 93 volts.

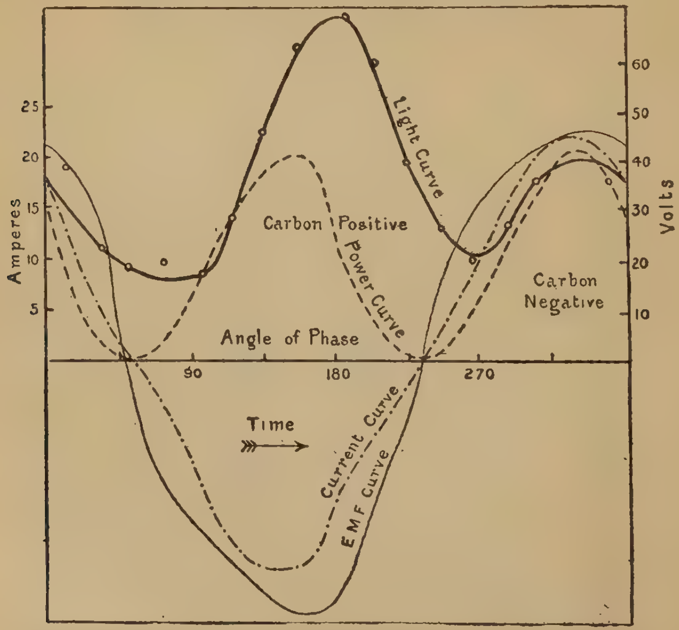
Angle of Phase.	Current through Arc (instantaneous value).	Angle of Phase.	Intensity of Light from Crater of lower Carbon.
0		359	92
20	+15·9	18	92
40	+11·7	36	98
60	+ 1·6	53	84
80	− 4·4	70	70
100		87	76
120		105	74
140		123	207
160		142	175
180		160	280
200		179	265
220	− 6·5	199	193
240	0·0	218	76
260	+ 4·4	238	84
280	+10·4	257	71
300		277	66
320		296	101
340		315	98
360		334	107
		340	120

Fig. 3.



The ordinates of the firm line marked "Light Curve" represent the periodic variation of illuminating-power of the centre of the *true arc* of an alternating-current arc. The other curves marked E.M.F., Current, and Power represent the variation of potential difference of the carbons, current, and power in the arc. Carbons, 15 mm. cored; ∼ = 83·3; amperes = 14; volts = 39; length of arc, 0·55 cm.; volts at alternator terminals = 104. (See Table I. A.)

Fig. 4.—Intensity of Light from Crater.



Frequency = 83·3 ∼. Length of arc = 0·42 cm. (See Table I. B.)

Fig. 5.—Intensity of Light from Arc.

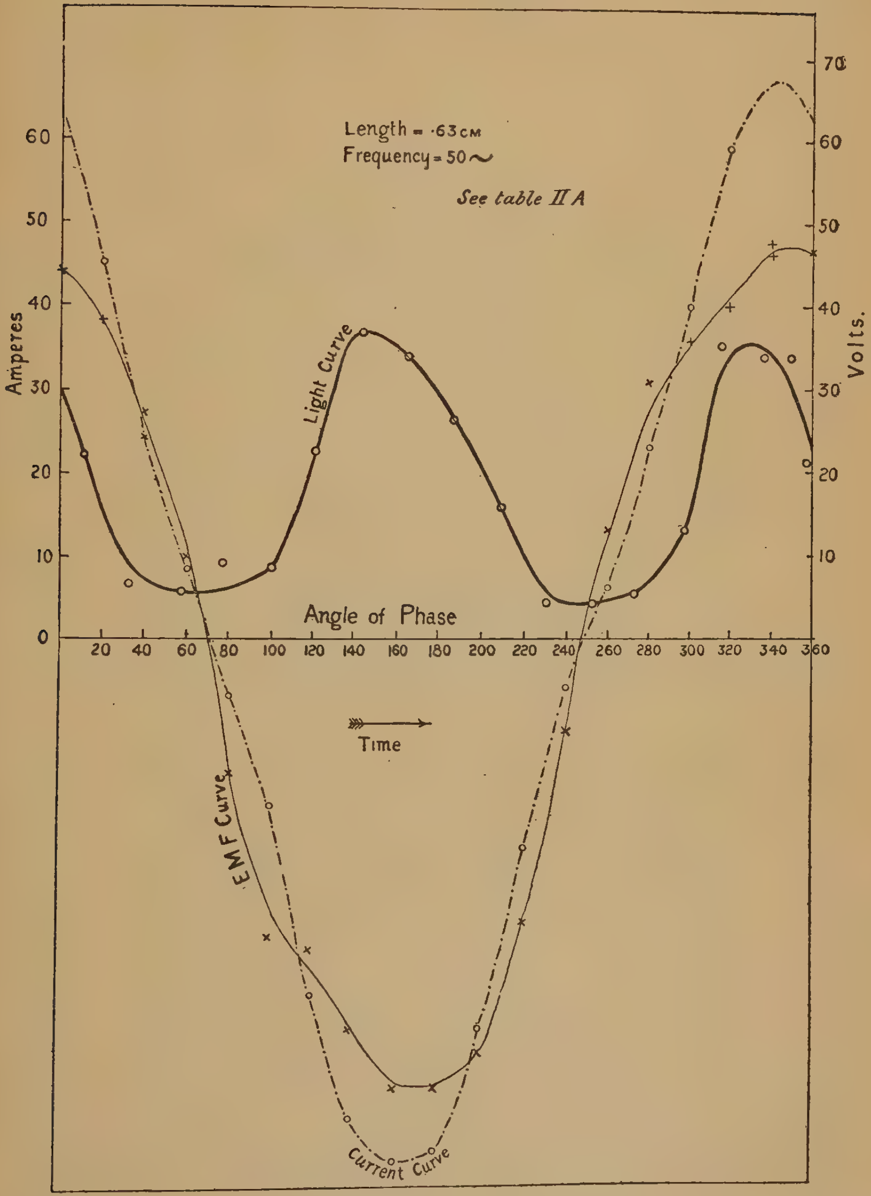
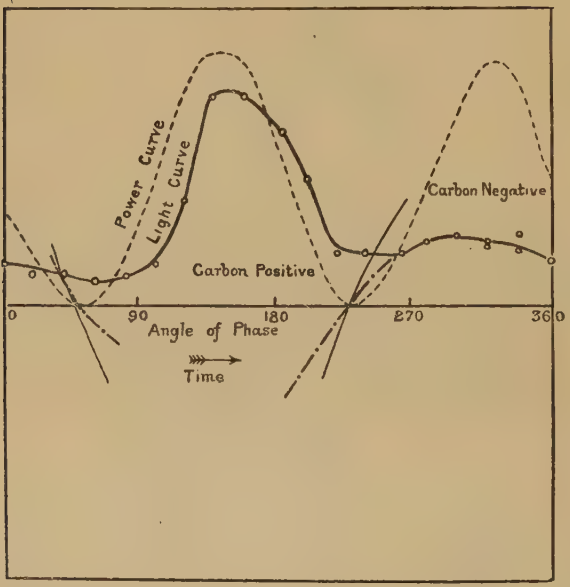


Fig. 6.—Intensity of Light from Crater.



Frequency = 50 ∼. Length of arc = 0·63 cm. (See Table II. B.)

Fig. 7.—Intensity of Light from Centre of Arc.

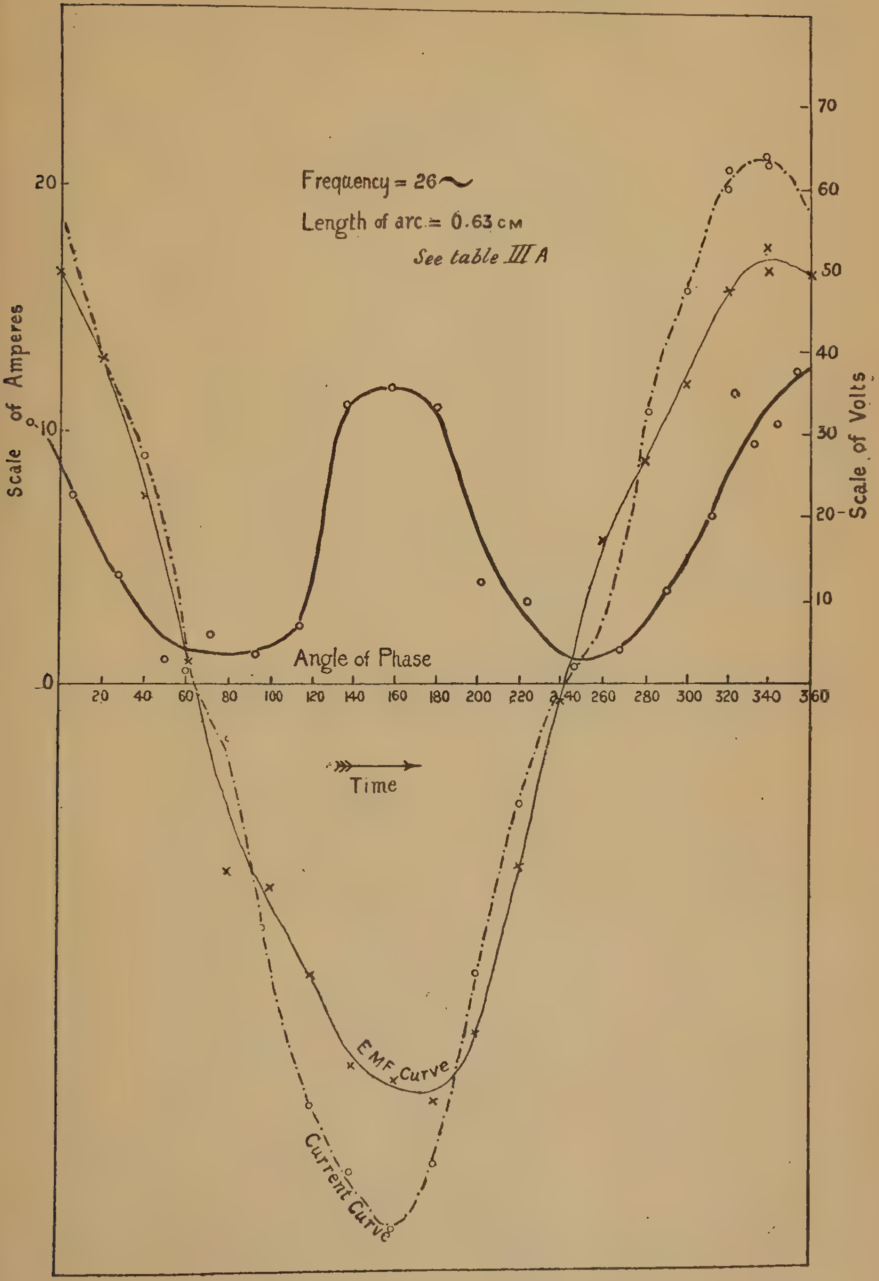


Fig. 8.—Intensity of Light from Lower Carbon.

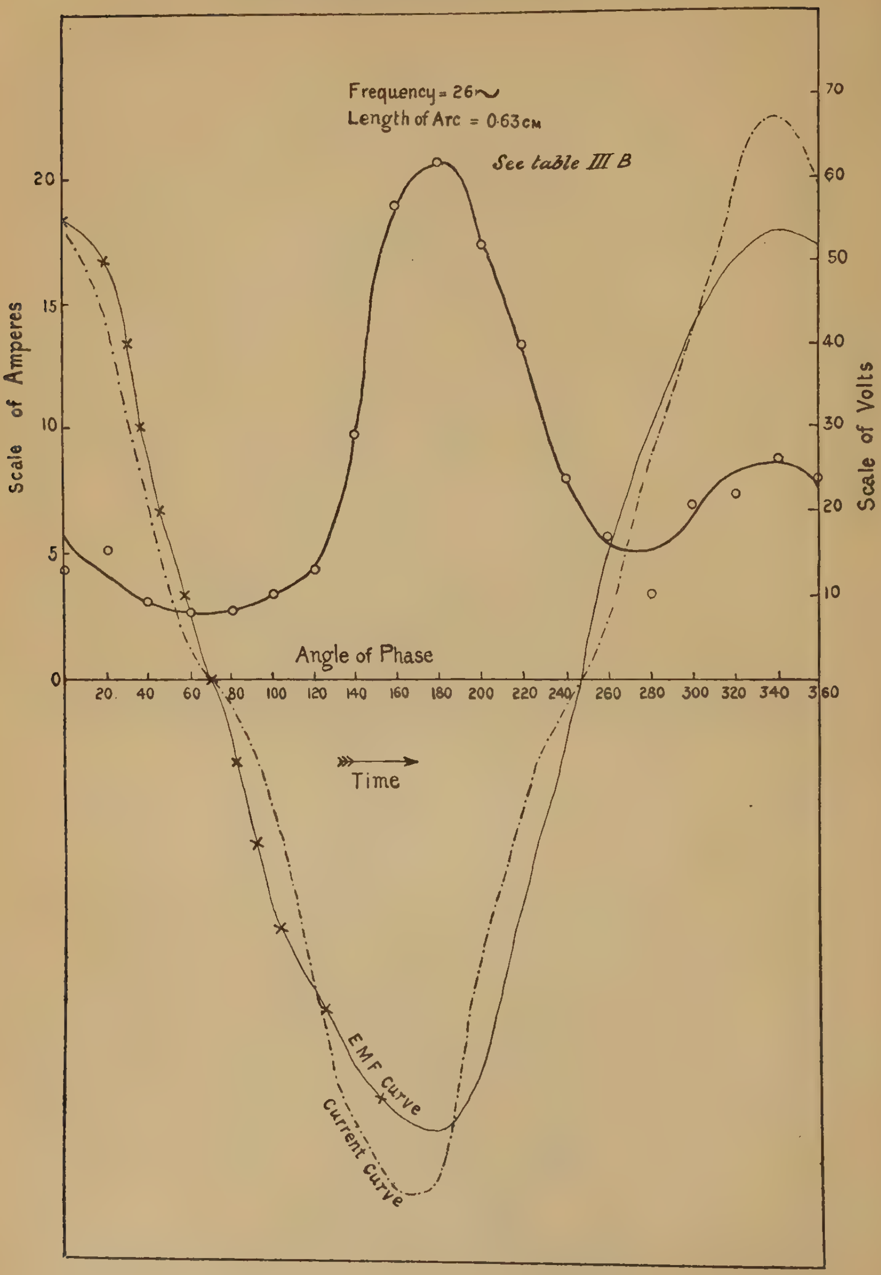


Fig. 9.—Intensity of Light from Centre of Arc.

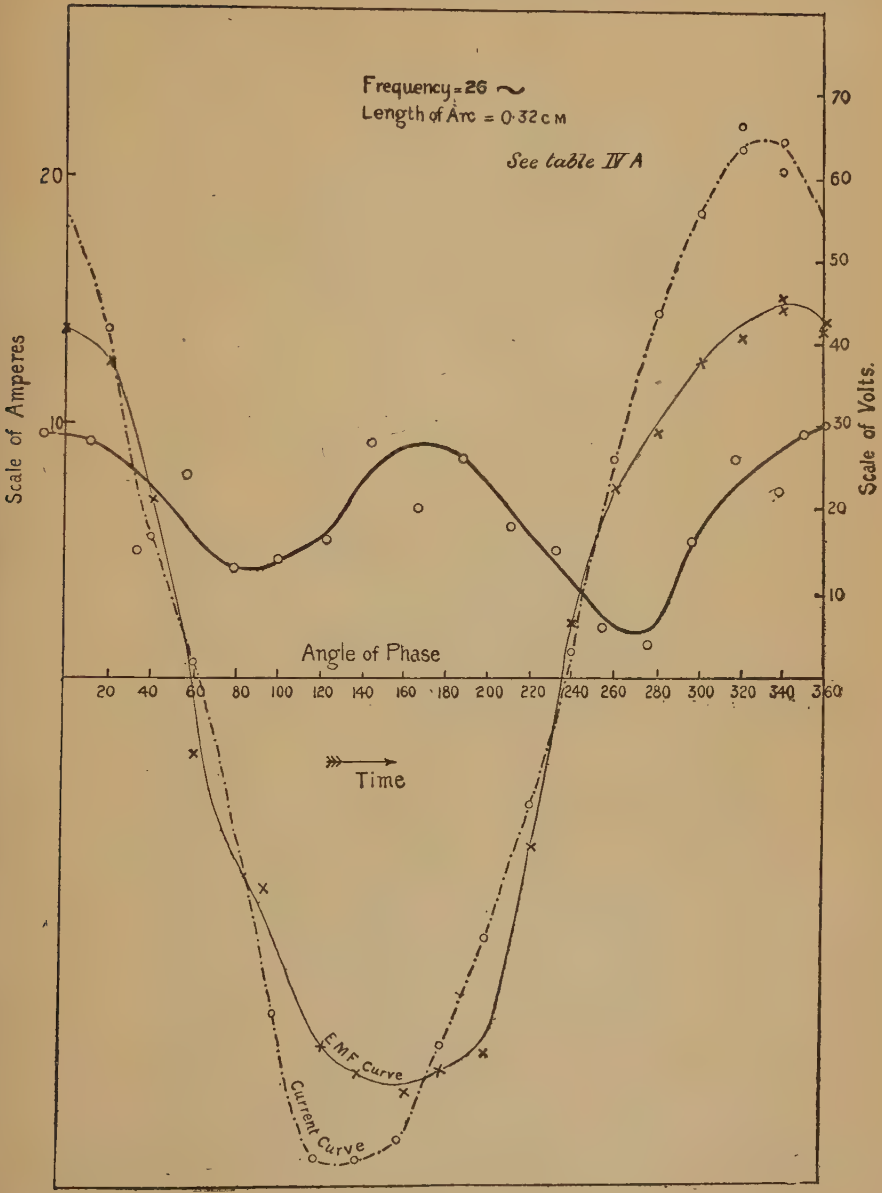
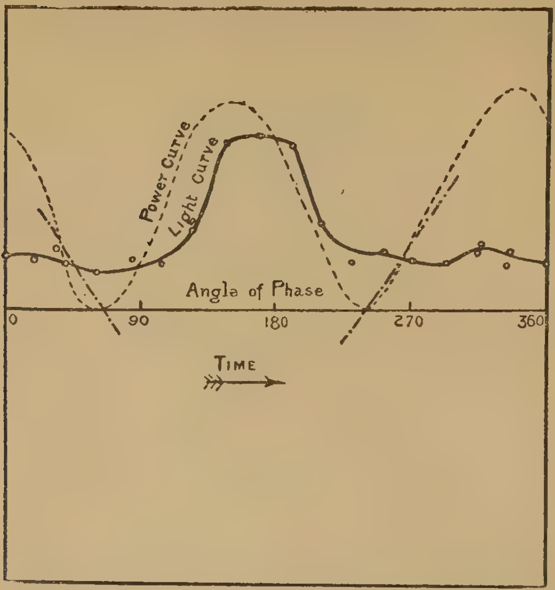
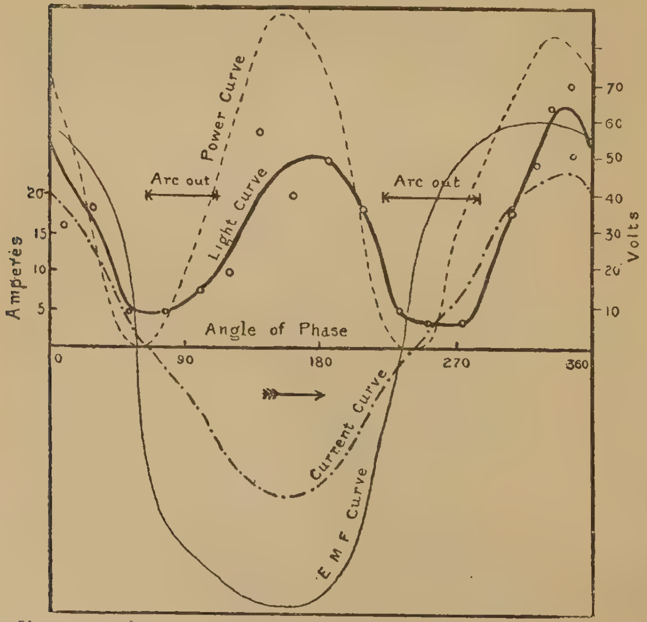


Fig. 10.—Intensity of Light from Centre of Arc.



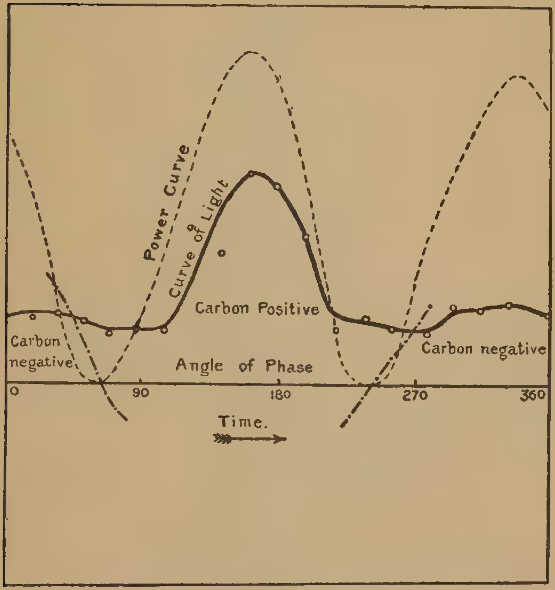
Frequency=50 ~. Length of arc=0·32 cm. (See Table IV. B.)

Fig. 11.—Intensity of Light from Centre of Arc.



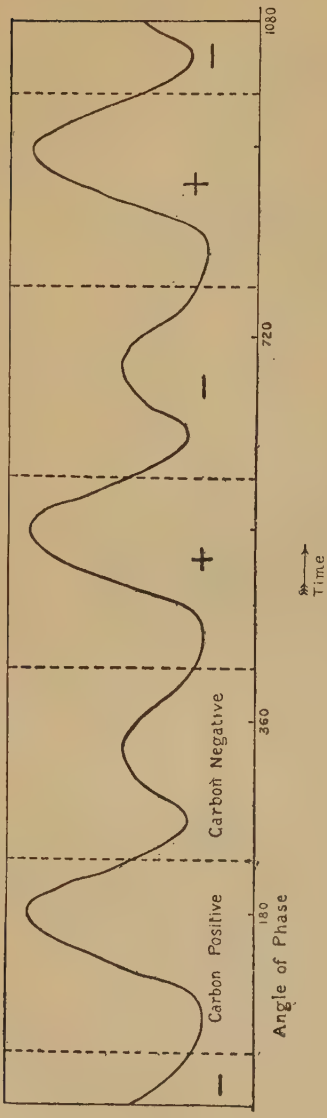
Frequency=50 ~ Length of arc=1·2 cm. (See Table V. A.)

Fig. 12.—Intensity of Light from Crater.



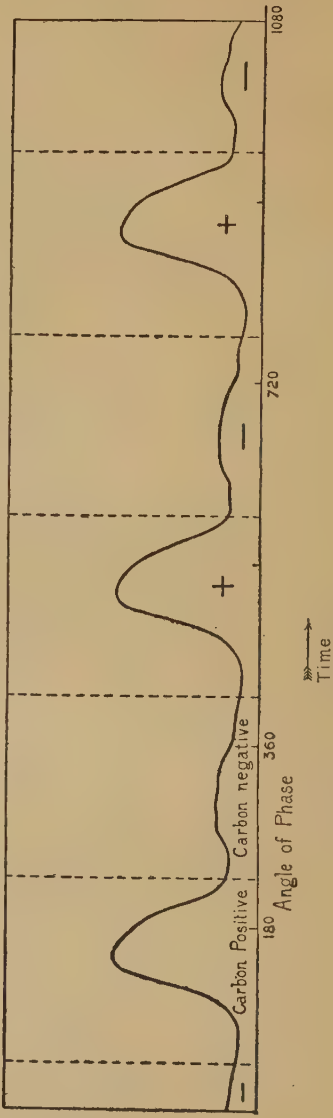
Frequency=50 ~. Length of arc=1·2 cm. (See Table V. B.)

Fig. 13.—Variation of Light from Crater of Alternating Current Arc.



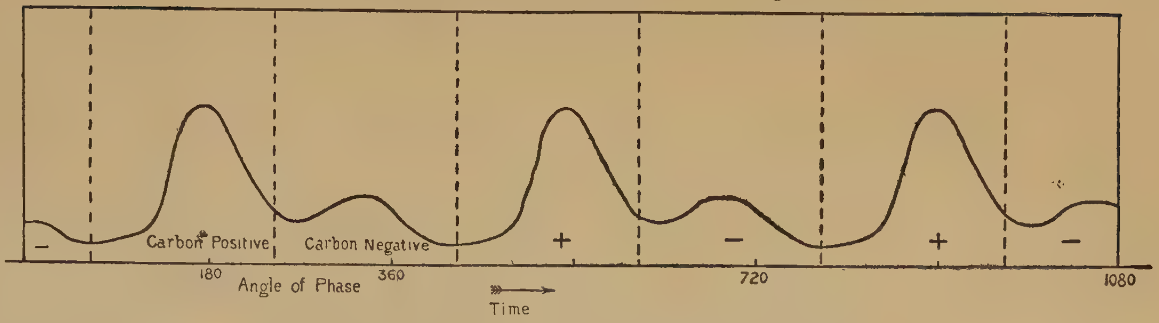
Frequency = 83·3. Length of Arc = 0·42 centim. (Table I. B.)

Fig. 14.—Variation of Light from Crater of Alternating Current Arc.



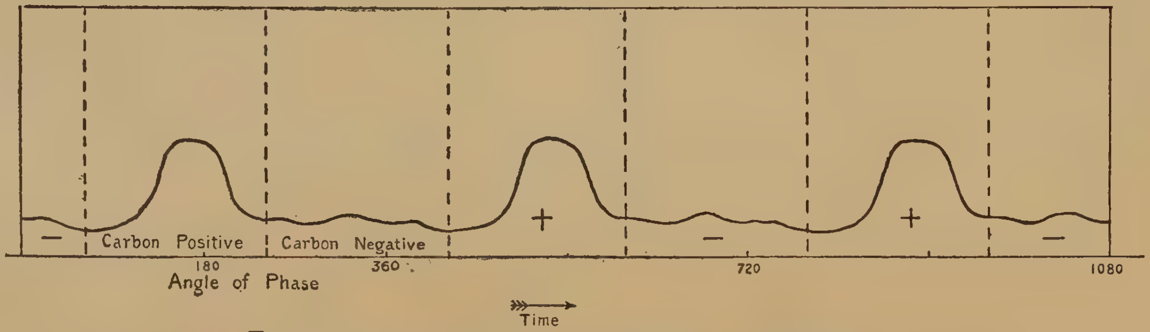
Frquency = 50. Length of Arc 0·63 centim. (Table II. B.)

Fig. 15.—Variation of Light from Crater of Alternating Current Arc.



Frequency = 26. Length of Arc = 0·63. (Table III. B.)

Fig. 16.—Variation of Light from Crater of Alternating Current Arc.



Frequency = 50. Length of Arc = 0·32 centim. (Table IV. B.)

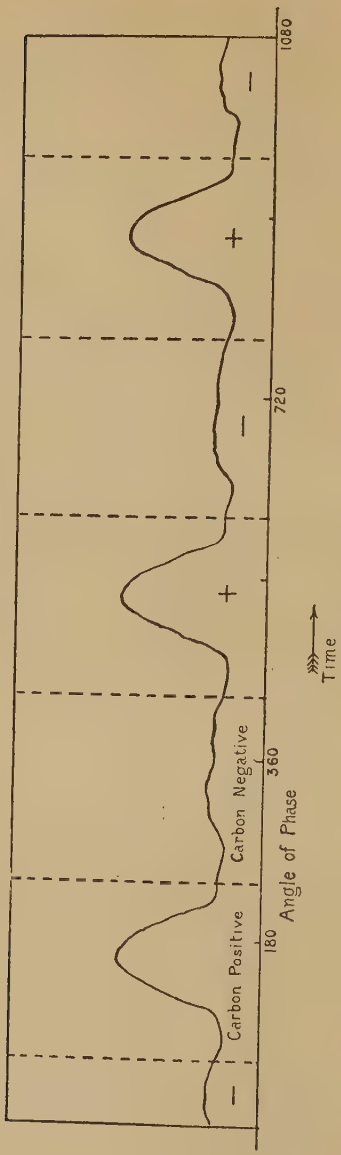


Fig. 17.—Variation of Light from Crater of Alternating Current Arc.

Frequency = 50. Length of Arc 1·2 centim. (Table V. B.)

Fig. 18.—Curves of Current, Potential Difference, Power, and Resistance of 360 Watt Alternating Current Arc.

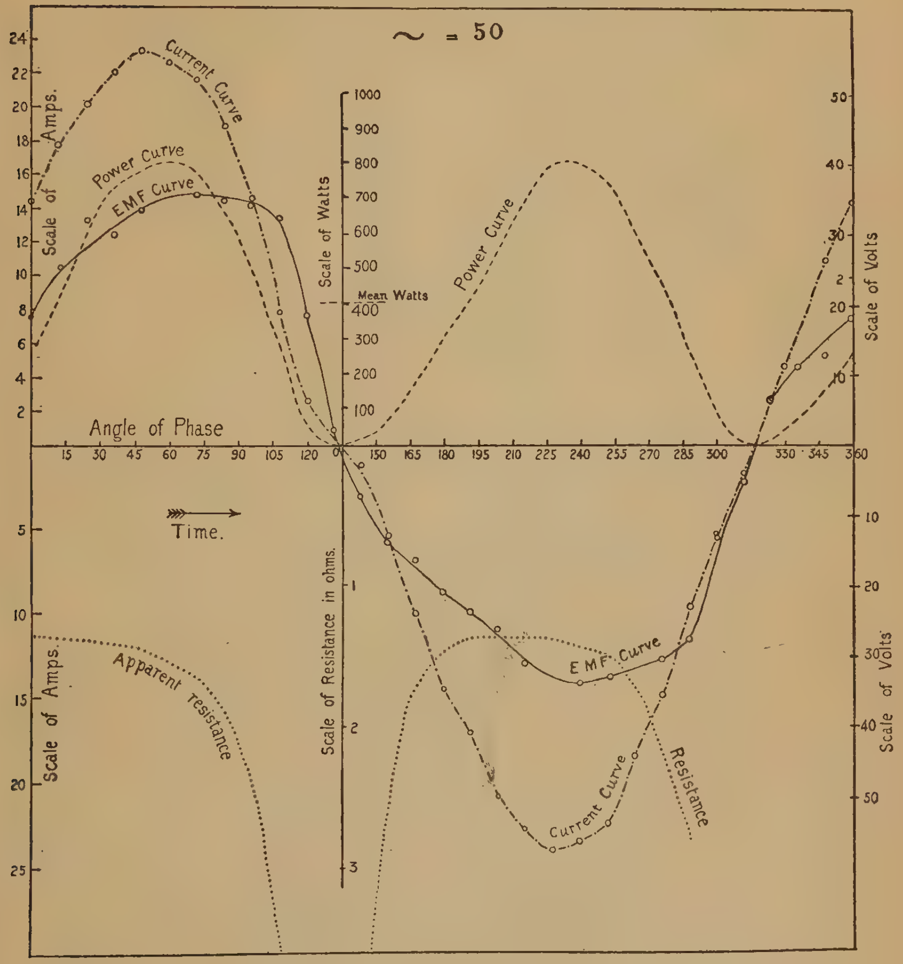


Fig. 19.—Curves of Current, Potential Difference, and Power of 500 Watt Alternating Current Arc.

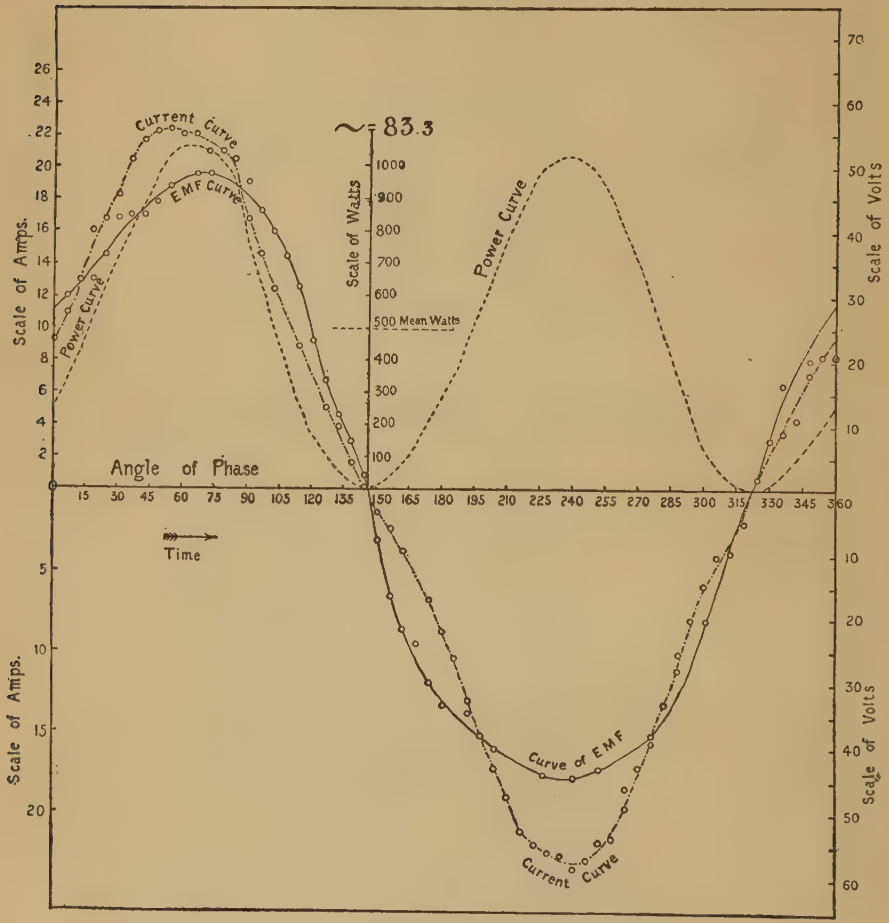
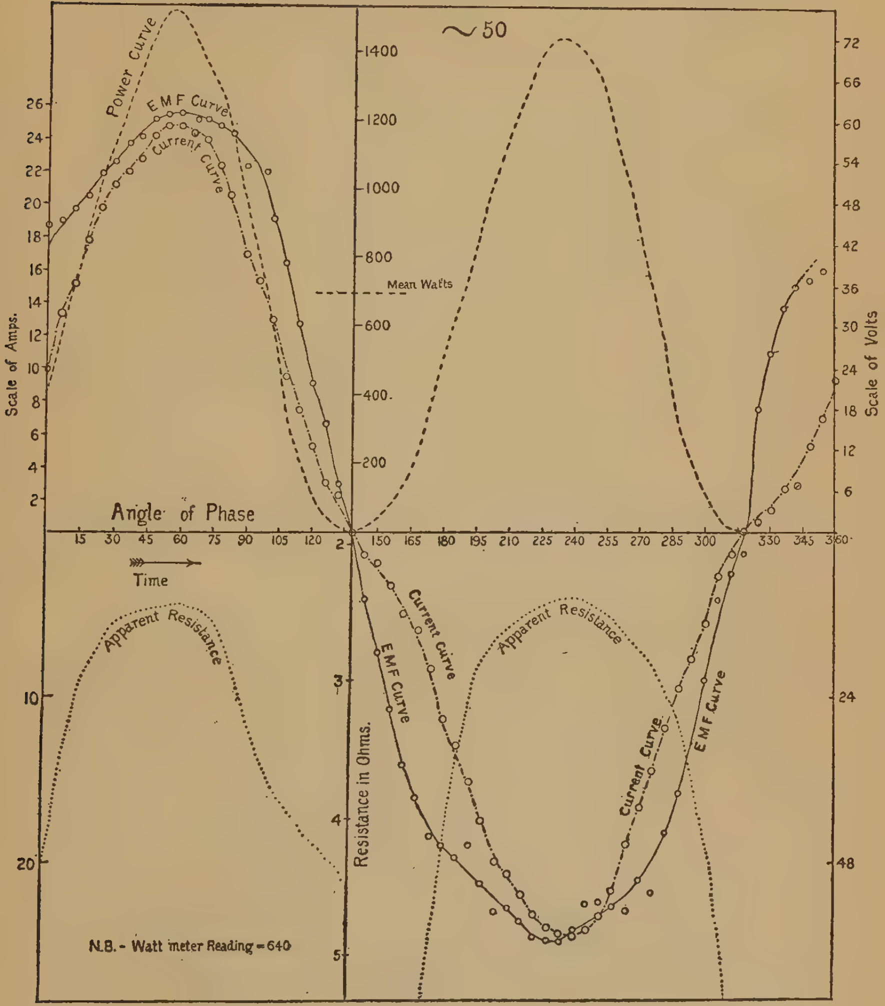


Fig. 20.—Curves of Current, Potential Difference, Power, and Resistance of 600 Watt Alternating Current Arc.



The second portion of the experimental work here recorded consisted in an investigation of the efficiency of the alternating-current arc as a light-giving agent, when compared with the continuous-current arc taking up the same mean power. The question in all its generality, whether alternating-current arcs are less efficient as light-giving agents for a given amount of power expended in them than continuous-current arcs absorbing the same power, is not one which we have attempted to settle, for the simple reason that a far more extended series of experiments would be necessary before it would be possible to say whether an alternating-current arc can or cannot be made to give the same mean spherical candle-power as a continuous-current arc absorbing the same mean power. Limitations of time compelled us to reduce our investigation to one definite problem. Taking alternating-current arcs, formed with the length, voltage, and the current and the carbons as described below, and taking continuous-current arcs also with the length, current, and voltage and carbons most usually employed in practice, we have investigated the relative magnitude of the mean spherical candle-power produced by these arcs for equal expenditure of powers in the arcs varying from 200 to 600 watts. This, it will be observed, is an investigation which has nothing whatever to do with the relative mechanical or electrical efficiency or excellence of the arc-lamp mechanisms, but it is purely a physical measurement made of the two arcs themselves under the conditions which are found to obtain in practice. In order to settle this question, an alternating-current arc lamp was taken with good regulating qualities ; it was furnished with cored carbons 15 mm. in diameter, and being placed under the conditions above described, in which the power supplied to the arc could be regulated and measured, a series of observations was made of the mean spherical candle-power of the arc for different powers expended in the arc and for two frequencies of 83·3 and 50 ∼. A similar set of experiments was made with the continuous-current arc, using in one case the same carbons—15 mm. carbons, both cored—and in the other case the positive carbon 15 mm. cored and a negative carbon 9 mm. solid. These sizes were chosen because the 15 mm. cored and 9 mm. solid sizes are those that are frequently employed in continuous-current arc lighting.

The arrangements for obtaining the mean spherical candle-power consisted of a mirror—the coefficient of reflexion of which had been determined—which was employed to reflect the light from the arc proceeding in different directions to the horizon into the photometer, the ray always falling upon the mirror at an incidence of 45°. The standard of comparison was an incandescent lamp, which was worked at rather a high temperature in order to diminish the difficulty of making the photometric comparison by diminishing as much as possible the colour-difference in the lights compared. This incandescent lamp was kept constantly standardized against another standard incandescent lamp worked at a rather lower temperature. The means of comparison was a Sugg's Star Disk Photometer. It was found that by focussing the eye to a point rather nearer the eye than the images of the Star Disk as seen in the two mirrors, the difficulty of discriminating between a small difference in the brightness of the two images in spite of a small colour-difference was to a great extent diminished. We abandoned as perfectly useless any comparison of the two lights in terms of red and green candle-power. By the employment of the reflecting wattmeter and carbon resistance as above described, it was perfectly possible to keep the power expended in the arc constant to a certain number of watts throughout long periods. In the case of the alternating-current arc lamp experiments, the alternating-current arc lamp was turned round its vertical axis so as to take a series of observations quickly in different directions, but at the same angle to the horizon, and the mean of these observations was taken as the effective illuminating-power in any angular direction; at the same time, the current through the arc and the potential-difference of the carbons were observed. These observations having been taken, involving many hundreds of photometric measurements, the results were set out in a series of photometric diagrams, as shown in figs. 21 to 24, which delineate the respective form of the photometric curves for the two arcs and for different wattages expended in the arc. These photometric diagrams were then integrated, and the mean spherical candle-power calculated in the usual way by means of a Rousseau's diagram, and finally the results embodied in one complete table and diagram, as given below (pp. 160, 161).

Fig. 21.—Photometric Curves of Continuous Current Arc.
Both carbons 15 mm. cored.

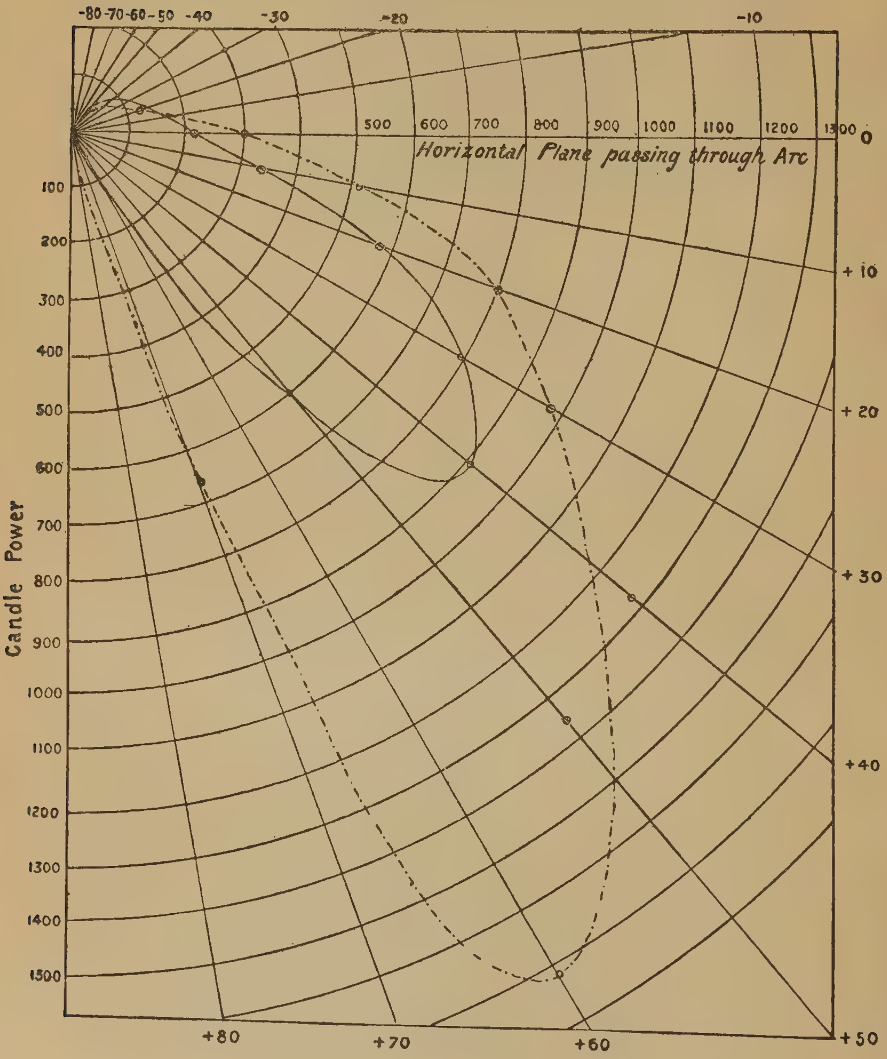


Fig. 22.—Photometric Curves of Continuous Current Arc.

Negative carbon 9 mm. uncored.
Positive carbon 15 mm. cored.

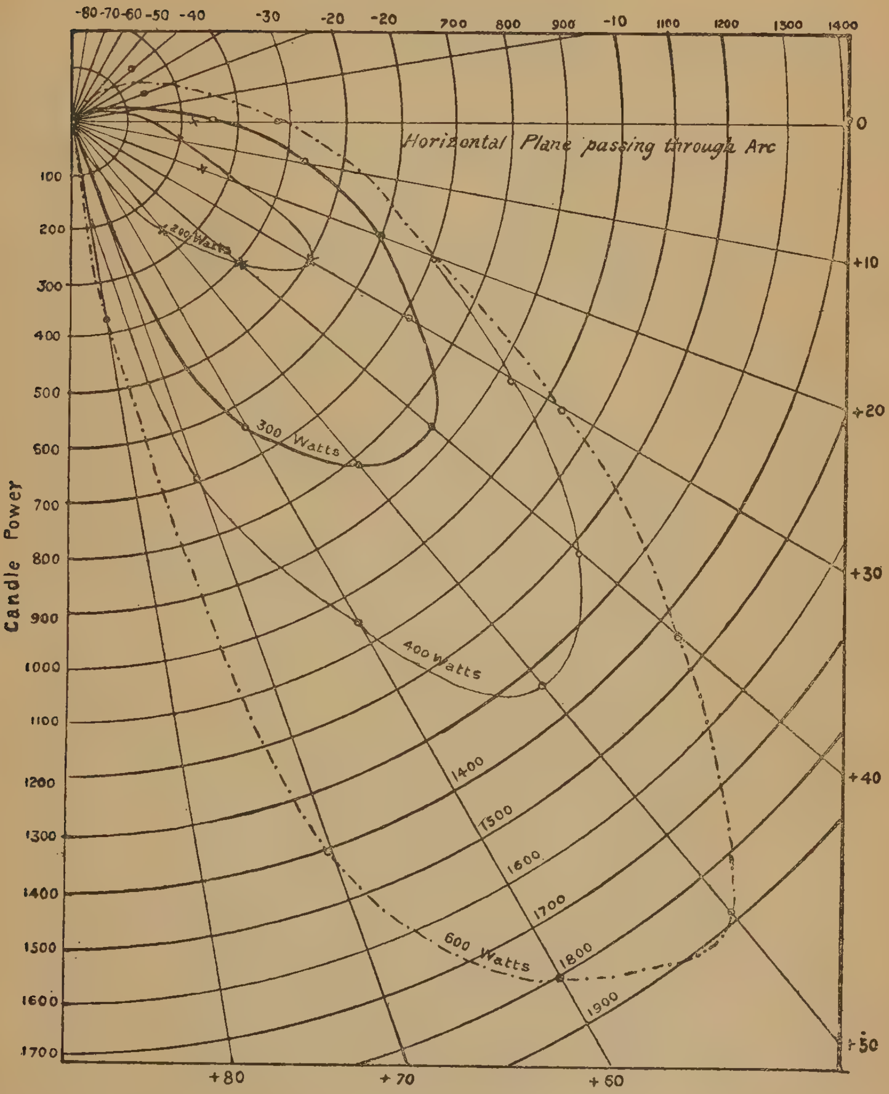


Fig. 23.--Photometric Curves of Alternating Arc Lamp ∼ 83·3.

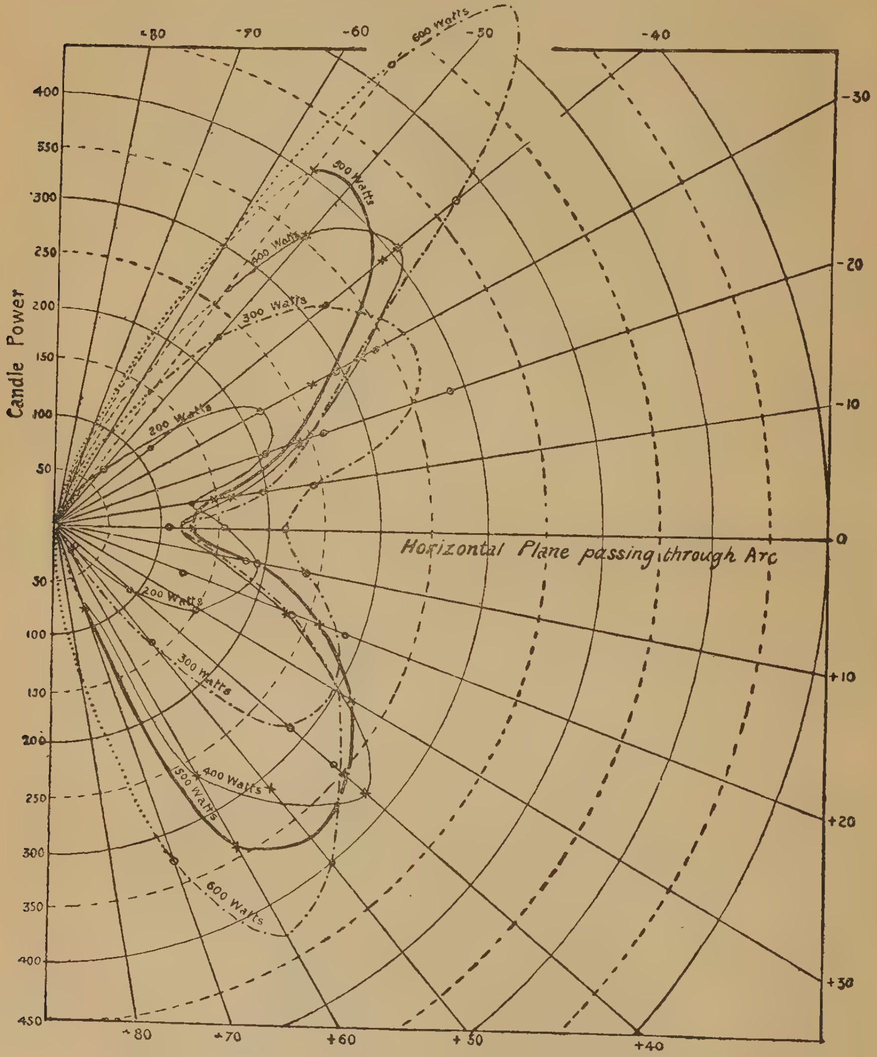
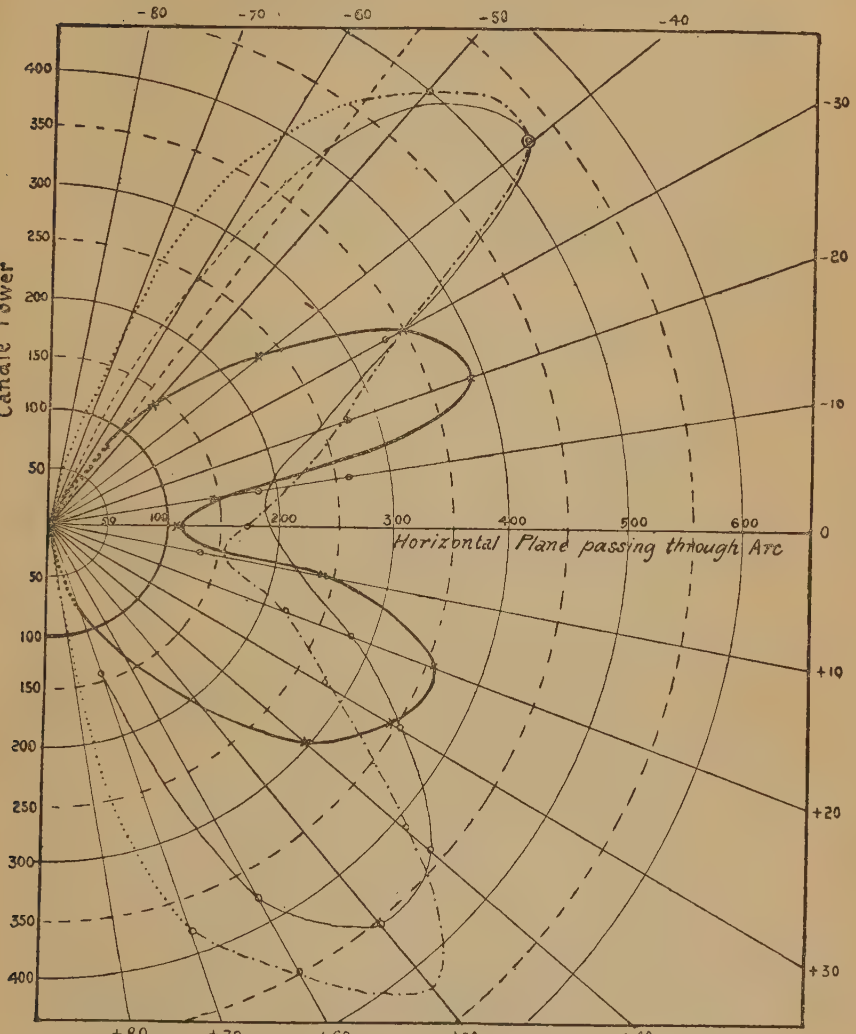


Fig. 24.—Photometric Curves of Alternating Arc Lamp ∼ 50.



	Continuous-Current Arc.						Alternating-Current Arc. Both carbons 15 mm. cored.							
	+Carbon 15 mm. cored. −Carbon 9 mm. solid.				Both carbons 15 mm. cored.		Frequency 83·3 ~.					Frequency 50~.		
Power expended in the arc in watts	582	380	292	*215	607	313	601	501	404	305	*233	596	459	308
Mean spherical candle-power	675	455	372	181	582	344	307	274	256	250	144	326	322	254
Potential difference of carbons in volts	56	44	36	26	60	40	37·5	34	28	21	15·3	39	31	22
Current through arc in amperes	10·6	8·7	8·1	8·2	10·2	8	16·2	15·1	15·1	15·5	15·4	16	15·1	14·6
Length of arc in centimetres	0·715	1·0	0·07	0	0·807	0·061	0·628	..	0·125	0·016	0·01	0·7	..	..
Power factor	1·0	0·25	1·0	1·0	1·0	1·0	..	..	0·96	0·96	0·99	0·97	0·99	0·97

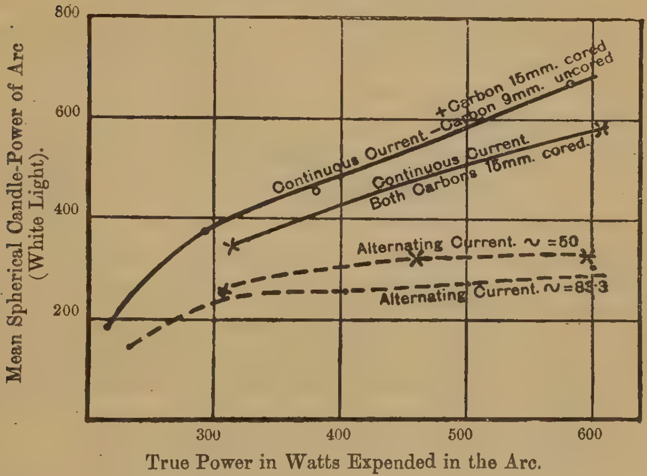
* In these observations the arc was hissing during the whole of the time.

In reading the above table it must be noted that it was the power, measured in watts, taken up in the arc which was kept constant. The current and difference of potential of the carbons were controlled so as to secure this result. The current and volts were approximately constant, and had the mean values given in the table, but the product of these mean values does not quite agree with the true mean power measured in watts taken up in the arc.

The results in the Table are graphically embodied in the diagram in fig. 25, from which it will be seen that, taking the alternating-current arc as employed, the total mean spherical candle-power is always considerably less than that of a continuous-current arc, taking the same mean power. Lowering the frequency seems to increase the efficiency of the alternating-current arc, as one might naturally assume it would do, and it is obvious that increasing the diameter of the lower carbon of the continuous-current arc would diminish its total mean spherical candle-power at any given wattage. This table and diagram therefore, we think, settle the question that for a given expenditure of power in the arc a greater amount of mean spherical illumination can be obtained, if that power is supplied in a continuous-current form than if it is supplied in an alternating form. It has been suggested that, with proper carbons and under proper conditions, the alternating and continuous-current arc should give the same mean spherical candle-power for the same expenditure of power, as has been shown to hold true in the case of the incandescent lamp by the investigations of Profs. Ayrton and Perry. We think, however, that no *à priori* reasoning can apply in the case of the arc. It is perfectly clear that, owing to the

Fig. 25.

(From 'Electrician,' Dec. 20, 1895, p. 247.)



interval of cooling that elapses, the mean temperature of the two carbons in the case of the alternating-current arc must be less than the temperature of the positive crater of the continuous-current arc, and that therefore the result we have obtained is only what might be expected. If the question is asked, how do we account for the difference in efficiency? the answer must be that the energy absorbed in the case of the alternating-current arc is radiated at a lower temperature, and the two arcs were therefore exactly in the same condition as regards comparison that two incandescent lamps would be, both taking the same total power but worked at different temperatures, and therefore different watts per candle, and therefore giving different mean spherical candle-powers with the same total power expended in them.

It may be observed in all cases the alternating-current arcs we employed in our experiments took 16 to 17 amperes, and the power was varied by varying the potential-difference of the carbons, and the alternating-current arc lamp used was one which effected this variation automatically, even although the power expended in the arc was varied from 200 to 600 watts. In order to complete the comparison of the continuous and alternating-current arcs, it will be necessary to compare the behaviour as regards illuminating-power of alternating-current arcs, taking the same mean power but formed with larger currents and less carbon potential-differences; that is to say, comparing alternating arcs of equal power-absorption, but taking very different currents and therefore having different lengths. We hope to extend this investigation to cover these additional questions at some future time.

The above observations have necessitated an enormous number of photometric and electrical measurements, and we have in the above work been very efficiently aided by Messrs. L. Birks, W. H. Grimsdale, A. M. Hanbury, E. N. Griffiths, and others, to whom our thanks are due.

Discussion.

Prof. Ayrton said the behaviour of the alternate-current arc was of great interest, for the power supplied could not be measured by simply multiplying the current by the electromotive force, since the current lags behind the volts.

The resistance, *i. e.* the ratio of the current to the E.M.F., also lags, but the authors do not appear to have made any attempt to measure the *true* resistance. The authors were to be congratulated on the guarded tone they had adopted as to the bearing of these experiments on the question of the relative efficiency of the alternating and continuous current arcs. In a previous communication, one of the authors had stated that the alternating-current arc must necessarily be a less efficient light-producing agent than the continuous-current arc. Although the last set of curves given in the paper might appear to support this supposition, he (Prof. Ayrton) felt that the difference obtained was probably due to the fact that the alternating-current arc was not being worked under proper conditions. The quality of the carbons and the length of the arc have a most important influence on the efficiency of an arc. At present, our knowledge is not sufficient to allow of our stating definitely whether or not an alternating-current arc can be made as efficient as a direct-current arc, but there is no doubt that it will be possible to get much better results than are at present attainable.

Prof. S. P. THOMPSON said that when the fact of the existence of the difference in phase between the current and volts in an alternating arc was first published, he made some experiments which showed that there was a lag, and not a lead, *i. e.*, that the arc acted as if it possessed self-induction. The resistance also lagged, and he thought this lag might be due to a thermal lag. The temperature of the arc will lag behind current, both when it is increasing and when it is decreasing; and if the resistance of the arc depends on the temperature of the vapour in the arc, then the resistance would also lag behind the current. It was not possible from *a priori* reasoning to say whether or no an alternating-current arc could ever be obtained of an efficiency equal to that of the direct-current arc. With suitable carbons, length of arc, current, and volts, it seemed to him that it might be possible to obtain an equal efficiency. The light-giving process in an arc is not merely an irreversible degradation of electric energy into heat; for the difference of potential between the carbons may be written $V = a + Cbl$; where a may be regarded as a back electromotive force, and bl as a true resistance. The first term of this expression does not vary with the length of the arc (l), but the second term does. Multiplying through by the current (C), the equation

$$\text{Watts expended} = \mathrm{C}a + \mathrm{C}^2bl$$

is obtained. It is the first of the terms on the right-hand side, which is a reversible effect, and corresponds to the power expended in driving the current against a back electromotive force, on which the light given out chiefly depends, due to something occurring at the crater surface.

Mr. Blakesley asked whether Prof. Thompson's idea of the existence of a reversible part in the process was not a strong argument in favour of the direct-current arc.

Prof. Ayrton said that in two communications made to the Congress held at Chicago it was shown that even with direct-current arcs there was a certain length of arc for which the efficiency was a maximum. Mrs. Ayrton had quite recently found that the efficiency of arc-lamp carbons altered with time. Prof. Thompson's suggestion as to a thermic lag was a valuable one.

Prof. Fleming, in his reply, said that when comparing two agents where there were so many variables, it was practically necessary to restrict the investigation. In their case they had kept the mean power constant and had left the other variables to take care of themselves.

XII. *On some Experiments with Incandescent Lamps.*

By Sir D. Salomons.*

[Abstract.]

35 A large electro-magnet is excited by means of a continuous current, while an alternating current is passed through an incandescent lamp. On bringing the lamp near the magnet the filament is set in vibration, which, if the lamp is brought sufficiently near the magnet, is sufficiently intense to break the filament. The number and position of the nodes formed

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* Read February 28, 1896.

in the vibrating filament are found to be independent of the natural period of the filament, but depend on the frequency of the alternating current.

Discussion.

Prof. S. P. Thompson asked whether it was not found that the number of segments into which the filament is divided depend to some extent on the natural period of the filament.

Prof. Ayrton said that the magnetic leakage was very large with the arrangement adopted, and he would like to know whether this method was a more sensitive one for mapping out the field than those ordinarily employed. In an instrument designed by Prof. Perry and himself, an alternating current was passed through a wire stretched between the poles of a magnet, and the tension of the wire was altered till the vibrations set up were a maximum. The natural period of the wire, and hence the frequency of the alternating current, was then deduced from the tension, &c. In Prof. Ewing's magnetic-curve tracer, on the other hand, the natural period of the stretched wire was made as different as possible from the period of the alternations which were to be observed, so that the natural vibrations of the wire did not influence the result. The author's arrangement appeared to him (Prof. Ayrton) to be intermediate between these two, and it would seem that the natural period of the filament would, to a certain extent, influence these results.

Prof. Perry suggested that the lamp might be held in a very steady support, so that after the large vibrations due to the natural period of the filament had died out, the vibrations of the period of the alternations alone could be observed.

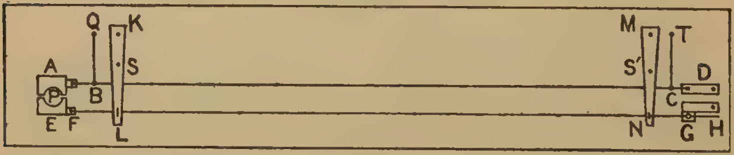
Sir D. Salomons, in his reply, said that the arrangement was not intended for making *measurements*. A lamp had been fixed in a steady clamp and the current passed for many hours, but the character of the vibrations remained unaltered. He had found the vibrating filament useful for microscopic work where any surface rather than a line of light was required.

XIII. *An Addition to the Wheatstone Bridge for the Determination of Low Resistances. By* J. H. Reeves, *M.A., City and Guilds of London Central Technical College**.

While it can be assumed with fair certainty that even in moderately equipped laboratories there will be found one sensitive galvanometer and a good set of resistance-coils arranged in the form of a Wheatstone bridge, it is by no means so common to find a convenient method for determining low resistances, such as, for example, a Kelvin bridge. The piece of apparatus which forms the subject of this paper is a comparatively cheap addition to the ordinary bridge, and enables the resistances of exact metre-lengths of even thick wires to be directly measured with almost as much ease as larger resistances can be determined with the ordinary Wheatstone bridge. The method also possesses the advantage that all the measurements are made in terms of a standard wire with fixed contacts, and, therefore, not subject to the wear which accompanies the frequent use of a slider; further, in the case of copper wires no temperature-correction is needed.

The apparatus is represented in fig. 1. On a mahogany baseboard are stretched close to one another two wires—one, ABCD, being the standard of comparison, while the other, EFGH, is the wire to be tested.

Fig. 1.



A and E are two massive pieces of brass which can be joined together by a plug P. D and H are smaller pieces of brass with binding-screws attached.

The standard BC is permanently fixed to A and D, whilst two clamps F and G fixed respectively to E and H form the means of fixing in its place the wire to be tested.

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* Read March 13, 1896.

KL and NM are two brass springs which pass over but do not touch BC. They are provided with binding-screws at K and M, and with two knife-edges L, N exactly one metre apart, which press on the wire FG when in position, while two screws S, S′ will raise the springs when required for inserting or removing this wire.

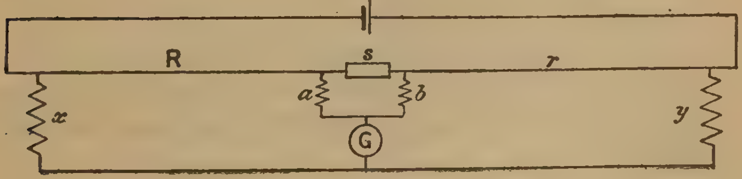
At points B, C in the standard are soldered two short lengths of wire terminating in binding-screws Q and T. The gauge of the standard wire, which is of copper, and the distance apart of B and C are so chosen that the resistance between B and C is 0·01 ohm at the temperature of the wire at the time of its final adjustment, which should be noted. The method of determining these points is given later on.

The arrangement is a variation of the Kelvin bridge, with this difference: that in the latter the measurements are made by varying the length between the knife-edges of sliders which press on the standards, of which several are required: whilst in the former only one standard resistance is employed, and the measurements are effected by the alteration of the other resistances in the arrangement. If the whole apparatus had to be bought, the Kelvin bridge would be the cheaper, but the new Addition utilizes the box of coils, which as beforementioned is sure to be available, and only requires in addition one standard to be made and adjusted.

The theory of the Kelvin bridge is to be found in the textbooks. In Gray's 'Absolute Measurements,' for example, vol. i. p. 359, it is proved that if fig. 2 represents the usual arrangement, and, if R, r, x, y, a, b, s are as marked, the following relationship must hold in order to obtain balance:—

$$\frac{R}{r} = \frac{x}{y} = \frac{a}{b}.$$

Fig. 2.



Further, it is shown that when s is small compared with a and b the accuracy of the equation

$$\frac{R}{r} = \frac{x}{y}$$

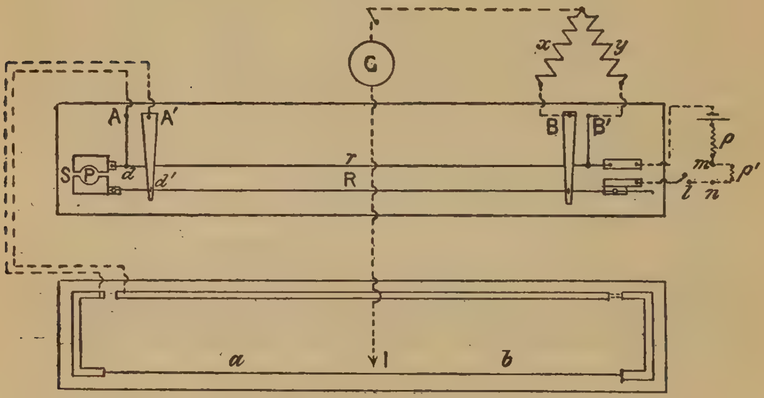
is not affected by a small want of equality between the ratios

$$\frac{a}{b} \text{ and } \frac{x}{y}.$$

In addition to the galvanometer and set of coils there is required a slide-wire bridge, which may be of a rough description but should have a resistance of an ohm or so. At the end of the paper it is shown how this latter and the Addition can be united in one.

The whole arrangement is joined together as shown in fig. 3, the dotted lines indicating the temporary connexions. By comparing this with figs. 1 and 2, no detailed description is necessary, as R, r, x, y, a, b, s are lettered to correspond.

Fig. 3.



The battery may with advantage be a storage-cell with two resistances in series with it. One, ρ, may be an adjustable carbon resistance, while ρ' may be constructed of wire and should have a considerable resistance. This latter terminates in mercury cups m and n, so that by joining these cups together with a short connector the resistance can be easily short-circuited.

The resistances x and y are resistances unplugged from the box of coils forming the ordinary Wheatstone bridge; y is the 1000-ohm coil of one of the ratio arms, and x is in the adjustable arm. If such a set of coils be not available, y may be any 1000-ohm coil, and x any box of coils containing resistances up to 5000 ohms. The two leads joining x and y to B and B′ respectively should be as stout and short as possible. The resistances from d, d' to I form the $a\ b$ of fig. 2.

To make a measurement, insert the wire whose resistance is required in its proper clamps, taking care that it lies quite straight between them. Its diameter and material being known, the resistance of one metre of it can be approximately calculated. Calling the resistance R_1, choose x_1 in the box so that

$$\frac{x_1}{1000} = \frac{R_1}{0{\cdot}01}.$$

Remove the plug P, and, including ρ' in the battery circuit, obtain balance by shifting the slider I, the galvanometer being also shunted if necessary. The arrangement is now an ordinary Wheatstone bridge,

$$\therefore \quad \frac{a + R_1}{b + r} = \frac{x_1}{y} = \frac{R_1}{r},$$

or

$$\frac{a}{b} = \frac{R_1}{r} = \frac{R}{r} \text{ approximately.}$$

Next insert the plug P, short-circuit ρ' so that now a strong current passes when the battery circuit is closed, and obtain balance by altering x. Let the new value be x_2. Then, since approximately $\frac{a}{b} = \frac{R}{r}$, we have, by the formula quoted above,

$$\frac{R}{r} = \frac{x_2}{y},$$

i. e.,

$$R = \frac{x_2}{y} \times r \text{ very approximately.}$$

In any case this new value of R is very much nearer the true value than the approximate value R_1. If the value of x has to be but little altered, it may be taken as the true value of R, but if, on the other hand, the first estimate was considerably in error, and, in consequence, a large alteration has to be made in x in order to obtain balance, the much closer value thus obtained must be taken as the first approximation, and both experiments repeated.

Now, since the wires are so close to one another, their temperature can be called the same, and, if the tested wire be of copper, their temperature-corrections are the same. Therefore, if R be a certain fraction of r at one temperature, it will be the same fraction at any other. But in the actual bridge on which the experiments described later on were first made, this temperature was 17°·7 C. Therefore, substituting numerical values in the last equation,

$$R = \frac{x}{1000} \times \cdot 01 = \frac{x}{100{,}000} \text{ ohms at } 17^\circ\!\cdot\!7 \text{ C.}$$

Of course, if R be not made of copper, the temperature must be noted and allowed for.

The accuracy and sensibility of this method depend on three conditions :—

(1) The accuracy of the ratio $\frac{x}{y}$;
(2) The accuracy of the value of r;
(3) The sensibility of the galvanometer.

As far as condition (1) is concerned, x and y being taken from a good box have an error of certainly less than 0·1 per cent.

Condition (2) is discussed later, and the effect of condition (3) can be best seen by reference to tests made in the laboratory of the Central Technical College with the apparatus arranged as in fig. 3, except that, at the start, the battery consisted of a single Daniell cell, the galvanometer being a four-coil reflecting one having a resistance of 687 ohms and a sensibility of about 400 scale-divisions per microampere.

Table I.

Tests made on high conductivity copper, $R = \frac{x}{100,000}$ ohms.

No. of wire. S.W.G.	Diameter. mils.	x.	Observation.	R.	Specific Gravity.	Specific Resistance.	Conductivity. Matthiessen's Standard.
22	28·07	4324	½ div. deflexion left.	0·04325	8·841	1·616	100·3
		4326	Small ,, right.				
		4328	1½ div. ,, right.				
20	35·5	2618	1 ,, ,, left.	0·02621	8·918	1·572	103·1
		2620	Small ,, left.				
		2622	½ div. ,, right.				
18	48·8	1421	½ ,, ,, left.	0·014215	8·912	1·602	101·1
		1422	½ ,, ,, right.				
16	64·2	802	3 ,, ,, left.	0·008032	8·907	1·572	103·1
		803	½ ,, ,, left.				
		804	2 ,, ,, right.				
14	80·1	515	2½ ,, ,, left.	0·005163	8·888	1·566	103·4
		516	Small ,, left.				
		517	1 div. ,, right.				
12	103·2	314	1 , ,, left.	0·003145	8·904	1·585	102·2
		315	1 ,, ,, right.				
—	106·9	291	1 ,, ,, left.	0·002916	8·940	1·580	102·6
		292	½ ,, ,, right.				
—	103·8	308	1½ ,, ,, left.	0·003086	8·870	1·588	102·4
		309	1 ,, ,, right.				
—	104·2	309	2 ,, ,, left.	0·003098	8·891	1·580	102·6
		310	½ ,, ,, right.				

Matthiessen's standard has been taken to be a specific resistance (resistance per cubic centimetre at 0° C.) of 1·620 International Microhms.

Towards the end of this table it will be seen that even a sensitive galvanometer barely allowed an accuracy of 0·1 per cent. The Daniell cell was therefore removed, and a small accumulator with a resistance of 2 ohms in series with it substituted. The last three wires were then again tested, and a marked improvement was at once seen, for now an alteration of 1 ohm in x produced a difference of deflexion of 10–15 scale-divisions, thus giving the interpolated figure correct to the first place given, *i. e.*, R correct to 1 part in 3000.

To further test the capabilities of this method, a strand cable composed of 7 copper wires, each of No. 16 S.W.G., was taken, the strands being flattened to pass under the springs, which had not been constructed for a wire of such a large diameter. The results, using the same cell and resistance, are shown in the next table.

Table II.

x.	Observation.	R.
117	6 divs. deflexion left.	
118	1½ " " left.	0·001183
119	3 " " right.	

Here only 4½ divisions correspond with a change in x of 1 ohm : thus an error in reading of ½ a division means an error in R of 0·1 per cent. The effect of using a still larger current was next tried, the 2-ohm coil being removed, and the two resistances ρ, ρ' of fig. 3 substituted. An ammeter was also included in the battery-circuit, so that the actual current could also be measured.

Now, on passing a strong current both wires heat, and if the wire r has the smaller diameter it will heat the faster. Therefore its resistance will also increase the faster, and hence the resistance of the wire R will apparently diminish, and *vice versa*. In order to see how much this diminution might amount to, the resistance ρ' was short-circuited, and ρ was adjusted until the battery-current was 6 amperes, and the results are shown in Tables III. and IV.

Table III.

With the current only continued long enough to measure the deflexion : current 6 amperes.

x.	Observation.	R.
118	3 divs. deflexion right.	0·0011812
119	23 " " left.	

TABLE IV.

After the current had flowed continuously for 3 minutes:
6 amperes as before.

x.	Observation.	R.
119	28 divs. deflexion left.	
118	4 " " left.	0·0011784
117	20 " " right.	

The difference between these two results is about $\frac{1}{4}$ per cent., while the deflexions are more than sufficient for an accuracy in R of 0·1 per cent.

A few days later these experiments were repeated with a current of 2 amperes. The slight discrepancy in the value of R was due to the fact that in the interval the cable, which was not quite straight, had been removed. On replacing it a slightly less length must have been included between the knife-edges. As, however, these experiments were being made solely to find the *sensibility* of the measuring arrangement, no attempt was made to straighten it. The results are shown in Table V., experiment (*a*) being for the short duration of current, and (*b*) the result after the current had flowed continuously for 3 minutes.

TABLE V.

Current 2 amperes.

Experiment.	x.	Observation.	R.
(*a*)	119	16 divs. deflexion left.	0·001178
	118	3 " " left.	
	117	10 " " right.	
(*b*)	118	4 " " left.	0·001177
	117	9 " " right.	

From the above results it will be seen, firstly, that 13 divisions correspond with 1 ohm difference in x, thus making the fourth significant correct, in other words, an error in R is less than 0·1 per cent.; and secondly, the passage of a current of 2 amperes only produced in 3 minutes a difference of like amount.

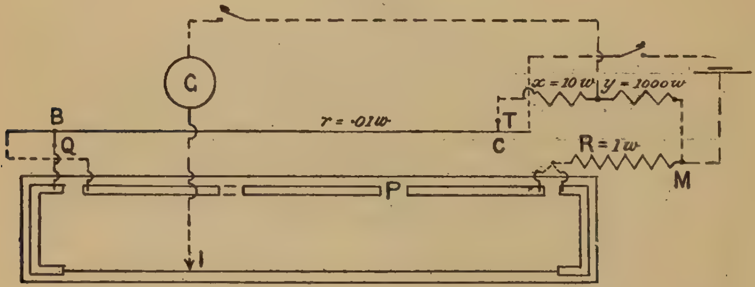
No larger cable was tried, nor was any wire smaller than No. 22 S.W.G., but between these limits an accuracy of 0·1 per cent. was easily obtained so far as the *sensibility of the method* was concerned.

The accuracy of the standard now remains to be considered. This can be best seen from a description of the method by which the correct length between the points B, C was determined.

In order that this length might be of a convenient amount to suit the dimensions of the base-board, it should have been made of copper wire No. 17 S.W.G.; but a piece of this gauge which was bought for the purpose proving unsatisfactory, a piece of No. 16 was taken and stretched till of the required diameter.

One, Q B, of the two short arms of fig. 1, was soldered on, the other, T C, being left loose. The whole was then screwed to a rough board, and the wire annealed by passing through it a current of about 50 amperes till it was too hot to touch and then allowing it to cool, the operation being repeated ten times. The arrangement represented in fig. 4 was next made.

Fig. 4.



By following out this figure, it will be seen to be the same arrangement as fig. 3, except that now the place of the plug is taken by the gap P. The standard was a 1-ohm coil of manganin wire, constructed by Messrs. Nalder, Bros. Its terminals rested in mercury-cups from which the various leads were taken, the ends of all wires dipping in the mercury being freshly amalgamated. The leads joining the 10- and 1000-ohm coils together and to the binding-screws T and M respectively were short and stout.

The arm T C was then put approximately in position, and good contact at C was maintained by pressure. To avoid thermoelectric effects this pressure was made with a piece of wood, and in order to see if any such effects were present a commutator, not shown in the figure, was put in the battery-circuit. It was then found that if balance had been obtained for one direction of the current, a reversal produced no appreciable deflexion, thus showing that this piece of wood had entirely prevented such effects. As a precaution, however, the current was reversed each time balance was obtained, and in no instance was any deflexion caused by the reversal.

The experiments were then conducted in the same order as before. With the gap open, balance was obtained by moving the slider I. The gap was then closed and balance obtained by altering the position of the arm T C. The gap was next opened, and the slider moved till again no deflexion was noted. This motion was very slight, and the alteration, after closing the gap, was not found to have produced any observable want of balance. The correct position of C was thus determined.

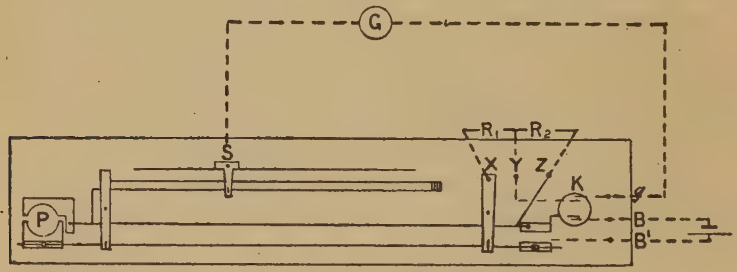
In order to see with what degree of accuracy this position had been arrived at, the arm was shifted 1 millim. in both directions, and the want of balance was indicated by a deflexion on either side of zero of one division; hence, as the length between B and C was over one metre, the error in the position of C was well under 0·1 per cent.

The arm TC was then soldered to the wire, and the tests repeated. No deflexion was observable with the ratio $\frac{10}{1000}$, but on adding a 1-ohm coil to the 1000, making the ratio $\frac{10}{1001}$, the want of balance produced nearly 2 divisions deflexion. The wire was then screwed in its proper place and there tested in a similar fashion with the same result, viz., with the ratio $\frac{10}{1000}$ no deflexion was noted, while with the ratio $\frac{10}{1001}$ a deflexion of over 1 division showed the want of balance. A thermometer lying beside the wire indicated 17°·7 C. Thus the resistance between B and C was 0·01 ohm at 17°·7 C., with an error of less than 0·1 per cent.

Thus on all three conditions an accuracy of 0·1 per cent. was obtained. This simple apparatus is, therefore, capable of measuring the resistances of metre-lengths of wires between the limits of No. 22 S.W.G. and a stranded cable of 7 No. 16's (and probably over a still greater range) with an accuracy *throughout* of 0·1 per cent., an accuracy quite sufficient for all commercial purposes.

Although in the above simple form of The Addition a slide-wire bridge is necessary, it may happen that this latter is not available. At a slight additional cost this slide-wire can be made an integral part of the apparatus. In fig. 5 is represented a rather more elaborate design. The slide-wire is easily recognized. The contact-piece of the slider S must be made adjustable so as to be able to press on either part of this wire. The circle K represents an ordinary Wheatstone-bridge key.

Fig. 5.



It will be seen that this design is very self-contained. All the connexions to be made consist of: a battery, with its supplementary resistance, to the binding-screws B, B', a galvanometer to g and S, and two resistances between X, Y and Y, Z. The former, marked R_1, is an adjustable box of coils, and the latter, R_2, is a single coil of 1000 ohms resistance.

Discussion.

Prof. A. Gray (communicated) said that the author had in his arrangement combined the fixed standard employed in Matthiessen and Hocking's modification of the ordinary bridge with the greater celerity of working arising from the smaller number of operations to be performed when the Kelvin bridge is used. Prof. Gray thinks that he, and probably others, have used a method similar to that of Mr. Reeves, but that the paper is of great utility since it shows how time may be saved and existing apparatus utilized.

Prof. Ayrton said that the advantage of the method described lay in the fact that it was independent of the resistances at the contacts. In Carey Foster's method, however, the coils had to be interchanged, and inaccuracy might be introduced owing to the varying resistance of the mercury contacts. Unless the mercury cups and the copper plates at the bottom were cleaned every day, and the contacts re-amalgamated, the resistance of the mercury cups was very variable. With regard to the question of Matthiessen's standard, it is to be remembered that the specific conductivity of copper has been steadily increasing. This increase was particularly noticeable in the copper prepared by the Elmore process, where, during the deposition of the metal, an agate burnisher is kept continually passing over the surface. Fitzpatrick had explained the rise in conductivity of copper by supposing that the density of the copper now supplied was greater than that of the copper used by Matthiessen, and this explanation seemed quite satisfactory. Mr. Reeves's experiments, however, have conclusively shown that this is not the true explanation; it was now possible to obtain copper in large quantity having a conductivity of 103 on Matthiessen's scale.

The Chairman (Prof. Carey Foster) explained how, when using his method, the accuracy of the result depends not on the elimination of the small resistances at the mercury cups, but on the constancy of these resistances. Matthiessen and Dr. Russell found that the specific gravity of copper was apt to be low, on account of the presence of dissolved oxide, and they were the first to pass hydrogen gas through the molten metal to remove this oxide.

Mr. Appleyard gave a simple diagrammatic sketch of the author's arrangement, and also pointed out that better results would probably be obtained with a galvanometer of 1 or 2 ohms resistance.

Mr. Campbell said that it ought to be definitely settled whether Matthiessen's standard was the conductivity per unit volume or per unit mass. Since copper was always bought by weight, he, as a practical man, strongly advocated the adoption of the mass conductivity; further, in this case the measurement of the specific gravity would be avoided.

XIV. *On Kathode Rays. By* Herr Puluj*.

[Abstract.]

Herr Puluj sent for exhibition some Röntgen photographs taken by means of a form of Crookes tube which he had described in a memoir published in 1889. With this tube he has succeeded in obtaining impressions with exposures of only two seconds.

Herr Puluj considers that the particles of matter torn from the kathode, which convey negative electrostatic charges, by impact on the glass walls or on screens, equalize their electric charges, and in this process call forth not merely a disturbance of the material molecules, but also of their ether envelopes. Each portion of the glass or screen bombarded by the kathode stream becomes the starting-point of ether waves, which, according to their oscillation period and oscillation character, are either visible rays (phosphorescence) or invisible Röntgen rays. The oscillations of the invisible rays may take place in the longitudinal direction, but no convincing argument has up to now been brought forward to support this view.

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* Read March 13, 1896

XV. *Note on Permeability to Röntgen Rays.*

By Messrs. ACKROYD *and* KNOWLES*.

[Abstract.]

THE Authors have exposed a plate on which a number of pieces of metal, oxides, and sulphates were placed to the Röntgen rays in order to see whether the permeability of bodies to these rays depends on the atomic or molecular weight of the body. In each case it was found that the opacity increased with the molecular weight.

DISCUSSION.

Mr. BLAKESLEY said that he considered the Röntgen rays to be the propagation of electrostatic strain through space. With reference to the non-refrangibility of these rays, he had observed in one of the photographs exhibited by Mr. Swinton a dark line at the edge of the shadow of a wooden pencil, which might have been due to the refraction of the rays by the wood. Mr. Blakesley has, however, found that this line is due to the varnish on the pencil. Some Röntgen photographs of quartz and ebonite rods not only did not exhibit these dark lines, but there was a very slight indication of a bright line just on the edge of the shadow, which would indicate that the refraction of these rays was less in the rods than in the surrounding medium.

Mr. EDSER exhibited some photographs taken with Mr. Jackson's form of tube, in which a concave kathode is employed. Mr. Edser said that the whole of the tube on the kathode side of the anode plate phosphoresced, so that the Röntgen rays seem to partake of the character of diffused light.

Prof. AYRTON said Mr. Jackson had found that the kathode rays form a parallel beam, and do not first come to a focus and then again spread out.

The CHAIRMAN said some observations made by Mr. Porter agreed with those of Mr. Edser.

Mr. BLAKESLEY described the tube used by Puluj, in which a mica screen coated with green calcium sulphide is placed between the kathode and the anode.

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* Read March 13, 1896.

Mr. GARDNER said that there seemed to be some confusion, for, when a concave kathode is employed, the kathode rays are brought to a focus and then again diverge. The phosphorescence on the inside of the glass had been shown by Lenard to be due to electricity travelling round the inside surface of the glass.

Mr. PIDGEON asked if any one had tried the effect of mounting the photographic film on a metal plate.

The CHAIRMAN said that Captain Abney had found that if the film was mounted on a ferrotype plate no action took place.

Prof. PERRY said he, for one, was of opinion that the Röntgen rays were undulatory. Prof. Larmor had given an explanation which seems to agree with the observed facts. This explanation supposes that the intermolecular spaces respond to vibrations of a certain frequency. The reason no refraction or diffraction effects had been observed was probably because of the extreme smallness of the wave-length of the undulation.

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XVI. *Notes on the Electro-magnetic Theory of Moving Charges.* *By* W. B. MORTON, *B.A.*[*]

1. THIS subject has been brought into prominence recently by the use which Mr. Larmor has made of moving electrons in his dynamical theory of the æther. The matter was investigated in 1881 by Prof. J. J. Thomson[†], who showed that a point charge moving so slowly that the electric displacement it carries is not sensibly disturbed generates magnetic force like a current element according to Ampère's rule; and by Mr. Heaviside[‡], who investigated the matter more generally in 1889, and showed that in steady rectilinear motion at any speed less than that of light, the lines of displacement continue to be radial but are concentrated

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[*] Read March 27, 1896.

[†] Phil. Mag. April 1881, July 1889; Recent Researches, pp. 16-23.

[‡] Electrical Papers, ii. pp. 504-518; Electromagnetic Theory, i. pp. 269-274.

towards the plane perpendicular to the direction of motion. The displacement at distance r, in a direction making an angle θ with the line of motion is proportional to

$$\frac{1}{r^2\left(1-\frac{u^2}{V^2}\sin^2\theta\right)^{\frac{3}{2}}},$$

where u is the velocity of the moving charge and V the velocity of light. The lines of magnetic force are circles round the line of motion.

2. This solution of course represents the state of affairs at a great distance from a small charged conductor of any shape. It would also give us the distribution of charge on a moving sphere if it were correct to assume that the lines of displacement meet the charged surface at right angles. This assumption was made by Prof. Thomson and, at first, by Mr. Heaviside, but the latter, quoting a suggestion of Mr. G. F. C. Searle, subsequently pointed out that when there is motion the electric force is no longer derived from a potential function, and as a consequence does not meet the equilibrium surface at right angles. Substituting the correct surface condition, he showed that the charged conductor, whose motion would give at all points the radial distribution found for a point charge, was not a sphere but a spheroid of certain ellipticity.

3. It seemed of some interest to inquire what the distribution of charge on a moving sphere would be. The surface-density at a point of the surface is now the normal component of the displacement at that point. By carrying the investigation a step further I have found that, if the conductor be a sphere or any ellipsoid, the ordinary static arrangement of charge is unaltered by the motion; *i.e.* the number of tubes of displacement leaving each element of the surface is unchanged, but the tubes no longer leave the surface at right angles. We may imagine that the motion has the effect of deforming the tubes, keeping their ends on the conductor fixed. The proof of this, involving a consideration of the general case, is here given and is followed by a note on the energy of a moving charge in a magnetic field.

4. Suppose we have any distribution of charge moving

with uniform velocity u parallel to the axis of z, and that the field has assumed its steady configuration. We shall denote $1-\frac{u^2}{V^2}$ by k^2, V being the velocity of light. Then since we have a steady state,

$$\frac{d}{dt}=-u\frac{d}{dz}.$$

Also, since each element of charge produces a magnetic field with no z-component, we have $\gamma=0$ in the general case also. Using these two data, the equations connecting the displacement (f, g, h) and the magnetic force (α, β, γ) become

$$-\frac{d\beta}{dz}=-4\pi u\frac{df}{dz},$$

$$\frac{d\alpha}{dz}=-4\pi u\frac{dg}{dz},$$

$$\frac{d\beta}{dx}-\frac{d\alpha}{dy}=-4\pi u\frac{dh}{dz},$$

$$\frac{dg}{dz}-\frac{dh}{dy}=-\frac{u}{4\pi V^2}\frac{d\alpha}{dz},$$

$$\frac{dh}{dx}-\frac{df}{dz}=-\frac{u}{4\pi V^2}\frac{d\beta}{dz},$$

$$\frac{df}{dy}-\frac{dg}{dx}=0.$$

These equations together with

$$\frac{df}{dx}+\frac{dg}{dy}+\frac{dh}{dz}=0$$

are satisfied by

$$f=-\frac{d\phi}{dx}, \quad g=-\frac{d\phi}{dy}, \quad h=-k^2\frac{d\phi}{dz},$$

$$\alpha=4\pi u\frac{d\phi}{dy}, \quad \beta=-4\pi\frac{d\phi}{dx},$$

where ϕ is any function satisfying

$$\frac{d^2\phi}{dx^2}+\frac{d^2\phi}{dy^2}+k^2\frac{d^2\phi}{dz^2}=0.$$

These results have been obtained by Prof. Thomson and Mr. Heaviside. The particular case of a point charge, e, is got by putting

$$\phi = \frac{e}{4\pi\sqrt{k^2(x^2+y^2)+z^2}}.$$

Evidently in the general case ϕ must vanish at infinity.

5. Mr. Heaviside points out that ϕ=constant is the condition holding at a surface of equilibrium. The matter may be stated thus:—If we suppose the field to terminate at the surface of a conductor, inside which the vectors vanish, we must see that the "curl" relations of the field are not violated for circuits which lie partly inside the empty space enclosed by the conductor. In particular, if there is a vector whose line integral round every circuit in the field vanishes, the lines of this vector must meet the surface at right angles. Otherwise we should have a finite value for the integral round a circuit drawn close to the surface outside and completed inside. In other words, if a vector is derived from a potential function, this function must be constant over the surface. In the ordinary static case it is the electric force (X, Y, Z) which is so derived; but in the case of a steadily moving field it is the vector $\left(X, Y, \frac{Z}{k^2}\right)$ which meets the surface at right angles.

6. Let $F(x\,y\,z)=C$ be the equation of the charged surface. Then $\phi(x\,y\,z)$ has to be constant over this surface and satisfy

$$\frac{d^2\phi}{dx^2}+\frac{d^2\phi}{dy^2}+k^2\frac{d^2\phi}{dz^2}=0.$$

Put $z=k\zeta$, then ϕ is a function of x, y, ζ, which is constant when $F(x, y, k\zeta)=C$, and which satisfies

$$\frac{d^2\phi}{dx^2}+\frac{d^2\phi}{dy^2}+\frac{d^2\phi}{d\zeta^2}=0.$$

Therefore if we regard $(x\;y\;\zeta)$ as Cartesian coordinates of a point, ϕ is the potential at external points of an electrostatic free distribution on the surface $F(x, y, k\zeta)=C$. The components of electric force due to this distribution, at a point

$(x\ y\ \zeta)$ on the surface, are

$$-\frac{d\phi}{dx},\quad -\frac{d\phi}{dy},\quad -\frac{d\phi}{d\zeta}.$$

This force acts in the normal to the surface, and is proportional to the surface-density at $(x\ y\ \zeta)$, which we shall call σ'. Therefore

$$\left(\frac{d\phi}{dx},\ \frac{d\phi}{dy},\ \frac{d\phi}{d\zeta}\right)=-\mathrm{A}\sigma'\left(\frac{d\mathrm{F}}{dx},\ \frac{d\mathrm{F}}{dy},\ \frac{d\mathrm{F}}{d\zeta}\right)\Big/\sqrt{\left(\frac{d\mathrm{F}}{dx}\right)^2+\left(\frac{d\mathrm{F}}{dy}\right)^2+\left(\frac{d\mathrm{F}}{d\zeta}\right)^2}$$

But

$$\frac{d}{d\zeta}=k\frac{d}{dz};$$

therefore, denoting differentiation with respect to $x\ y\ z$ by subscripts 1 2 3,

$$(\phi_1,\ \phi_2,\ \phi_3)=-\mathrm{A}\sigma'(\mathrm{F}_1,\ \mathrm{F}_2,\ \mathrm{F}_3)/\sqrt{\mathrm{F}_1^2+\mathrm{F}_2^2+k^2\mathrm{F}_3^2}.$$

Now let σ be the surface-density at $(x\ y\ z)$ on the moving conductor $\mathrm{F}(x\ y\ z)=\mathrm{C}$, then equating σ to the normal component of $(f\ g\ h)$,

$$\sigma=\frac{f\mathrm{F}_1+g\mathrm{F}_2+h\mathrm{F}_3}{\sqrt{\mathrm{F}_1^2+\mathrm{F}_2^2+\mathrm{F}_3^2}};$$

or putting in the values we have found for $(f\ g\ h)$ in terms of ϕ,

$$\sigma=-\frac{\phi_1\mathrm{F}_1+\phi_2\mathrm{F}_2+k^2\phi_3\mathrm{F}_3}{\sqrt{\mathrm{F}_1^2+\mathrm{F}_2^2+\mathrm{F}_3^2}}$$

$$=\mathrm{A}\sigma'\sqrt{\frac{\mathrm{F}_1^2+\mathrm{F}_2^2+k^2\mathrm{F}_3^2}{\mathrm{F}_1^2+\mathrm{F}_2^2+\mathrm{F}_3^2}}.$$

Now the perpendicular from the origin on the tangent plane to $\mathrm{F}(x\ y\ z)=\mathrm{C}$ at the point $(x\ y\ z)$ is

$$p=\frac{x\mathrm{F}_1+y\mathrm{F}_2+z\mathrm{F}_3}{\sqrt{\mathrm{F}_1^2+\mathrm{F}_2^2+\mathrm{F}_3^2}},$$

and the perpendicular from the origin on the tangent plane to $\mathrm{F}(x,\ y,\ k\zeta)=\mathrm{C}$ at $(x\ y\ \zeta)$ is

$$p'=\frac{x\dfrac{d\mathrm{F}}{dx}+y\dfrac{d\mathrm{F}}{dy}+\zeta\dfrac{d\mathrm{F}}{d\zeta}}{\sqrt{\left(\dfrac{d\mathrm{F}}{dx}\right)^2+\left(\dfrac{d\mathrm{F}}{dy}\right)^2+\left(\dfrac{d\mathrm{F}}{d\zeta}\right)^2}}$$

$$=\frac{xF_1+yF_2+k\zeta F_3}{\sqrt{F_1^2+F_2^2+k^2F_3^2}}$$

$$=\frac{xF_1+yF_2+zF_3}{\sqrt{F_1^2+F_2^2+k^2F_3^2}};$$

$$\therefore\ \frac{\sigma}{\sigma'}=\frac{Ap}{p'}.$$

If now $F(x\,y\,z)=C$ is an ellipsoid, then we know that $\sigma' \propto p'$, therefore also $\sigma \propto p$, that is the arrangement of charge on the moving ellipsoid is the same as if it were at rest.

7. Applying the above to the ellipsoid $(a\,b\,c)$, we find that ϕ as a function of $(x\,y\,\zeta)$ is the potential of a free distribution on the ellipsoid

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{k^2\zeta^2}{c^2}=1;$$

$$\therefore\ \phi=C\int_\mu^\infty\frac{d\lambda}{\sqrt{(a^2+\lambda)(b^2+\lambda)\left(\frac{c^2}{k^2}+\lambda\right)}}$$

$$=Ck\int_\mu^\infty\frac{d\lambda}{\sqrt{(a^2+\lambda)(b^2+\lambda)(c^2+k^2\lambda)}},$$

where μ is given by

$$\frac{x^2}{a^2+\mu}+\frac{y^2}{b^2+\mu}+\frac{\zeta^2}{\frac{c^2}{k^2}+\mu}=1,$$

or

$$\frac{x^2}{a^2+\mu}+\frac{y^2}{b^2+\mu}+\frac{z^2}{c^2+k^2\mu}=1.$$

Determining the value of the constant C so that the density at a point shall be $\frac{ep}{4\pi abc}$, we get

$$\phi=\frac{e}{8\pi}\int_\mu^\infty\frac{d\lambda}{\sqrt{(a^2+\lambda)(b^2+\lambda)(c^2+k^2\lambda)}}.$$

Putting $b=a$, $c=ka$, we get

$$\phi=\frac{e}{4\pi\sqrt{k^2(x^2+y^2)+z^2}}.$$

Showing that, as Mr. Heaviside pointed out, the field of a point charge is given when the conductor is an oblate spheroid whose axes have the ratio 1 : k.

For a sphere the integral becomes

$$\phi = \frac{e}{8\pi k'a} \log \frac{\theta + k'a}{\theta - k'a}$$

where

$$k' = \sqrt{1-k^2} = \frac{u}{\mathrm{V}},$$

and θ is given by

$$\frac{k^2(x^2+y^2)}{\theta^2 - k'^2a^2} + \frac{z^2}{\theta^2} = 1.$$

To test the value of ϕ let us make k' approach zero, *i.e.* the motion becomes infinitely slow. θ is then $=r$.

Then
$$\phi = \frac{e}{8\pi a} \operatorname*{Lt}_{k'=0} \frac{\log (r + k'a) - \log (r - k'a)}{k'}$$

$$= \frac{e}{8\pi a} \cdot \frac{2a}{r} = \frac{e}{4\pi r}.$$

8. The mutual energy of a moving charge and external magnetic field has been given by Mr. Heaviside for the case of motion which is very slow compared with the velocity of radiation. It is $eu\mathrm{A} \cdot \cos(u\mathrm{A})$, where A is the circuital vector potential of the external field. Mr. Larmor, in the second part of his "Dynamical Theory" (Phil. Trans. 1895, p. 717), concludes that the same expression holds good for motion at any speed. He seems, however, to overlook the fact that in the general case the displacement-currents in the medium—being no longer derivable from a potential function—will make their appearance in the result as well as the convection-current eu.

If (F G H) is the vector potential, the part of the energy corresponding to the displacement currents will be

$$\int (\mathrm{F}\dot{f} + \mathrm{G}\dot{g} + \mathrm{H}\dot{h})\, d\tau,$$

which in the case we have been considering becomes

$$-u\int\left(\mathrm{F}\frac{df}{dz} + \mathrm{G}\frac{dg}{dz} + \mathrm{H}\frac{dh}{dz}\right)d\tau$$

$$= u\int\left(\mathrm{F}\frac{d^2\phi}{dx\,dz} + \mathrm{G}\frac{d^2\phi}{dy\,dz} + k^2\mathrm{H}\frac{d^2\phi}{dz^2}\right)d\tau.$$

But by a well-known transformation, when we take the integral through all space, we have

$$\int\left(\mathrm{F}\frac{d^2\phi}{dx\,dz}+\mathrm{G}\frac{d^2\phi}{dy\,dz}+\mathrm{H}\frac{d^2\phi}{dz^2}\right)d\tau$$

$$=\int\frac{d\phi}{dz}\left(\frac{d\mathrm{F}}{dx}+\frac{d\mathrm{G}}{dy}+\frac{d\mathrm{H}}{dz}\right)d\tau$$

$$=0 \text{ since } (\mathrm{F\,G\,H}) \text{ is circuital.}$$

∴ The expression for this part of the energy reduces to

$$-u(1-k^2)\int\mathrm{H}\frac{d^2\phi}{dz^2}d\tau=-\frac{u^3}{\mathrm{V}^2}\int\mathrm{H}\frac{d^2\phi}{dz^2}d\tau.$$

Therefore if the velocity u ceases to be negligible in comparison with V, we have a correction of the *second* order in the ratio $\frac{u}{\mathrm{V}}$ in addition to the expression involving the convection-current simply. It also appears from the above that the force on the moving charge cannot, unless this term be neglected, be expressed in terms of the magnetic intensity at the charge, but will depend on the entire field.

XVII. *A Further Examination of the Edison Effect in Glow Lamps. By* J. A. Fleming, *M.A., D.Sc., F.R.S., Professor of Electrical Engineering in University College, London*[*].

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§1. The experiments described in this paper had for their object the further examination of an effect which can be produced in certain forms of electric incandescence lamps and to which attention was first drawn by Mr. Edison in 1884. This effect may be generally described as follows :—A carbon filament incandescence lamp having the ordinary horse-shoe loop carbon has a metallic plate placed in the exhausted bulb, the plate being carried on a platinum wire sealed through the globe, and fixed so as to stand up between the legs of the horse-shoe (see fig. 1). If the lamp is set in action at the usual incandescence by a continuous current of the proper strength, and a suitable sensitive galvanometer is connected between the insulated metal plate and the *positive* terminal of the lamp, it will in general be found to indicate a current of some milliamperes flowing through it. The direction of this current is *from* the positive electrode of the lamp through the galvanometer *to* the insulated metal plate, or wire. When the same galvanometer is connected between the *negative* pole of the lamp and the middle plate, unless it is very sensitive, it indicates no current. This effect was very carefully examined

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[*] Read March 27, 1896.

Fig. 1.

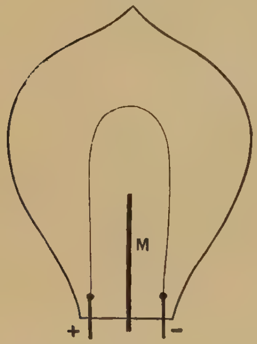
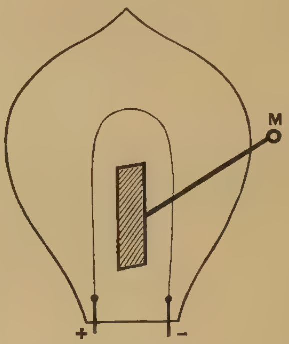


Fig. 2.



by Mr. W. H. Preece in 1885, and he subjected it to a systematic examination by the aid of a number of lamps having such metal plates placed in various positions*. By this observer a number of very interesting facts were collected, the result of which was to point out the general nature of the phenomenon. A sufficient number of new questions were, however, suggested by the information so acquired to invite further inquiry. Whilst confirming and re-examining the experimental results obtained by Mr. Preece, some facts that had previously escaped notice presented themselves, which it is the object of this paper to describe.

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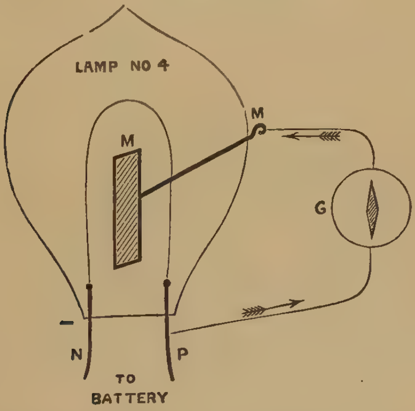
* "On a Peculiar Behaviour of Glow-Lamps when raised to High Incandescence," by W. H. Preece, F.R.S. Proceedings of the Royal Society, 1885, p. 219.

§ 2. The first experiments were made with a lamp of the form shown in fig. 2, similar to some used by Mr. Preece in his experiments. A metal plate, generally of aluminium, is supported on a platinum wire sealed through the bulb or glass receiver, the plate being so fixed that its plane is at right angles to the plane of the loop of the carbon, and as nearly as possible midway between the legs. The plate therefore projects between the legs of the horse-shoe carbon, and the carbon conductor arches over it without touching it. The plate is entirely insulated from the carbon.

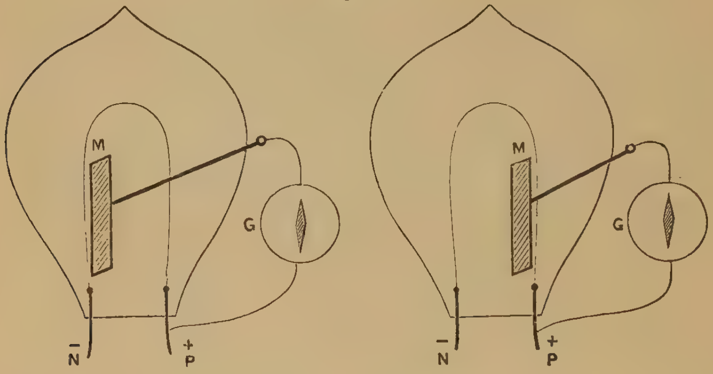
The preliminary experiments with this normal type of middle-plate lamp consisted in determining the effective potential-difference between the third terminal and one or other of the two electrodes of the carbon filament when the lamp was subjected to varying steady electromotive forces sufficient to raise the temperature of the carbon from dull red to vivid incandescence, and in determining the magnitude of the current flowing in a circuit connecting the middle plate with one or other of the electrodes of the lamp.

Fig. 3.



§ 3. *Experiment* 1.—An ordinary carbon filament electric lamp, having the horse-shoe shaped conductor, had a platinum plate (see fig. 3) about 2·5 centimetres long by 1·5 centimetres wide welded to a platinum wire, sealed through the side of the bulb. The plate was so placed as to project between the legs of the carbon conductor, having its plane at right angles to the plane of the horse-shoe, and initially fixed about halfway between the two legs. This lamp will be described in the subsequent paragraphs as Lamp No. 4. Under a steady electromotive force of 40 volts, this lamp took 3·7 amperes when working at the normal temperature corresponding to about 3·5 watts per candle-power. When a milamperemeter having a resistance of 6372 ohms was joined between the base P of the *positive* leg of the carbon (see fig. 4) and the middle plate M, a current was found passing through the galvanometer from the terminal P to the plate M. This current had a magnitude of about 3 milliamperes when the carbon was in the normal state of incandescence.

Fig. 4.



If the milamperemeter was connected between the *negative* electrode of the lamp and the middle plate, no current perceptible by this galvanometer was found. On replacing the milamperemeter by a more sensitive Elliott mirror galvanometer (resistance 7142 ohms), it was found that a small current passed through it, when joined in between the negative electrode of the lamp and the middle plate, but that this current had a magnitude hardly exceeding ·0001 of a milliampere when the lamp was at its normal incandescence.

In order to avoid repetition, it may be here said that, unless otherwise stated, the terminal of the lamp in connexion with the positive pole of the working battery will be spoken of as the positive electrode of the lamp; that in connexion with the negative pole of the battery as the negative electrode. For brevity's sake, the half of the carbon filament between the centre of the filament and the positive electrode will be called the *positive leg*, and the other half the *negative leg*.

§ 4. A preliminary series of experiments was made with lamp No. 4 by placing the lamp in a photometer and determining the watts per candle-power and the current taken by the lamp corresponding to various working electromotive forces, taken over the whole range of electromotive force from that necessary just to render the filament incandescent to the highest the lamp could with safety endure. In any subsequent experiments, the simple measurement of the potential-difference between the electrodes of the lamp enabled the rate of dissipation of energy in the filament and the watts per candle-power to be deduced. It may here be remarked that in the preliminary experiments some difficulties arose from the occlusion of residual gas by the middle metal plate, but finally this was overcome, and the vacuum in these experimental bulbs made and preserved as perfect as in good ordinary commercial lamps. The following results were then obtained with this lamp No. 4. The lamp was raised to various degrees of incandescence by varying the working volts by the aid of a rheostat in series with the lamp.

The milamperemeter was employed to measure the effective potential-difference between the positive electrode of the lamp and the middle plate and then, tabulating against the working volts of the lamp the current in milliamperes flowing through the galvanometer, the potential-difference between the middle plate and the positive electrode of the lamp was calculated from these figures. The results are given in the table below (Table No. 1).

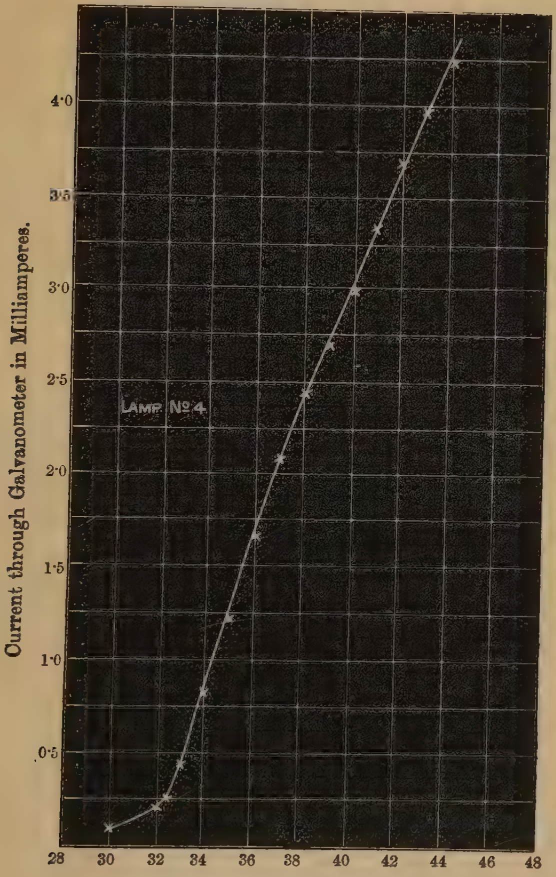
Table No. 1. Lamp No. 4. *Milamperemeter.*

Table showing the volts between the middle plate and the positive electrode, and the current flowing through a galvanometer of 6372 ohms resistance connecting them, taken for various working voltages of the lamp.

Working volts of lamp.	Volts between plate and positive lamp electrode.	Current in milliamperes through galvanometer.	Working volts of lamp.	Volts between plate and positive electrode.	Current in milliamperes through galvanometer.
30	·54	·085	36	10·7	1·69
32	1·2	·190	37	12·7	2·01
32 5	1·6	·25	38	14·9	2·36
33	2·8	·44	39	17·1	2·71
33·5	4·7	·74	40	18·9	2·99
34	5·3	·84	41	21·4	3·37
34·5	7·1	1·12	42	23·4	3·71
35	7·8	1·23	43	25·2	3·99
			44	26·8	4·25

The results given in table No. 1 are plotted in curve No. 1, in which horizontal abscissæ represent to scale the working volts of the lamp and vertical ordinates the milliampere currents through the galvanometer. It will be seen that the curve representing the current from lamp electrode to plate takes a rather sharp turn upwards at a point corresponding to 33 working volts, and this occurs when the lamp is working at about 7·8 watts per candle-power. Beyond this point the curve is very approximately a straight line. Accordingly, at and beyond the volts at which the carbon filament becomes fairly well incandescent, the effective potential-difference between the middle plate and the positive lamp electrode is very nearly a linear function of the lamp voltage; and at the normal working volts, viz. 40 volts, this potential-difference between the middle plate and the positive electrode so determined is apparently about half that between the lamp terminals, the plate being nearly midway between the carbon legs. The results given in Table 1 are the mean of several observations, but it was noticed that when the lamp was maintained at a steady voltage, the potential-difference between the middle plate and the positive electrode would often *jump suddenly* from one value to another. This effect renders it difficult to obtain the stable values of the plate and positive electrode potential-difference. Corresponding to any definite

Table No. 1.—Curve No. 1.



Working Volts of Lamp.

steady voltage on this lamp, the current may have one or other of two values, but not always permanently preserving either; a galvanometer deflexion indicating say 10 volts between the plate and positive electrode of the lamp will often slowly increase until after a few minutes it is 12 or 14 volts, yet all the time the working volts on the lamp are remaining perfectly constant. It will then often suddenly jump perhaps to 22 volts, and then slowly decrease to 19 volts, or so. This tendency of the potential-difference between the middle plate and positive lamp electrode to jump from a low to a high value, or *vice versâ*, is most marked in lamps in which the plate is about half-way, and symmetrically placed, between the legs of the carbon. We shall speak of these two values as the high and low value of the current through the galvanometer, and defer until later a discussion of some other causes tending to make the current pass from a high to a low value or the reverse, as well as its possible explanation. In Table No. 2 are tabulated a set of observations on the same lamp No. 4, showing these double values which the potential-difference and current may have, and it may be here noted that in the previous Table No. 1, the higher values have been taken in those cases in which double values exist.

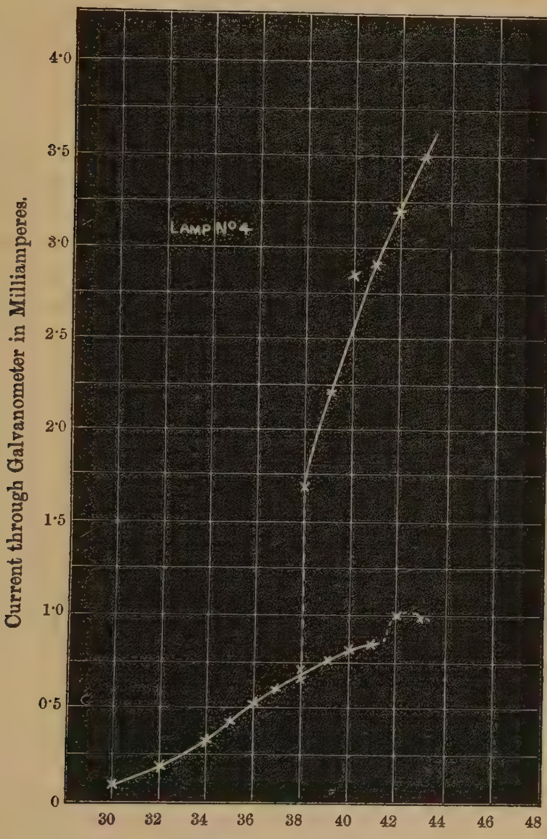
TABLE NO. 2. LAMP NO. 4. *Milamperemeter.*

Table showing the multiple values of the potential-difference between the middle plate and positive electrode of the lamp corresponding to various given working voltages.

Working volts of the lamp.	Volts between the middle plate and positive electrode of the lamp.	Current through the graded galvanometer in milliamperes.	Working volts of the lamp.	Volts between the middle plate and positive electrode of the lamp.	Current through the graded galvanometer in milliamperes.
30	·6	·095	39	4·3	·761
32	1·1	·174	„	14·0	2·22
34	2·0	·317	40	5·1	·809
35	2·6	·412	„	18·1	2·87
36	3·2	·507	41	5·3	·841
37	3·9	·618	„	18·4	2·91
38	4·3	·682	42	6·4	1·01
„	4·5	·714	„	20·0	3·17
„	11·0	1·74	43	6·2	·983
			„	22·0	3·49

These observations are plotted in Curve No. 2, in which the abscissæ represent the working volts of the lamp and the ordinates the current in milliamperes flowing through the galvanometer connecting the positive electrode and the middle plate. It is seen that corresponding to any working pressure

TABLE No. 2.—CURVE No. 2.



Working Volts of Lamp.

above 38 volts for this lamp, which is equivalent to 4·2 watts per candle-power, there are two possible values of the effective potential-difference between the middle plate and the positive electrode. As the working voltage of the lamp is gradually raised, the reading of the galvanometer inserted between the middle plate and positive electrode is also increased, but there is a great tendency to jump from a certain low value to a higher one, and this occurs when the working pressure of the lamp is preserved steady. There is also an effect produced by the presence of a magnet near the lamp bulb. When the current is at the low value corresponding to any working voltage, the galvanometer reading does not seem to be perceptibly altered by bringing a magnet near the lamp, but when it is at its high value, the reading is sometimes increased for a little, showing a steady deflexion, and then immediately falls to its low value.

§ 5. *Experiment* 2.—The difference of potential between the middle plate and the positive electrode of the lamp depends to a considerable extent upon the position of the middle plate. Supposing the plate to be placed with its plane perpendicular to the plane of the carbon horse-shoe and then moved to various positions between the two legs of the carbon, it is found that the difference of potential between the plate and the positive electrode will have different values according to the position of the plate. This fact was elucidated by means of the same lamp No. 4 as used above. By carefully tapping the lamp, the supporting platinum wire carrying the platinum middle plate could be bent so as to displace the plate from its symmetrical position as regards the two carbon legs, and bring it nearer to one or other of the legs. In several different positions the current flowing through the milamperemeter, when connected between the middle plate and positive electrode, was measured, the lamp being kept meanwhile at the same working electromotive force.

Estimating as nearly as possible the fractional distances, the plate was placed at distances from the negative leg equal to $\frac{1}{10}$, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{9}{10}$ of the whole distance between the positive and negative legs, and the lamp being taken through a definite cycle of volts, the potential-difference between the middle plate and the positive electrode was measured with the milamperemeter. The results are collected in the following tables. The diagrams in fig. 6 represent the horse-shoe carbon loop and the middle plate M in various positions, the galvanometer G being inserted between the plate M and the positive electrode P. By the phrase "whole distance" in the following tables is meant the whole distance or width of the space between the positive and negative carbon leg.

TABLE NO. 3. LAMP NO. 4. *Milamperemeter.*

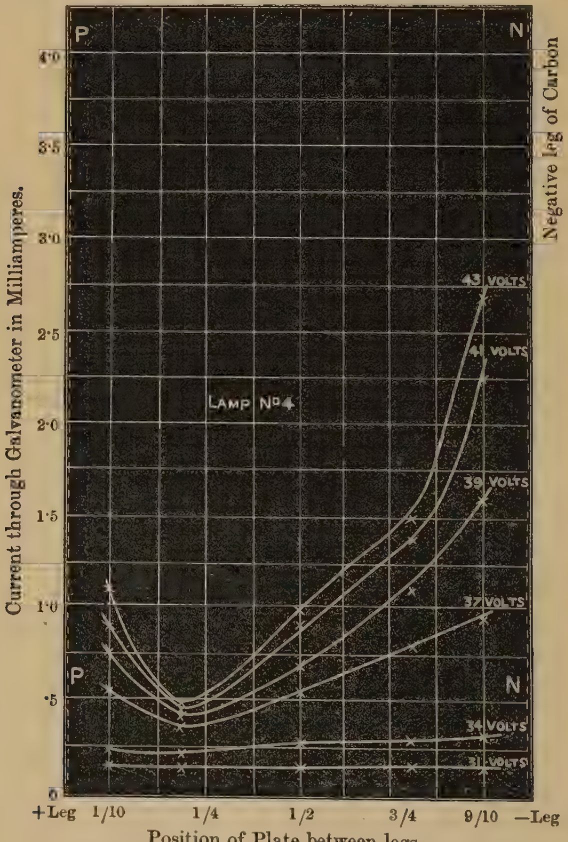
Table showing the potential-difference between the middle plate and positive electrode of the lamp at various positions of the plate and at various working voltages.

Working volts of the lamp.	Volts between the middle plate and positive electrode.	Current through the galvanometer in milliamperes.	Working volts of the lamp.	Volts between the middle plate and positive electrode.	Current through the galvanometer in milliamperes.
Middle plate at $\frac{1}{10}$ of whole distance from negative leg.			Middle plate at $\frac{3}{4}$ of whole distance from negative leg.		
31	·9	·142	31	·9	·142
34	2·2	·317	34	1·4	·222
37	6·0	·952	37	2·2	·349
39	10·2	1·61	39	2·5	·396
41	14·2	2·24	41	2·9	·460
43	16·8	2·66	43	3·0	·476
Middle plate at $\frac{1}{4}$ of whole distance from negative leg.			Middle plate at $\frac{9}{10}$ of whole distance from negative leg.		
31	·9	·142	31	·9	·142
34	1·8	·285	34	1·5	·238
37	5·1	·809	37	3·4	·539
39	7·0	1·11	39	4·8	·761
41	8·7	1·38	41	5·7	·904
43	9·5	1·50	43	6·8	1·07
Middle plate $\frac{1}{2}$ of whole distance from negative leg.					
31	·9	·142			
34	1·7	·269			
37	3·5	·555			
39	4·3	·682			
41	5·6	·888			
43	6·2	·983			

The results of this Table No. 3 are plotted in the curves No. 3. These curves are to be interpreted as follows :—The two vertical lines P and N represent the two legs of the

Table No. 3.—Curve No. 3.

The middle plate was moved along into different positions between the two carbon legs indicated by the horizontal distances, and at each position the current between the middle plate and positive electrode of the lamp is represented by the vertical ordinate of a curve. The several curves correspond to different working volts on the lamp.

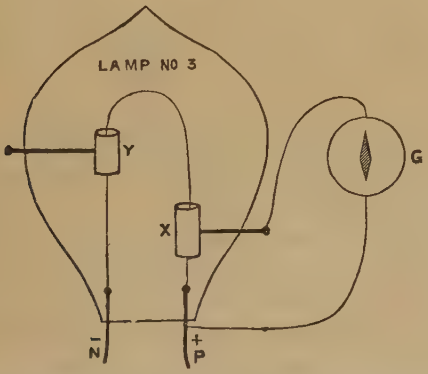


carbon horse-shoe. At various distances on the way from P to N the milliampere current through a galvanometer connected between the middle plate, placed at that point, and the positive electrode of the lamp is represented by the magnitude of the vertical ordinate of each curve. For every one of the different voltages at which the lamp is worked, there is therefore a curve representing by its ordinates this current strength through a galvanometer inserted between the middle plate, placed at these positions, and the positive electrode of the lamp, and it is seen that there is a *minimum value* for this current at a position equal to $\frac{3}{4}$ of the whole distance between the legs reckoned from the negative leg.

Imagine the middle plate therefore connected through a galvanometer with the positive electrode of the lamp, and let the middle plate be first placed close to the positive leg and then moved continuously nearer towards the negative leg. The current through the galvanometer would first fall off as the plate receded from the positive leg, and after reaching a minimum at a point about $\frac{1}{4}$ of the whole distance between the legs reckoned from the positive leg, would rise up to a maximum when the middle plate was as nearly in contact with the negative leg as possible without actually touching it.

§ 6. *Experiment* 3.—In order to explore more thoroughly the action of the different portions of the incandescence carbon conductor in producing this effect, a lamp was taken having a horse-shoe shaped carbon, and a pair of small platinum cylinders, held on platinum wires sealed through the glass, so placed as to embrace without touching the carbon conductor. One of these cylinders, X, was placed so as to embrace the

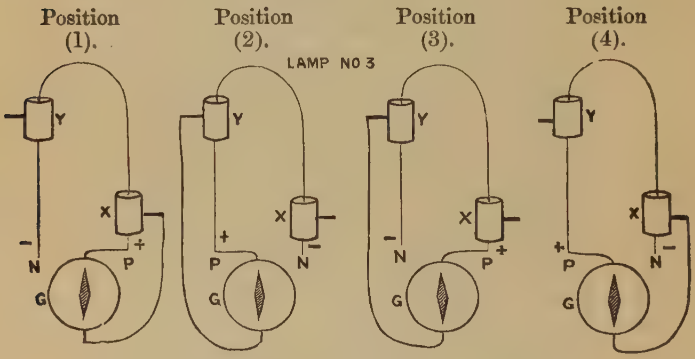
Fig. 5.



carbon near the bottom of the leg, and the other, Y, near the spring of the arch (see fig. 5). These small cylinders had a length of about 12 millims. and a diameter of about 8 millims. so that the distance from the carbon filament to the inner surface of the cylinder was about 3 or 4 millims. The lamp had a rather thick carbon, and at an electromotive force of 48 volts took a current of 1·32 amperes to raise it to its normal incandescence of 18·8 candles, corresponding to 3·3 watts per candle-power. This lamp will be alluded to as Lamp No. 3.

It is obvious that there are four possible arrangements in which a current can be obtained between an embracing cylinder and a positive electrode of the lamp. These are illustrated in fig. 6, in which the horse-shoe shaped line stands

Fig. 6.



for the carbon filament, X and Y are the platinum cylinders, P and N are the positive and negative electrodes of the lamp, and G is the galvanometer.

We will call these arrangements (1), (2), (3), (4), as figured. It will be seen that if we imagine the carbon filament straightened out, these four arrangements are equivalent to being able to slide a cylinder along the filament into four positions, and in each position measuring the potential-difference between the cylinder and the positive end of the carbon. We are thus able to place an embracing collecting-plate at four different places along the carbon conductor, and determine the potential-difference between this embracing cylinder and the positive electrode of the lamp. A series of experiments was made with lamp No. 3, in which the working volts of the lamp were raised to various values, and in each case the potential-difference between one of the cylinders X or Y and the positive electrode of the lamp was observed as before by means of the milamperemeter. The results are given in Table 4.

TABLE No. 4. LAMP No. 3. *Milamperemeter.*

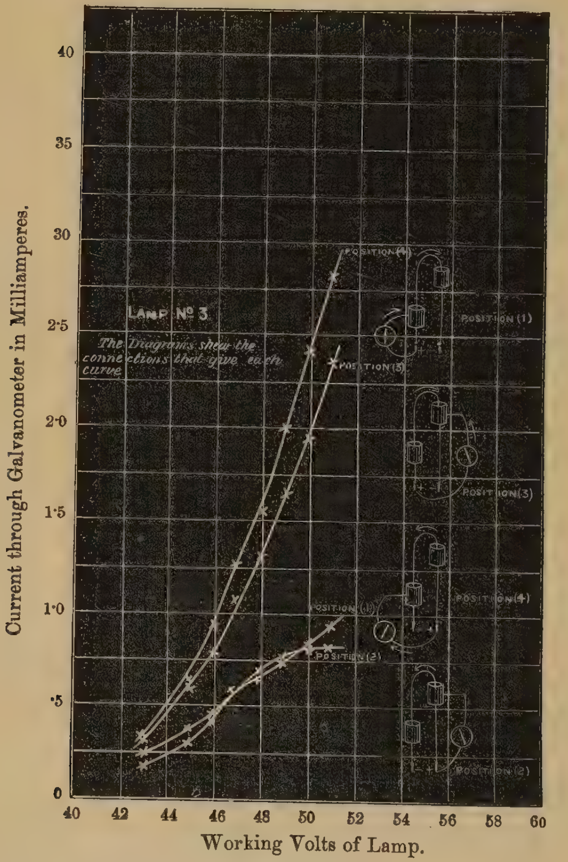
Table showing the potential-difference between a platinum cylinder embracing the carbon and the positive electrode of the lamp, and the current flowing through the galvanometer in milliamperes; for the four positions shown in fig. 6.

Working volts of the lamp.	Volts between cylinder and positive electrode.	Current in milliamperes through the galvanometer.	Working volts of the lamp.	Volts between cylinder and positive electrode.	Current in milliamperes through the galvanometer.
Position (1).			Position (2).		
43	1·6	·25	43	1·2	·19
45	2·4	·38	45	2·0	·31
46	2·9	·46	46	2·9	·36
47	3·7	·58	47	3·7	·58
48	4·2	·67	48	4·2	·67
49	4·7	·74	49	4·9	·78
50	5·2	·82	50	5·1	·81
51	5·9	·93	51	5·3	·84
Position (3).			Position (4).		
43	2·1	·33	43	2·2	·35
45	3·9	·62	45	4·0	·63
46	4·9	·78	46	5·9	·94
47	6·8	1·08	47	8·0	1·26
48	8·2	1·30	48	9·8	1·55
49	10·4	1·65	49	12·6	1·99
50	12·3	1·95	50	15·1	2·38
51	14·9	2·36	51	18·0	2·85

These observations are plotted in Curve No. 4.

This table of observations shows us that taking the lamp at any definite voltage, the potential-difference between the positive electrode and a cylinder embracing the carbon filaments is greatest when that cylinder is as low down near the foot of the negative leg of the carbon as possible. If the cylinder is placed near the top of the negative leg that potential-difference becomes less. If the cylinder is taken near the top of the positive leg or near the base of the positive leg it is least, and it seems to be a minimum when the cylinder

Table No. 4.—Curve No. 4.



surrounds the top of the positive leg, and is as far as possible away from the foot of the negative leg. We have here a confirmation of the fact observed with respect to lamp No. 4, viz.: that the potential-difference between the positive lamp electrode and the metal plate held somewhere near the incandescent carbon conductor is greatest when the plate is as near as possible to the foot of the negative leg or negative electrode.

In this lamp No. 3, in which in all positions the cylinder employed is placed very near some point on the incandescing conductor, the current through the galvanometer joining the positive electrode and the cylinder has never been observed to jump or to take double values as in the case of lamp No. 4.

If a sensitive galvanometer is connected to the two insulated cylinders X and Y, and if in addition there is a battery in series with this galvanometer, then no current can be detected in such an arrangement even when the battery has an electromotive force of 120 volts, whether the lamp carbon is incandescent or not. Just as in the case of lamp No. 4, the current obtained by connecting either cylinder with the negative electrode of the lamp is excessively small.

In order to obtain curves showing the mode of variation of the effective difference of potential between one or other of the metal cylinders X and Y and the positive electrode of the lamp, a set of observations was made on the lamp when submitted to various working voltages, and at the same time the milamperemeter was connected first between the positive electrode and cylinder X and then between the positive electrode and cylinder Y, with the following tabulated results (p. 204).

These results, when plotted out in curves in which horizontal ordinates represent the working volts of the lamp and the vertical ones the milliampere currents going through the galvanometer, in both cases have the same general form as curve No. 1 plotted for the case of lamp No. 4.

Table No. 5. Lamp No. 3. *Milamperemeter.*

Table showing the potential-differences between the positive electrode of the lamp and the two cylinders X and Y respectively, and the corresponding currents through the galvanometer.

Working volts of the lamp.	Volts between cylinder X and the positive electrode P of the lamp. Position (1).	Milliampere current through the galvanometer connecting X and P.	Volts between cylinder Y and the positive electrode P of the lamp. Position (3).	Milliampere current flowing through the galvanometer connecting Y and P.
34	·1	·015	·1	·015
35	·15	·023	...	......
36	·20	·031	·3	·045
37	...	......	·4	·060
38	·6	·095	·7	·11
39	·8	·126	·9	·14
40	1·05	·166	1·1	·17
41	1·3	·206	1·6	·25
42	1·8	·285	2·0	·31
43	2·1	·333	2·8	·44
44	2·6	·412	3·5	·55
45	3·0	·476	4·5	·77
46	3 6	·571	5·8	·92
47	4·2	·659	7·2	1·12
48	4·9	·791	8·8	1·39
49	5 6	·889	10·9	1·72
50	6·2	·984	13·6	2·15
51	...	......	15·1	2·39
52	...	......	18·6	2·95

§ 7. *Experiment* 4.—The magnitude of the current found on connecting any galvanometer between one of the cylinders and the positive electrode of the lamp was found to be dependent to some degree on the perfection of the vacuum. Lamp No. 3 when first made had not a very perfect vacuum. A series of measurements was, however, made with it, and the same repeated after re-exhaustion. The results are tabulated together below.

TABLE NO. 6. LAMP NO. 3. *Elliott Galvanometer.*

Table showing the relative values of the potential-difference between cylinder Y and the positive electrode for good and imperfect vacua in the case of lamp No. 3.

Working volts of the lamp.	Very high vacuum in the lamp.		Imperfect vacuum in the lamp.	
	Watts per candle-power taken up in carbon.	Volts between cylinder Y and positive electrode.	Watts per candle-power taken up in carbon.	Volts between cylinder Y and positive electrode.
40	6·75	2·02	9·1	4·17
41	5·75	2·89	7·9	7·25
42	5·30	3·62	6·5	8·39
43	4·90	4·54	5·9	9·80
44	4·50	6·08	4·3	11·96
45	4·15	7·50	4·0	13·76
46	3·84	9·25	3·7	15·08
47	3·55	10·33	3·4	16·21
48	3·30	13·2	3·2	18·28
49	3·07	16·3	3·0	22·05
40	2·90	18·4	2·8	22·99
51	2·74	21·8	2·6	23·93
52	2·58	24·4	2·4	28·26

The imperfection of the vacuum is indicated by the higher watts per candle-power absorbed at low voltages, and we see that at any given working pressure the potential-difference between the positive electrode and the cylinder Y embracing the top of the negative leg is *greater* when the vacuum is imperfect than when it is very good. The presence of residual air tends to bring down the potential of the embracing cylinder more nearly to that of the carbon at the point adjacent to it.

§ 8. *Experiment* 5.—A series of observations was next made in which the potential-difference between the middle plate and the positive electrode was determined by the aid of a condenser. If a condenser of capacity C in microfarads is charged to a potential of V volts and discharged through a ballistic galvanometer, we can determine the ballistic constant of the galvanometer. A second observation of a like nature in which the "throw" of the same galvanometer is observed when the same condenser is charged by contact with two points concerning which we require to know the potential-difference, gives us the means of calculating the electrostatic potential-difference in volts. A condenser of ·987 microfarad capacity carefully determined was charged to a potential of 54 volts and discharged through a certain ballistic galvanometer having a needle whose periodic time of vibration was about three seconds. The resulting "throw" of the galvanometer was 5° 30′. Hence a discharge of $54 \times \cdot 987 = 53\ 3$ microcoulombs through the galvanometer produces a "throw" of 5° 30′. Neglecting a very small correction for the logarithmic decrement, in this case not of importance, we have for the ballistic constant R the value

$$53{\cdot}3 = \mathrm{R} \sin \tfrac{1}{2}(5^\circ\ 30') = \mathrm{R} \times \cdot 04798\ ;$$

hence

$$\mathrm{R} = 1110.$$

The same condenser was then connected between the middle plate and positive electrode of lamp No. 4 and then discharged through the same ballistic galvanometer. The lamp was subjected to a working pressure of 39 volts as determined by a corrected voltmeter attached to the electrodes of the lamp. The charge of the condenser was sent through the ballistic galvanometer, and a "throw" of 4° obtained. If v is the potential-difference between the middle plate and positive electrode of the lamp, we have the following equation for v in terms of the ballistic constant and angle of "throw":

$$\cdot 987\, v = 1110 \sin 2^\circ$$

$$= 1110 \times \cdot 0349$$

or

$$v = \frac{111 \times 349}{987} = 39 \text{ nearly.}$$

The potential-difference between the middle plate and the positive electrode as determined by this method is therefore *exactly the same* as the potential-difference between the positive and negative electrodes of the lamp. In other words, when the filament is brought to full incandescence, the middle metal plate is brought to the same potential as the negative electrode of the lamp. This observation was repeated with several other lamps having middle plates in various positions and of various forms, and always with the same result, viz., that the potential of the middle plate when insulated is brought down nearly to that of the negative electrode.

§ 9. *Experiment* 6.—In order to confirm the results obtained by the condenser method and to eliminate all the conditions which necessarily exist when we attempt to measure potential-difference galvanometrically, an electrostatic method of measuring the potential-difference at any instant between the metal plate and the positive electrode of the lamp was next used. For this purpose a Kelvin multicellular electrostatic voltmeter was employed to determine the potential-difference between the positive and negative electrodes of the lamp and between the positive electrode of the lamp and the middle plate, with the following results:—

A lamp (No. 4) having the plate fixed between the carbon legs was raised to various working voltages and the potential-differences above mentioned taken.

TABLE NO. 7. LAMP NO. 4. *Kelvin Electrostatic Voltmeter.*

Working volts of the lamp.	Static potential-difference in volts between middle plate and positive electrode of the lamp.
41	41
58·1	58·7
61·0	61·5

These observations confirm conclusively the previous results. The insulated metal middle plate is in this case brought to the *same potential* as the base of the negative leg of the carbon; and hence, on measuring electrostatically the potential-difference between that metal plate and the positive electrode of the lamp, we find it to be the same as the potential-difference between the two electrodes of the lamp.

§ 10. *Experiment* 7.—In order to see if this was the case when the metal collecting-plate had a very small surface placed at some distance from the negative electrode of the lamp, the lamp called No. 1 was employed. In this lamp a platinum wire threaded through the turns of a double spiral 100 volt carbon lamp (see § 11). The lamp was raised to various working voltages, and the electrostatic voltmeter employed to measure at the same time the static potential-difference between the positive electrode of the lamp and the platinum wire, with the following results:—

Table No. 8. Lamp No. 1. *Electrostatic Voltmeter.*

Working volts of the lamp.	Static potential-difference in volts between platinum wire and positive electrode of the lamp.
62	53
79	75·5
97	85
118	107

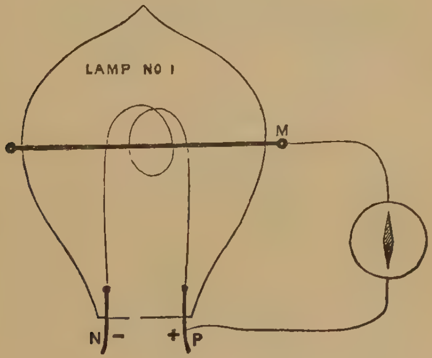
The figures in the above Table No. 8 show that when the surface of the collecting-plate is very small and is placed some distance from the base of the negative leg of the carbon it is brought down only to the potential of some point (probably the nearest point) on the carbon conductor, and that therefore the potential-difference between the plate and positive electrode of the lamp is somewhat less than the potential-difference between the working terminals of the lamp. At the same time, however, the electrostatic voltmeter shows no measurable potential-difference between the negative terminal of the lamp and the platinum wire, and the most sensitive galvanometer between these points gives no indication of any current.

By means of the electrostatic voltmeter it was, however, ascertained that in those cases in which the metallic plate presented considerable surface (several square centimetres) and was placed so that some portion of it was not removed by more than a centimetre or two from the base of the negative leg of the carbon, it was brought down almost immediately to the potential of the negative terminal of the lamp. If the middle plate is placed at a little distance from the carbon loop then, on testing by the condenser method, it is found that the plate is not instantly brought down to the potential of the negative terminal, but that some few seconds have to elapse before this is the case.

§ 11. A series of experiments was then undertaken in order to determine the effect of varying (1) the surface, and (2) the position of the metal plate in the bulb, and in these experiments the plate was sometimes of platinum and sometimes of aluminium. In all cases the vacuum was a very perfect one, any occluded gases in the plates being got rid of by special means.

Experiment 8.—A normal 100-volt carbon-filament lamp, having a carbon filament coiled in a spiral of two turns (see fig. 7) had a short stout platinum wire (·024 inch diam.) sealed across the bulb so as to thread through, without touching, the spirals of the carbon. The lamp at 100 volts took 1·54 amperes and gave an illumination of 40 candles, equivalent to a power absorption of 3·9 watts per candle-power. The vacuum was very good. This lamp will hereafter be called Lamp No. 1. As before, no current could be detected by a galvanometer when joined up between the platinum wire and the negative electrode, but when the galvanometer was connected between the platinum wire and the positive elec-

Fig. 7.



trode of the lamp a current of some milliamperes was found passing through it. As in the case of lamp No. 4, this lamp was characterized by a great tendency to change suddenly the value of the current flowing through the galvanometer when the working volts on the lamp were kept perfectly constant. In the first series of observations the milamperemeter was employed to measure the current flowing between the positive electrode of the lamp and the platinum wire when it was connected between these points, and at and beyond a working-pressure of 90 volts or so the galvanometer would often jump suddenly from one reading to another, when the lamp working volts were kept perfectly constant.

In the following table, No. 9, are collected the results when the working pressure of the lamp was gradually raised from 80 to 100 volts :—

Table No. 9. Lamp No. 1. *Milamperemeter.*

Table showing the potential-difference between the positive electrode and the platinum wire, and the current flowing through the galvanometer connecting them, for various voltages of the lamp.

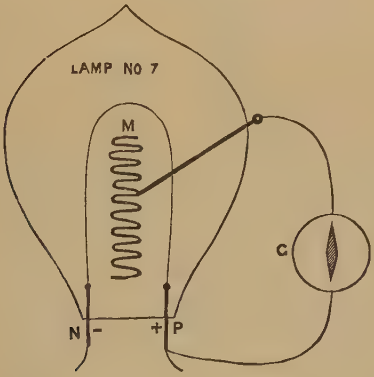
Working volts of the lamp.	Volts between platinum wire and positive electrode.	Milliampere current through the galvanometer.	Working volts of the lamp.	Volts between platinum wire and positive electrode.	Milliampere current through the galvanometer.
80	·1	·016	92	1·7	·270
"	·2	·032	"	2·4	·381
82	·3	·048	93	1·9	·302
83	·2	·032	"	2·6	·413
84	·4	·064	94	3·2	·509
86	·4	·064	"	9·4	1·49
"	·7	·114	95	4·9	·78
87	·5	·080	96	5·8	·92
88	·8	·127	97	7·1	1·13
"	·9	·142	98	8·3	1·32
90	1·0	·159	99	8·9	1·41
"	1·3	·206	100	8·6	1·37
91	1·4	·222	"	10·3	1·64

These figures show that at any definite working electromotive force of the lamp the current between the positive electrode and the middle plate has very variable values, and that it suddenly changes from one value to another without any apparent reason, the working volts of the lamp remaining constant all the time.

If the surface of the collecting-plate is large, say several square centimetres, the potential-difference existing between it and the positive electrode is not found to be so much reduced by attempting to measure it with a galvanometer of about 6000 ohms resistance as it is when the collecting wire presents, as in this lamp No. 1, only a small total surface of about one square centimetre.

§ 12. *Experiment* 9.—A horse-shoe carbon filament, taking 1·3 ampere of current at a working-pressure of 42·5 volts, had a middle plate made of a long piece of platinum wire bent up in a zigzag shape so as to form a rectangular-shaped grating (see fig. 8). The object of this was to ascertain whether a middle plate offering a surface pierced with many apertures was as effective in producing the current as a solid plate of about the same general outline. Practically it was found that this was the case.

Fig. 8.

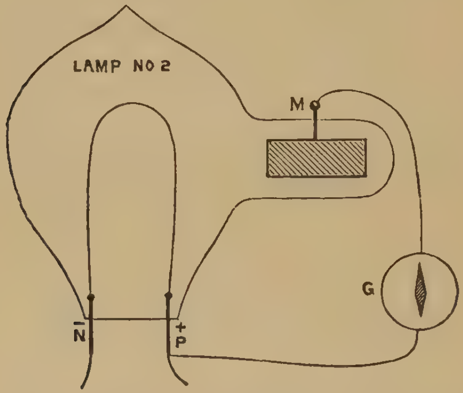


The magnitude of the currents obtained at various working voltages are of the same magnitude approximately as in the case of a lamp like No. 4, that is to say some 3–4 milliamperes at full incandescence.

§ 13. A set of experiments was then undertaken with the object of examining the special effect of varying the position of the middle plate, and a series of lamps was used in which platinum or aluminium plates held on platinum wires were placed in the lamp bulb, or in tubes opening into it, in various positions. These lamps are generally 50-volt lamps of usual type, and had single horse-shoe shaped filaments.

Experiment 10.—A lamp-bulb had a side tube blown on it (see fig. 9) and a plate about 6 centims. long and 1·5 centims. wide welded to a platinum wire was sealed into it. The platinum plate was placed vertically and edgeways in the side tube and the side tube was in such a position that the plane of the platinum plate coincided with the plane of the horse-shoe filament. This lamp, called henceforth No. 2, when worked at 48 volts took 1·3 amperes of current and gave a light of 17·5 candles, equivalent to a power-consumption of 3·55 watts per candle-power. The vacuum was very good. In the case of this lamp the current between the positive electrode of the lamp and the platinum plate was found to be numerically very much smaller at the usual working pressure of the lamp than was found to be the case in those lamps in which the middle plate was placed between the carbon legs or in the form of a cylinder embracing the carbon.

Fig. 9.



The current obtained at any definite working voltage was considerably greater when the leg of the carbon nearest the plate was the *positive* leg than when it was the *negative* leg. A series of observations were taken using the lamp at different voltages and measuring with the Elliott galvanometer the potential-difference between the platinum plate and positive electrode of the lamp, and these results were as tabulated below in Table No. 10.

TABLE NO. 10. LAMP NO. 2. *Elliott Galvanometer.*

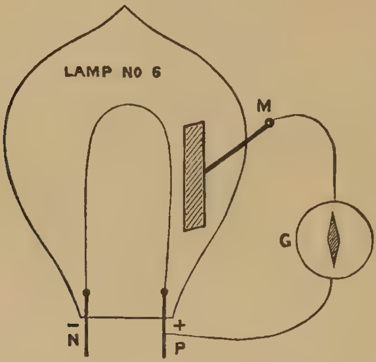
Table showing the potential-difference between the platinum plate and the positive electrode of the lamp at various working voltages. Positive leg of carbon nearest the platinum plate.

Working volts of the lamp.	Watts per candle-power.	Volts between platinum plate and positive electrode.	Working volts of the lamp.	Watts per candle-power.	Volts between platinum plate and positive electrode.
43	6·26	·088	48	3·55	·199
44	5·80	·111	49	3·32	·236
45	4·42	·144	50	3·12	·289
46	4·10	·156	51	2·94	·352
47	3·80	·172			

If we compare together the results obtained with this lamp No. 2, in which a plate is placed edgeways on and outside the carbon loop, with the results obtained in the case of lamp No. 3, in which the plates embraced the carbon in the form of cylinders, we see the difference produced by the change of position of the plate. Both these lamps, No. 2 and No. 3, are 48-volt lamps when working at normal incandescence. Referring to Table No. 7 in § 7, we see that for lamp No. 3 at 48 volts the voltage difference of the positive electrode and the platinum cylinders was respectively 13·2 and 18·3 volts as measured with the Elliott galvanometer, and this indicated a current of about 1·3 and 1·8 milliamperes flowing through the resistances from the positive lamp electrode to the metal plate; but in the case of lamp No. 2, at 48 volts the potential-difference between the platinum plate and the positive lamp electrode was only ·2 volt, and this corresponded to a current of ·03 milliampere nearly. Accordingly the current is greatly diminished when the collecting-plate is placed edgeways to and someway outside the loop of the carbon. At normal incandescence the current between the positive lamp electrode and the middle plate when joined by the galvanometer is about ·03 or ·04 milliampere when the *positive* leg of the carbon is nearest the middle plate, but only about ·02 or ·03 milliampere when the negative leg is nearest the plate.

§ 14. *Experiment* 11.—In order to compare the previous results just given with those obtained when the collecting-plate was placed broadside to and yet outside the carbon loop, a lamp was made as in fig. 10 in which an aluminium plate was held on a platinum wire just outside one leg of the carbon and with its plane perpendicular to the plane of the horse-shoe.

Fig. 10.



The aluminium plate was 5 centims. long and 1 centim. wide, and distant from the nearest leg of the carbon about ·5 centim. This lamp therefore differed from lamp No. 4 in having the plate outside the carbon loop rather than between the legs. It may be noticed that in this lamp the current here obtained by joining the positive lamp electrode to the plate through a galvanometer was slightly greater when the leg nearest the plate was *negative* than when it was the positive leg, whereas in the case of lamp No. 2 it is just the reverse. This lamp exhibited also the same effect as lamp No. 4, in that the current flowing between electrode and plate is very liable to "jump" from one value to another even when the lamp is kept at constant volts. The following tabular result of the observations shows this. This lamp is called Lamp No. 6 and was a 50-volt horse-shoe carbon lamp, taking 1·33 amperes of current at a working electromotive force of 50 volts.

TABLE No. 11. LAMP No. 6. *Milamperemeter.*

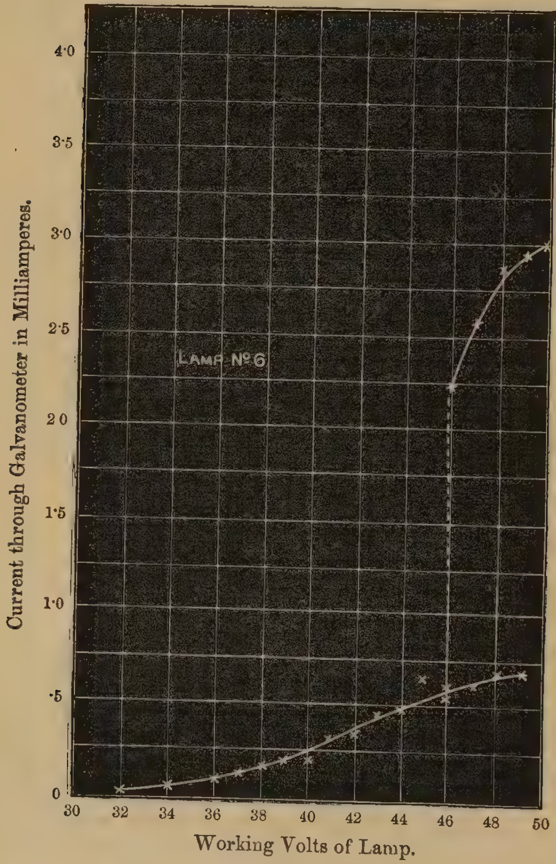
Table showing the potential-difference between the positive electrode of the lamp and the aluminium plate, and the current in milliamperes flowing through a galvanometer connecting them. Vacuum good.

Working volts of the lamp.	Potential-difference of the positive electrode and plate in volts.	Milliampere current through the galvanometer.	Working volts of the lamp.	Potential-difference of the positive electrode and plate in volts.	Milliampere current through the galvanometer.
32	·1	·016	46	3·8	·601
34	·3	·047	,,	3·5	·555
36	·6	·095	,,	14·1	2·23
37	·8	·126	47	3·8	·602
38	1·0	·158	,,	16·2	2·57
39	1·2	·190	48	4·1	·650
40	1·7	·221	,,	17·7	2·80
41	2·0	·317	49	4·2	·666
42	2·3	·364	,,	18·3	2·90
43	2·8	·444	50	19·5	3·03
44	3·1	·491	,,	20·0	3·17
45	4·0	·634	,,	4·1	·650

The results in Table No. 11 are plotted in curve No. 5.

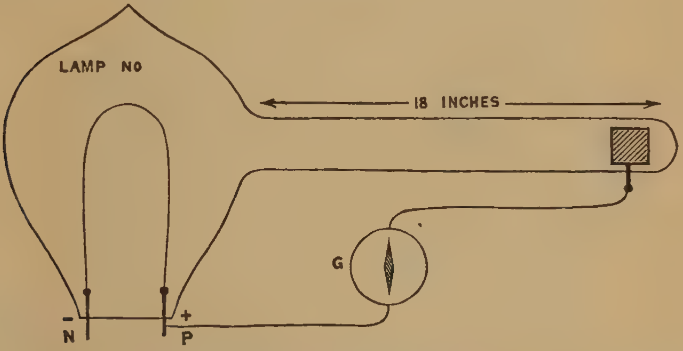
This table shows that when the lamp is kept at a constant voltage the current through the galvanometer jumps from one value to another. The fluctuation of the current takes place when the negative leg of the carbon is the one farthest from the plate. When the leg adjacent to the aluminium plate is the negative one then the current is steady at any definite voltage of the lamp.

Table No. 11.—Curve No. 5.



§ 15. From the above experiments it is clear that the current obtained when a galvanometer is connected between a metallic plate and the positive electrode of the lamp is greater in proportion as the collecting-plate is larger and in proportion as it is brought into close proximity to the base of the negative leg of the carbon. Also that a plate so placed is brought down to the potential of the negative electrode. It seemed desirable to see how far the removal of the collecting-plate to a great distance from the negative leg would influence these results, and experiments were accordingly tried with a tube of the form shown in fig. 11.

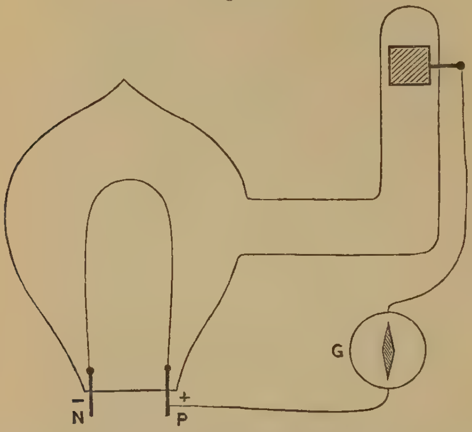
Fig. 11.



Experiment 12.—In this case a glass tube about eighteen inches long and three-quarters of an inch in diameter was attached to a lamp bulb. The end of the glass tube farthest from the bulb was closed and an aluminium plate welded to a platinum wire was sealed in near this closed end. The plate had a length of about 3 centimetres and a width of about 1 centimetre. The tube formed an extension of the bulb-space, and accordingly this arrangement formed a device by which a metal plate could be removed to a distance of some eighteen inches from the incandescent conductor contained in the bulb. On placing this lamp on a circuit and bringing the carbon to normal incandescence and connecting the terminals of the Elliott galvanometer respectively to the aluminium plate and the positive electrode of the lamp, a very small current was found to be passing through it, not, however, exceeding one ten-thousandth of a milliampere. When the galvanometer was joined in between the aluminium plate and the negative leg of the carbon no current whatever could be detected with this galvanometer, which was sufficiently sensitive to show one hundred-thousandth of a milliampere. We thus find that the removal of the plate to a distance of some eighteen inches from the incandescent conductors practically extinguishes the phenomenon.

Experiment 13.—Another similar bulb was provided having a side tube blown on it of half the length, viz. about 9 inches long. At the end of this tube was placed a small aluminium plate as before, and the tube was bent up about the middle at right angles (see fig. 12). When the carbon conductor in the bulb, which was that of an ordinary 50-volt 16 candle-power lamp, was rendered incandescent by being connected to a circuit of appropriate electromotive force, and the Elliott galvanometer connected in between the aluminium plate and the positive electrode of the lamp, a current of not more than about one twenty-thousandth of a milliampere was detected. The fact that the "Edison effect" was extinguished when the collecting-plate was placed at the extremity of an elbow-tube was first observed and recorded by Mr. Preece.

Fig. 12.



§ 16. The effect of position and size of the plate having been examined, the next step which naturally suggested itself was to determine the effect of the different portions of the incandescent conductor in the production of it.

Experiment 14.—A lamp like No. 4 was provided, but in which one leg of the carbon was enclosed in a glass tube of the size of a quill. The glass tube was sealed on to the platinum wire and extended nearly up to the bend of the carbon (see fig. 13). This lamp, called No. 9, was placed on the circuit in such a manner that the shielded leg was the *positive* leg, and a series of observations taken as usual of the current flowing through the milamperemeter when connected between the middle plate, placed between the carbon legs, and the positive electrode of the lamp. The results are as tabulated below in Table No. 12. The lamp took 1·25 ampere of current at a working electromotive force of 42 volts.

Fig. 13.

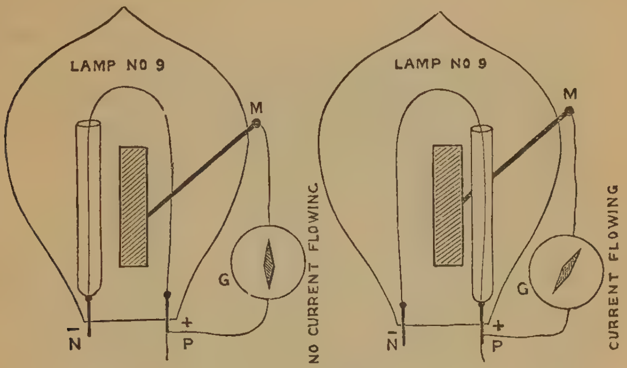


Table No. 12. Lamp No. 9. *Milamperemeter.*

Table showing the potential-difference between the middle plate and positive electrode of the lamp, and the current flowing through a galvanometer connecting them, when the *positive* leg of the carbon is shielded in a glass tube.

Working volts of the lamp.	Potential-difference of middle plate and positive electrode in volts.	Milliampere current through galvanometer.	Working volts of the lamp.	Potential-difference of middle plate and positive electrode in volts.	Milliampere current through galvanometer.
36	·2	·032	45	6·9	·936
38	·3	·048	46	7·5	1·19
40	·95	·12	47	9·7	1·53
41	1·2	·19	48	12·4	1·96
42	1·8	·29	49	15·1	2·39
43	3·1	·49	50	19·1	3·03
44	4·2	·66	51	23·1	3·66

So far the results are quite normal, and if the results in the above Table No. 12 are compared with those in Table 2 for a similarly constructed lamp with no tubular shield, we find that the magnitude of the current flowing from the positive electrode to the middle plate is in the two cases very much the same. The lamp No. 9 was then placed on the circuit in such a manner that the leg shielded by the glass tube was the *negative* leg, and a similar series of observations of the current between the positive leg (now the uncovered leg) and the middle plate was made. The results were as follows in Table No. 13 :—

Table No. 13. Lamp No. 9. *Milamperemeter.*

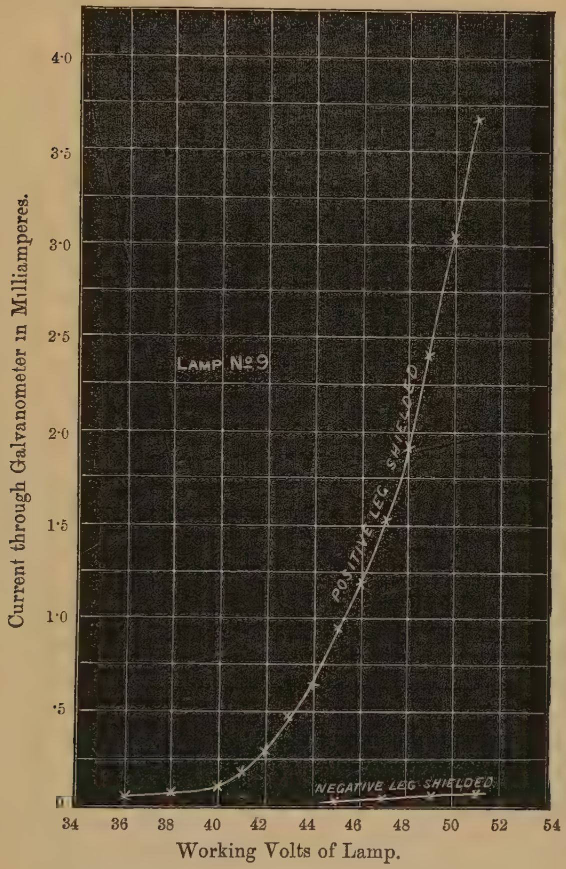
Table showing the potential-difference between the middle plate and positive electrode, and the current flowing through a galvanometer connecting them, when the *negative* leg of the carbon is shielded in a glass tube.

Working volts of the lamp.	Potential-difference of middle plate and positive electrode in volts.	Milliampere current through galvanometer.
45	·10	·016
47	·15	·024
49	·25	·039
51	·30	·044

On comparing the results in Table No. 13 and the previous one No. 12 we see what an immense reduction in the current flowing between the positive electrode and the middle plate is produced by shielding the *negative* leg. Hence the action in virtue of which the current is produced is greatly interfered with by enclosing or covering up the negative leg of the carbon. In this particular case at 51 volts, the current between the positive electrode of the lamp and the middle plate when the *negative* leg is covered up is only $\frac{1}{83}$ of that which it is when the *positive* leg is covered up. The results in Tables Nos. 12 and 13 are plotted together in Curve No. 6.

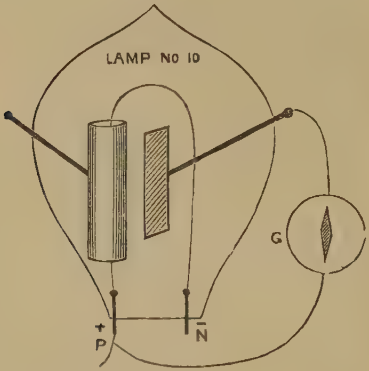
§ 17. The question remained to be settled whether the nature of the tubular screen in any way affected the results, and the glass tube was accordingly replaced by a metal (aluminium) tube and a lamp (No. 10) taken in which one of the carbon legs (see fig. 14) was surrounded by an aluminium cylinder extending nearly the whole length of the leg, and also a middle plate of aluminium was placed between the legs. Both the plate and the cylinder were held on platinum wires sealed through the glass. The lamp took 1·25 amperes of current at 41·5 volts.

Tables No. 12 and 13.—Curve No. 6.



Experiment 15.—The lamp was placed on a circuit so that the leg shielded by the aluminium cylinder was the *positive* leg. The milamperemeter was then connected between the positive electrode of the lamp and the middle plate, and the usual measurements made. It was found that the current "jumped" a good deal, and that high and low values of the galvanometer current occurred, even when the terminal voltage of the lamp was kept perfectly constant.

Fig. 14.

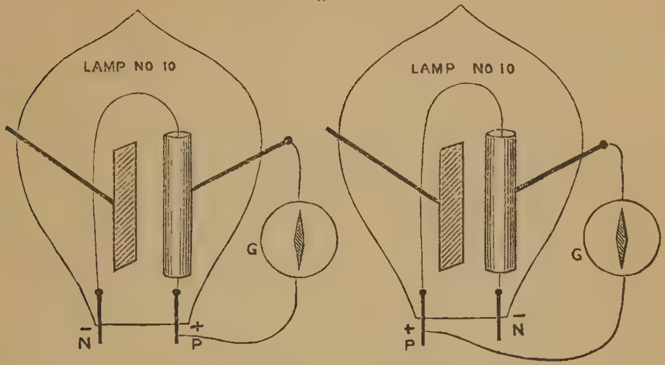


The lamp was then reversed on the circuit so that the shielded leg was the *negative* one, all other arrangements remaining the same. The current now between the positive electrode and the middle plate was practically zero, at any rate too small to be measured with this galvanometer. Hence we see that shielding the negative leg, whether by glass or a metallic cylinder, entirely cuts off the production of a current between the positive lamp electrode and the middle plate.

§ 18. *Experiment* 16.—Another series of experiments was made with the lamp No. 10 in which the galvanometer was connected between the positive electrode of the lamp and the aluminium cylinder (see fig. 15), the leg inside the cylinder being either the positive *or* the negative leg. In this case the middle plate remained unused and insulated and acted as a shield between the cylinder and the carbon leg which was not contained in the cylinder.

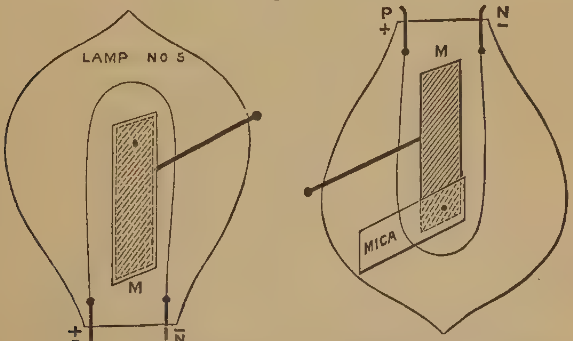
It was found that when the cylinder surrounds the negative leg and its surface is, therefore, as much exposed to it as possible the current is a maximum, but that when it includes the positive leg the current is greatly diminished, both by reason of the fact that it opposes less surface to the negative leg, and also because the middle plate acts as a shield between it and the negative leg of the carbon.

Fig. 15.



§ 19. *Experiment* 17.—In order finally to demonstrate that the negative leg of the carbon loop was the chief active agent in this production of a current between the middle plate and the positive electrode, a lamp like No. 4 was taken having a metal middle plate between the legs, and this middle plate had attached to it a mica screen (see fig. 16) a little

Fig. 16.



MIDDLE PLATE SHIELDED

MIDDLE PLATE EXPOSED

larger than the plate, and so fixed by a loose rivet that it could be shaken in front of the plate so as to shield one side of it, or shaken on one side so as to fully expose the plate. This device was in fact a removable shield attached to one surface of the metal middle plate, and when placed up against it, it shielded one surface from, and when jerked on one side it exposed that surface to, the carbon leg opposite to that surface. This lamp (called No. 5) was set on the circuit in the first place so that the leg of the carbon horse-shoe opposite to the mica-shielded side of the middle plate was the *positive* leg. If the carbon was brought up to an incandescence corresponding to about 3·5 or 4 watts per candle-power and the galvanometer connected between the positive electrode and the middle plate, then it was found that the effect on the galvanometer current which was produced by the interposition or withdrawal of the screen of mica between the positive leg and the plate was not very great. It reduced the current through the galvanometer from about ·44 milliampere to ·38 milliampere. If, however, the current flowing through the lamp carbon is reversed in direction so that the mica screen is interposed on that side of the middle metal plate which faces the *negative* leg, the result is very different. When the screen is down, the current flowing through the galvanometer from the positive electrode to the middle plate being as before ·44 milliampere, the interposition of the mica screen on the side of the plate facing the negative leg reduced the current at once to zero. We find therefore, that the interposition of a mica screen between the middle plate and the negative leg reduces to zero the current flowing between the positive lamp electrode and the middle plate. As in all other lamps with a middle plate set exactly between the legs, the current through the galvanometer joining the middle plate and positive electrode is very liable to "jump" from a low to a higher value or *vice versâ*. When the current has its higher value corresponding to any given voltage on the lamp terminals, the effect of screening is less marked, and although the interposition of the mica screen on the side facing the negative leg has an effect of reducing the current flowing through the galvanometer connected between the middle plate and positive electrode, it is not by any means reduced to zero.

§ 20. The foregoing experiments afford proof that the production of the current through a galvanometer joined between the positive electrode of the lamp and a metal plate placed somewhere in the vacuous bulb, is an effect due chiefly to the *negative* leg of the carbon, and that shielding the negative leg by enclosing it in a glass or metal tube, or covering with a mica screen that surface of the plate which is exposed to the negative leg, either quite prevents or greatly reduces the production of this current. The experiments also have shown that the magnitude of the current flowing through the galvanometer is increased by bringing the plate near to the base of the negative leg, or, better still, making the metal plate in the form of a cylinder and making this cylinder surround the *negative* leg near its base; and correspondingly it is diminished by removing it from the negative leg to a considerable distance, or by shielding this collecting-plate from the radiation from the negative leg of the carbon. The experiments with the condenser have also given evidence that when an insulated metal plate is sealed into a lamp, this plate is brought down either instantly or in a very short period of time to the potential of the *negative* leg near its base or to that of the *negative electrode* of the lamp. In looking for an explanation of these facts we are assisted by our previous knowledge that in carbon incandescence lamps, when working at an efficiency equivalent to 3 to 4 watts per candle-power, there is a gradual loss of carbon from all parts of the conductor. We know also that the carbon molecules which are projected from the conductor are thrown off into a space so highly vacuous that their mean free path is of a length comparable with, or greater than that of the dimensions of the glass bulb. The existence of molecular shadows in incandescence lamps [*] affords evidence that from intensely heated portions of the carbon conductor carbon molecules are projected in straight lines and move freely forward until they impinge against the glass. Commercial experience informs us that at and above a temperature corresponding to 3 watts per candle-power this loss of carbon becomes very rapid and thins away the filament in one place, or generally reduces the diameter of the carbon conductor. Hence we have every reason to believe that when in a normal state of incandescence the carbon conductor in a lamp is throwing off in all directions carbon molecules, and that in the vacuum usually obtained the mean free path of these projected molecules is comparable with the dimensions of the vessel containing the conductor. The whole of the experiments which are detailed here seem to be capable of consistent interpretation if we may justifiably make the hypothesis that these carbon molecules or atoms so projected from the conductor when intensely heated by the current flowing through it are all *negatively* charged. Some of the observed facts seem to point to the conclusion that the molecules projected from the incandescent conductor, whether they are portions of the conductor itself or molecules of the residual gases, respectively carry away negative charges proportional in magnitude to the potential of the conductor at the point from which they are thrown off. They may, therefore, be looked upon as condensers of small but definite electrostatic capacity charged to the potential (negative) of that part of the incandescent conductor at which they separate from it. We have then in addition to explain how it comes to pass that there are few or no projected molecules charged positively. Two suggestions may be made on this point: either the radiation of matter is wholly confined to that half of the conductor at a negative potential or the incandescent carbon molecule thrown off from the heated conductor cannot retain a positive charge. There is much to lead to the conclusion that from *all parts* of the incandescent carbon conductor there is a constant radiation of matter carrying a negative electric charge. The nearer down to the negative electrode of the carbon we select our point of observation the greater is this molecular charge found to be. It will be convenient to denote this conveyance of electric charge by moving charged molecules by the term *molecular electrovection*. We can then state the hypothesis thus—from all portions of the negative leg of the carbon loop a process of molecular electrovection is going on when the conductor is incandescent, the molecular charge being negative, and equal in potential to that of the point on the conductor from which it is projected.

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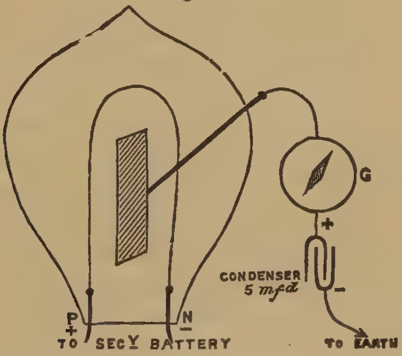
[*] See J. A. Fleming, 'Philosophical Magazine,' August 1885, p. 141.

§ 21. On the assumption that a molecular shower of negatively charged atoms was being projected against the middle plate when the conductor of the lamp was incandescent it was considered probable that a positively charged conductor connected to the middle plate would be discharged, and this was found to be the case.

Experiment 18.—A lamp of the form of No. 4, having a middle metal plate placed between the carbon legs, had its middle plate connected to one terminal of the Elliott galvanometer. The other terminal of the galvanometer was connected to one terminal of a condenser of 5 microfarads capacity. The other terminal of the condenser was connected by a wire to the gas pipes of the laboratory. The lamp was actuated by secondary batteries (see fig. 17) not very well insulated. If the condenser was charged to a potential of 50 volts so that the plate next the galvanometer was *positively* charged, then this positive charge was *instantly discharged* when the carbon was rendered incandescent. If, however, the plate of the condenser in connexion with the middle plate through the galvanometer was charged *negatively* the condenser was not discharged when the lamp was illuminated by rendering its carbon incandescent. It is a very striking experiment to see a condenser charged with this

Fig. 17.



amount (250 microcoulombs) of electricity instantly discharged when its positive coating is brought into connexion with the middle plate of such a lamp. The discharge may be brought about either by joining up the positive side of the condenser to the middle plate first, and then rendering the carbon of the lamp incandescent by switching on the lamp, or the lamp may be first of all illuminated and then the junction of the condenser effected. In both cases the middle plate when positively electrified is instantly discharged.

It was found that if the lamp carbon is rendered incandescent by a highly insulated secondary battery, then in order to produce the discharge, the plate of the condenser not in connexion with the middle plate, and which is *negatively* charged, must be somewhere connected with the battery circuit. It does not matter, however, whether the wire from the negative side of the condenser is in connexion with the positive or the negative pole of the secondary battery actuating the lamp; all that is necessary is that the negative side of the condenser should be in conducting connexion with the circuit of the incandescent carbon. The experiment may be interpreted by considering that this negative charge of the condenser can escape out of the incandescent conductor and discharge across the highly vacuous space to the positively electrified cool middle metal plate; but that a positive charge cannot be discharged out of the hot conductor, or, which amounts to the same thing, a negative charge cannot discharge across from the cool metal plate to the incandescent carbon which is positively charged. We have then a unilateral conductivity exhibited by this highly vacuous space bounded by two electrodes one of which is incandescent and the other of which is cold. Negative electricity is discharged at once out of the hot surface but not out of the cold, and a negative discharge can take place from hot to cold but not *vice versâ*. When the discharge of a charged condenser is effected by connecting the positive plate, through a galvanometer, with a metal plate sealed into the lamp and the negative plate with the lamp circuit, and then switching on the lamp, there is a curious instant of delay before the discharge begins. When the metal plate is placed very near the negative leg of the carbon the discharge of the condenser is complete in one instant. This the case when a lamp of the type No. 4 (fig. 18) is used. If, however, we employ a lamp of the type No. 2 (fig. 10), in which the metal plate is at some distance from the negative leg of the carbon, the discharge of the condenser is long drawn out and the electric charge in it is as it were tapped off slowly and not in one short sharp discharge.

Moreover this effect of discharging a condenser takes place only when the carbon is above a fair red heat. At brilliant incandescence and when the carbon is above a temperature corresponding to 3 watts per candle-power, the discharging power of a lamp of the type of No. 4 is very great. A condenser of 10 or 20 microfarads capacity charged to 50 volts is discharged instantly if its positive plate is connected to the metal plate placed not far from the negative end of the incandescent carbon conductor.

The foregoing results were confirmed with lamps of other types. Using, for instance, a lamp like No. 6 with the aluminium plate placed outside the carbon horse-shoe and near the leg, the same discharging power for positive electricity was found. It was not dependent on the direction of the current through the lamp carbon, although it seemed a little more vigorous when the leg nearest the plate was the negative leg. As above observed, the rate of discharge was much reduced when employing a lamp having the metal plate placed edgeways on to the carbon and some way from it, as, for instance, when employing a lamp of the form of No. 2.

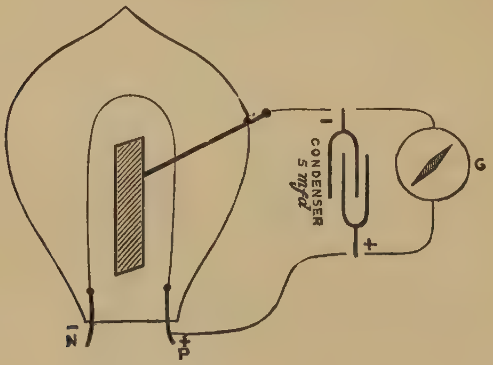
§ 22. *Experiment* 19.—A series of experiments was in this case also tried to determine the effect of shielding the negative leg of the carbon. The lamp No. 9 was employed, in which one leg of the carbon was enclosed in a glass tube connecting the positive plate of a charged condenser through a galvanometer with the middle plate of the lamp, and the negative plate of the condenser somewhere to the battery circuit; it was found that when the shielded leg of the carbon was the positive leg the condenser was discharged as before. If, however, the shielded leg or leg enclosed in the glass tube was made the negative leg, which could be done by reversing the current through the carbon conductor, then the condenser was not discharged when its positive plate was connected with the middle plate.

The same fact was less perfectly exhibited by employing the lamp with the middle plate having a removable mica shield on one side. We are thus able to assure ourselves that the active agent in producing this discharging effect upon a positively charged body connected to the middle plate is the negative leg of the carbon conductor. The experiments were varied in many ways, but all pointed to the conclusion that if a charged condenser is connected to two terminals, one of which is a metal plate and the other a carbon conductor, both enclosed in a high vacuum but yet separated from each other by an inch or so of distance, the condenser is discharged instantly when the carbon terminal is rendered highly incandescent, provided that the negative plate of the condenser is in connexion with it.

§ 23. If the condenser is left in contact with the middle plate under some circumstances, not only is it discharged if previously charged but is charged again in an opposite direction.

Experiment 20.—A condenser of 5 microfarads capacity perfectly discharged has its poles or terminals connected for one instant, one with the middle plate of No. 4 lamp and the other with the positive electrode of the lamp (see fig. 18).

Fig. 18.



On removing it and testing it with the galvanometer G it is found that the condenser plate in connexion with the middle plate of the lamp has received a negative charge and the other plate of the condenser a positive charge.

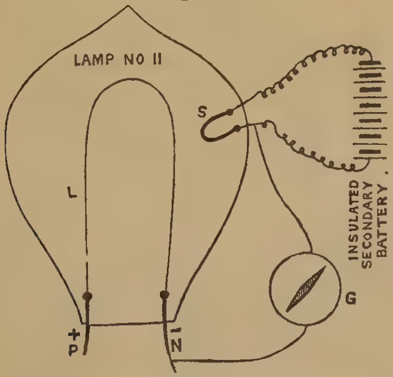
If, however, the condenser is connected between the negative electrode of the lamp and the metal middle plate of the lamp, on insulating and testing it we find it has not the slightest charge.

It is very astonishing to see how instantly a condenser of very large capacity is charged when one pole of the condenser is connected to the middle plate and the other to the positive electrode of the lamp.

§ 24. In considering the behaviour of the heated carbon electrode and the cool metal plate in their respective powers of discharging the positive or negative charge of the condenser, it seemed that the fundamental fact was the power of the heated surface to discharge negative electricity out of itself. Hence arose the question, how far the observed facts would be modified if the middle metal plate itself could be also heated. One way by which this might have been done would have been to have rendered this plate incandescent by heating it by radiant heat concentrated by means of a powerful mirror or lens. Some experiments tried in this way were not satisfactory, and consequently a method was adopted in which a middle plate of carbon could be rendered incandescent electrically.

Experiment 21.—A vacuum tube was provided with two carbon conductors (see fig. 19), one the ordinary carbon filament L of a 50 volt lamp, and the other the small carbon S of a 4 volt lamp. The smaller carbon was sealed in the

Fig. 19.



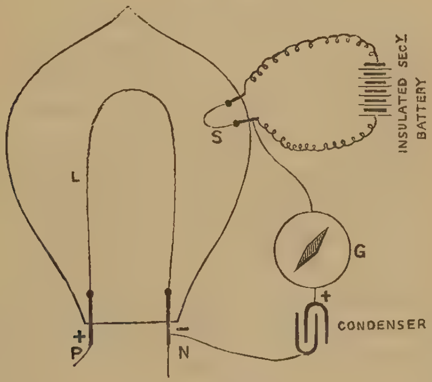
usual way through the glass and placed so as to stand symmetrically between the legs of the larger carbon loop.

The smaller carbon could be rendered incandescent by an insulated battery of fifteen secondary cells, appropriate resistance being introduced. The larger carbon also could be rendered incandescent by the proper electromotive force. If the smaller carbon was kept cold and employed simply as a third electrode or middle plate, all the phenomena previously described as happening with metal middle plates of aluminium or platinum took place. If the small (cold) carbon loop is connected through a galvanometer with the positive electrode of the larger carbon loop when this last is rendered incandescent by a current, we find as usual a current of a few milliamperes passing through the galvanometer from the positive electrode of the larger carbon to the small carbon. If the small carbon (still cold) is connected through the galvanometer to the negative electrode of the larger carbon we get no current. This is the normal effect, and it is the same for a cold carbon conductor used as a middle plate as for a metal middle plate.

Experiment 22.—The next experiment consisted in making this small carbon incandescent by an insulated secondary battery, appropriate resistance being inserted so that the carbon was brought to the normal condition of temperature as indicated by its incandescence. When this was done the galvanometer was inserted between the positive electrode of the large carbon loop and one of the electrodes of the small carbon loop. A current was obtained as before. On connecting the galvanometer between the *negative* electrode of the large carbon loop and one of the electrodes of the small carbon loop, a current of nearly equal value was now obtained. In this last experiment it was found to be immaterial whether the terminal of the galvanometer was joined to the positive or to the negative electrode of the small carbon loop. Hence we find that when the small carbon loop is not incandescent and is used as a middle plate or electrode, it is brought down together with the insulated battery attached to it to the same potential as the negative end of the large incandescent carbon, and we get as usual a current through a galvanometer connected between the positive electrode of the large incandescent carbon and any point on the small cold carbon, and no current between the negative electrode of the large hot carbon and the small cold one. On rendering the smaller carbon loop incandescent this is all changed. The smaller carbon, now hot, is not brought down to the potential of the negative ends of the larger carbon, and we get a current through a galvanometer connected between either positive or negative electrode of the large hot carbon and any point on the circuit of the smaller equally hot carbon.

§ 25. *Experiment* 23.—With this same vacuum tube having double carbons, further experiments were performed on the discharging power of the hot and cold electrodes for positive and negative electricity. The two carbons could be rendered incandescent either simultaneously or singly by two sets of insulated secondary batteries attached to each respectively. For the sake of distinction we shall speak of the large carbon loop in this bulb as the L loop and the smaller one as the S loop. A condenser of 5 microfarads capacity (see fig. 20) was employed, which was charged to a potential of about 50 volts. When the positive plate of this charged condenser was attached to the carbon L and the negative side to the carbon S, then on making L incandescent by its own insulated battery and keeping S cold, the condenser was found not to be discharged when insulated and tested by a galvanometer. If, however, the same charged condenser was

Fig. 20.

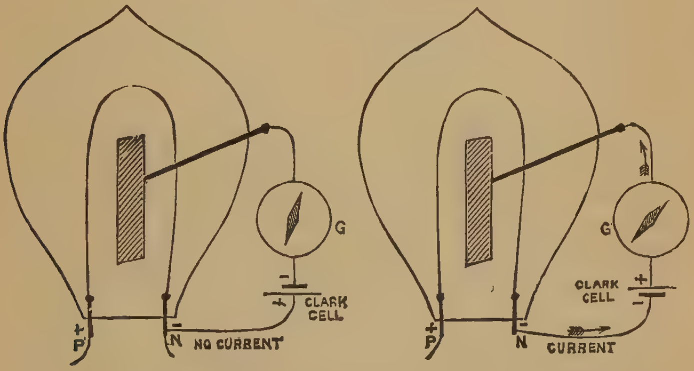


connected in the same way to the two carbons and the carbon S, to which the negative side of the condenser was attached, was made incandescent, the condenser was instantly discharged. If the direction of the charging of the condenser was reversed the same rule was found to hold good. The condenser was discharged if the *negatively* charged plate of the condenser was connected to the *incandescent* carbon loop, but not if it was connected to the cold carbon loop. Beginning with the condenser charged and connecting it in between the two carbon loops, neither of them being incandescent, then the condenser was discharged instantly if that loop to which the *negatively* charged side of the condenser was attached was rendered incandescent, but not discharged if the loop to which the positive side of the condenser was connected was rendered incandescent. If both loops were rendered incandescent simultaneously the condenser in any case was discharged, but apparently at an accelerated rate. These experiments show again that if two carbon electrodes are sealed into a high vacuum, negative electricity escapes very freely out of either electrode if it is rendered incandescent, but that the escape or discharge of positive electricity is not in the same way facilitated by heating the positive electrode. Accordingly a highly vacuous space bounded by two carbon electrodes separated by a distance less than the mean free path of the gaseous molecule at that pressure, presents a unilateral conductivity when one of these electrodes is cold and the other highly incandescent. For if the hot electrode is connected to a negatively charged body and the cold electrode to a positively charged body, discharge takes place across the vacuous space, but if the charges are reversed then no discharge takes place. The negative charge can escape from the heated electrode but not from the cold one.

§ 26. *Experiment* 24.—The question of the apparent unilateral conductivity of the vacuous space bounded by a hot and a cold electrode was then further examined by the aid of the lamp No. 6 formerly used.

In this lamp an aluminium plate is sealed into the vacuum and placed just outside the carbon horse-shoe. If a sensitive galvanometer (the high resistance Elliott galvanometer) is joined up between the metal plate and the negative electrode of the lamp, then, as in other cases when the lamp is in action, no current of a magnitude much greater than ·0001 of a milliampere is detected. If a single Clark standard cell is inserted in the galvanometer circuit (see fig. 21) with its negative pole attached to the middle plate and its positive pole to the galvanometer terminal, the current is barely if at all increased. In this case the negative pole of the Clark cell is in connexion with a cold metal electrode and the *positive* pole is in connexion through the galvanometer with the incandescent carbon electrode, and under these circumstances the galvanometer detects no current flowing. The position of the Clark cell is now reversed, and it is joined up so that its positive pole is in connexion with the middle plate and its *negative* pole in connexion, through the galvanometer, with the incandescent carbon electrode. It is then found that a considerable current of some few milliamperes in magnitude is flowing through the galvanometer. The direction of this current in the ordinary way of speaking is *from* the negative electrode of the lamp through the galvanometer *to* the metal plate sealed into the bulb. We thus find that a negative current of electricity can be made to flow across the vacuous space between the incandescent carbon and the metal plate *from* the hot carbon *to* the cooler metal plate, but not in the reverse direction. The space presents an apparently unilateral conductivity.

Fig. 21.

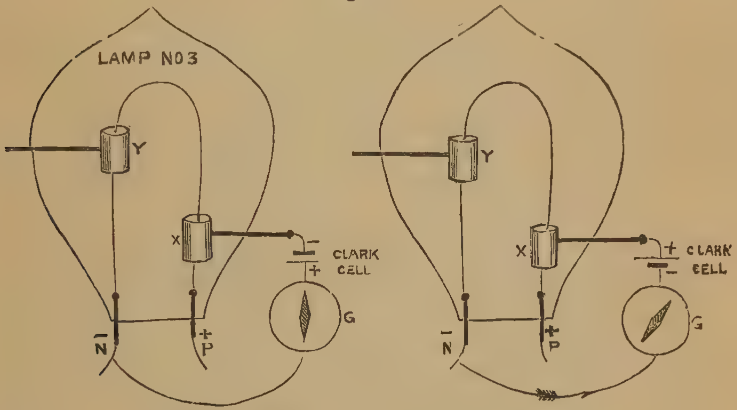


§ 27. *Experiment* 25.—The same experiment was repeated, only using instead of a Clark cell an insulated secondary battery of 25 small cells. When the secondary battery (see fig. 21) was connected with its negative pole to the metal plate and its positive pole through the galvanometer to the negative electrode of the carbon, no current greater than that found with the Clark cell similarly arranged was found ; but if the secondary battery was reversed and joined up with its positive pole to the middle plate and its negative pole through the galvanometer to the negative electrode of the incandescent carbon, then so strong a current flowed through the galvanometer that it could not be measured without shunting-down the galvanometer considerably. The same experiments were repeated with the lamps having the zigzag wire as a metal plate, No. 7, and the same general results obtained. These experiments therefore show that in a circuit which consists partly of a galvanometer-wire and partly of a highly vacuous space bounded by two electrodes—one a metal plate and the other an incandescent carbon surface,—the insertion of an electromotive force in one direction can produce a very sensible current, but that if the electromotive force is reversed then no current flows. The direction of the electromotive force must be such as to urge negative electricity from the hot surface to the cold across the vacuous space.

§ 28. *Experiment* 26.—In order to make use of different parts of the incandescent conductor as the electrode opposed to the metal plate, recourse was had to the lamp No. 3, with metal (aluminium) cylinders embracing without touching the carbon at two different places. These cylinders, as before, we will call X and Y (see fig. 22). Cylinder X was the one near the base of the positive leg of the carbon, and cylinder Y was the one near the top of the negative leg.

When the galvanometer was connected between the negative electrode of the lamp and the cylinder X surrounding the lower part of the positive leg, no perceptible current was found to be passing when the carbon was rendered incandescent. On inserting a single Clark cell in series with the galvanometer so that the negative pole of the cell was in connexion with the cylinder X and the positive pole of the cell through the galvanometer in connexion with the negative electrode of the lamp, hardly any perceptible current was found to be passing (see fig. 22). The Clark cell was then reversed, connecting the positive pole of the cell to the cylinder X and the negative pole through the galvanometer to the negative electrode of the lamp. On bringing the lamp into action, a considerable current of several milliamperes was found to be passing in such a direction that a current of positive electricity was flowing across the vacuous space from the metal cylinder to the hot carbon, or a negative current from the hot carbon to the cooler metal cylinder. On switching-off the lamp, there was a curious "kick" or "throw" of the galvanometer, indicating a sudden rush of current in the same direction as the steady current which the cell had been sending. These effects occurred also when the cylinder Y was employed, and the galvanometer with or without the cell in series joined in between the negative electrode of the lamp and the cylinder Y embracing the top part of the negative leg of the carbon (see fig. 22); but in the last case the steady current sent by the cell across the vacuous space between the cylinder and the hot carbon was only about a quarter as great in magnitude as when the cylinder X was employed. There was the same kind of "kick" of the galvanometer on breaking the lamp-circuit. These experiments evidently showed that the highly vacuous space between the hot carbon traversed by its own current, which rendered it incandescent, and the insulated cylinder possessed a sort of unilateral

Fig. 22.



conductivity, negative electricity from a separate source of small electromotive force being able to be forced through it from the hot carbon surface to the cooler metal surface, but not in an opposite direction.

§ 29. In the above-recorded experiments the carbon conductor was rendered incandescent by a unidirectional or continuous current in a highly perfect vacuum. In seeking for an hypothesis to connect them together, it became essential to ascertain how the effects would be modified if the vacuum was imperfect and if the current was alternating instead of continuous.

Experiment 27.—The fundamental experiment was therefore repeated with the normal type of lamp (No. 4), having a middle metal plate placed symmetrically between the legs of the carbon. A lamp of this type was set in action by an alternating current of suitable strength and of which the frequency was some 80 to 100 per second. On connecting the milamperemeter between *either* of the electrodes of the lamp and the middle plate, a *continuous* electric current was found flowing through the galvanometer. The direction of this current was such that positive electricity was found to flow from either lamp terminal to the middle plate of the lamp. In other words, a continuous current of negative electricity flowed out of the middle plate to one or other of the two terminals of the lamp, viz. to that terminal to which the other extremity of the galvanometer was joined. Hence, since in this case each leg of the carbon becomes in rapid succession positive *and* negative when the lamp is operated with an alternating current, the unilateral effect observed of a current flowing between the middle plate and the positive leg, when the current through the carbon is a continuous current, is here found to exist equally between the middle plate and *both* terminals of the lamp. This is only what might have been expected. The potential of the middle plate is then not the same as that of the base of either leg of the carbon, but something between the two depending upon the position of the plate.

§ 30. *Experiment* 28.—The effect of lowering the vacuum was also the subject of experiment. In a lamp with a highly perfect vacuum no current greater than about ·0001 milliampere is observed when a very sensitive high-resistance galvanometer is joined up between the negative electrode of the carbon and the insulated middle plate, and, as we have seen, the experiments with the electrostatic voltmeter showed that the plate was brought down under these circumstances to the potential of the base of the negative leg of the carbon. If, instead of employing a very perfect vacuum, a bad one is produced by imperfectly exhausting the lamp, then it is found that under these conditions the Elliott galvanometer indicates a not inconsiderable current of something approaching to a milliampere when joined in between the negative electrode of the lamp and the middle plate. Hence, when the vacuum is imperfect the equality in potential between the middle plate and the negative electrode is not maintained.

The direction of the current in this last case is such as to show that negative electricity is flowing through the galvanometer from the negative electrode of the lamp to the middle plate. In other words, negative charge is carried over from the plate to the positive leg of the carbon across the imperfectly vacuous space ; and the means by which this is effected is the residual air. This seems to afford proof that the normal effect of the molecular electrovection of negative electricity from the negative leg is due to carbon molecules, and that the presence of residual air exhibits itself, when present beyond a certain amount, in producing an effect which the carbon molecular electrovection cannot produce.

§ 31. *Experiment* 29.—It seemed very desirable to ascertain if the effect of molecular electrovection exists in the case of an incandescent platinum wire rendered vividly incandescent in a highly perfect vacuum. A bulb was accordingly constructed similar in every way to lamp No. 4, but having a platinum-wire horse-shoe conductor and a platinum middle plate. When this wire was rendered highly incandescent by a continuous current, a sensitive galvanometer (the high-resistance Elliott) showed a current of about one five-thousandth of a milliampere when connected between the positive electrode of the incandescent wire and the middle plate, but little or no current when connected between the negative electrode and the middle plate. This molecular *electrovection* current was thus very much less in magnitude than that observed in the case of the carbon filament lamps, but it is in the *same direction.* We are, however, enabled to state that at a condition of vivid incandescence just short of fusion a platinum wire *in vacuo* exhibits the same effects as a carbon filament, and that it can disturb the electrical condition of an insulated metal plate near it sealed into the same vacuum and tends to bring it down towards the potential of the negative end of that platinum wire.

On the hypothesis that all these effects are due to a scattering of negatively charged molecules from the incandescent conductor, we must affirm that the same process goes on in a platinum wire rendered incandescent in a vacuum, only that the radiation of matter is far greater in the case of the incandescent carbon than it is in the case of the incandescent platinum.

§ 33. If a lamp is selected having an insulated plate fixed between the legs of the carbon filament, it is found that under certain conditions the electric conductivity of the vacuous space between the plate and the negative leg is much affected by the presence of a magnetic field. If a galvanometer, preferably a movable coil galvanometer, having a resistance of about 500 or 600 ohms, is connected between the middle plate and the negative leg it will show but little current passing when the lamp is incandescent at normal temperature. If the volts on the lamp terminal are raised so that the filament is brought into a state of incandescence corresponding to about 2·5 or 3 watts per candle, then the galvanometer will show a small current passing through it. If then a horse-shoe magnet is held so as to create a magnetic field the direction of which is across the space between the plate and the negative leg, the current indicated by the galvanometer immediately decreases considerably. This happens irrespective of the direction of the field so long as it is across the direction of the line joining the negative leg and the middle plate. This indicates that the presence of this transverse field increases the resistance of the rarefied gas. The galvanometer current responds to the presence of the magnet in a manner which shows that the resistance to the flow of the current through the gas is increased by creating a magnetic field at right angles to the line of the current. The general fact that gaseous resistance is increased by such a transverse magnetic field has been already noted and described by Professor J. J. Thomson. The behaviour of bismuth as regards electrical resistance in a magnetic field is strikingly similar.

The "jumping" of the current from one value to a higher, which has been already mentioned, appears to be due to something equivalent to a sudden change in the resistance of the space between the negative leg and the middle plate when the lamp is in action and at high incandescence. The fact of sending a small current through this space seems to effect a change in the qualities of the rarefied gas as a conductor which makes it conduct better. There are certain after-effects in some cases which are strongly similar to the polarization of electrodes observed in the case of liquid electrolysis, and which seem to point to the validity of the view that gaseous conduction is effected by a similar process.

The experiments also confirm the opinion of Professor J. J. Thomson that gases, or at least certain gases in a rarefied condition, are very good conductors, and they show that the greatest part of the obstacle to conduction through a vacuum-tube is at the electrodes and may be largely removed by heating the kathode to incandescence.

Discussion.

Prof. S. P. Thompson said he would like to have some information as to the state of exhaustion of the lamps; whether this was such as is found in ordinary commercial lamps, or whether it more nearly approached that used by Crookes. A great change in the conductivity, &c., took place at an exhaustion slightly greater than that ordinarily found in incandescent lamps. It would be of interest to vary the size of the kathode, and to investigate whether the magnitude of the effects observed depended on the fall of potential per unit length along the filament. Another point was whether the position of the plate for which the effect was a minimum was the same for all lamps, or whether it changed with the volts and the length of the filament employed. Again, did the minimum occur at a certain fraction of the distance between the positive and negative leads, or, as was the case in some of the phenomena observed by Crookes, at a definite distance from either of the leads? These points might be investigated by means of a lamp with a straight filament, where the fall of potential per unit length along the filament could be made the same as with the loop-shaped filament, but the fall of potential per unit length in the vacuum would be different. The author's proposed experiment of heating the kathode by concentrating on it the rays of a lamp did not seem to him (Prof. Thompson) to differ materially from Crookes' experiment in which an incandescent wire, heated by a current, was used as the kathode.

Mr. SKINNER said that the heating of the kathode by means of a "burning-glass" could easily be carried out.

Mr. BLAKESLEY pointed out that it would be quite possible to produce an increase of the current by means of a magnet.

Mr. SEARLE said that Prof. J. J. Thomson had shown that a magnet affected the conductivity of a gas.

Prof. FLEMING, in his reply, said that no doubt the effects were largely dependent on the vacuum in the lamps. The lamps employed were exhausted to the ordinary commercial vacuum. Since it was found that the "treating" was more worn off the negative leg of the filament, and that a screen placed between the legs of the filament was more blackened on the side turned towards the negative leg, it would appear that the particles of carbon were shot off from the negative leg, and hence perhaps the charge was carried by these carbon molecules.

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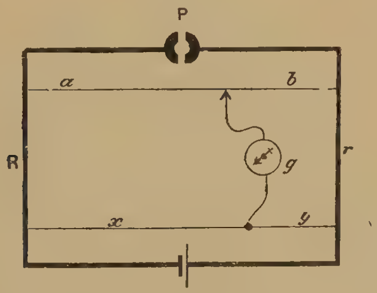
XVIII. *Adjustment of the Kelvin Bridge.*

By Rollo Appleyard*.

In a recent paper by Mr. J. H. Reeves† an alternative method of adjusting the ratios of the resistances in the Kelvin bridge was described; the following remarks may be regarded as a note upon that paper.

The measurement of a resistance by the arrangement adopted by Mr. Reeves involves two operations. It may be shown, however, that for certain purposes the two adjustments can be combined mechanically, and balance effected by a single test.

Fig. 1.



Consider the conductors R, r, a, b, x, and y in fig. 1. It is required to compare R with r. In the accepted form of the Kelvin bridge a, b, x, and y have fixed values, such that $ay=bx$, and balance is obtained by the one operation of varying R or r. Mr. Reeves prefers to keep R and r fixed; and, consequently, he has first to find provisional values of a, b, x, and y such that $ay \bumpeq bx$; and then to balance again, using these values, to find R : r; with the plug, this time, inserted at P.

But fig. 1 suggests that $a+b$, and $x+y$, or convenient parts of them, may each be formed into a slide-wire, and that these two slide-wires may be placed parallel to one another, with a double sliding-contact between them, as shown in fig. 2. Then, in whatever position the slider may be, the fundamental ratio $ay=bx$ is always maintained, and the first condition of the Kelvin bridge is mechanically fulfilled. The one adjustment consists in moving the double slider along the bridge until there is no deflexion of the galvanometer at g; in which case

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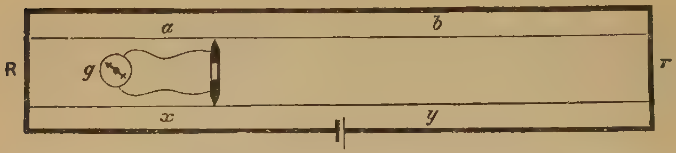
* Read April 24, 1896.

† *Supra*, p. 166.

Fig. 2.



$$\frac{a}{b}=\frac{x}{y}=\frac{R}{r}.$$

Since writing the above, I have referred to the original paper of Lord Kelvin (Proc. Roy. Soc. vol. xi. p. 313, 1861), and find that he proposes the use of parallel slide-wires for his auxiliary conductors; I have no doubt he had in view some such apparatus as that which I have here suggested. A Kelvin bridge with a single slide-wire was used by Matthiessen and Hockin in their differential method; it is described by Clerk-Maxwell in 'Elec. and Mag.' vol. i. p. 406 (1873).

Discussion.

Mr. Reeves said that apparently the author had completely missed the object of his (the speaker's) paper. For the object there aimed at was to make use of such sets of resistance-coils as are always to be found in any laboratory. In the author's arrangement it would be necessary to carefully calibrate the two wires, and also, since the resistances used must necessarily be small, to determine the resistance of the contacts.

Prof. Ayrton (communicated) said the author's suggestion was ingenious, but did not obviate the necessity for much of Mr. Reeves' "addition." Further, Mr. Reeves' proposal to employ ordinary resistance boxes was not made because such resistances are absolutely necessary, but because, since they are to be found in every laboratory, their use saves the expense of such a wire resistance, accurately calibrated, as Mr. Appleyard employed.

Mr. Appleyard, in his reply, said that his instrument was designed for use in a factory, where the time saved in making a series of tests was of more importance than the cost of the instrument.

XIX. *The Effect of Wave Form on the Alternate Current Arc.* *By* Julius Frith, 1851 *Exhibition Scholar*[*].

In the paper by Dr. Fleming and Mr. Petavel, recently read before the Physical Society, on the Alternate Current Arc[†], I think too little attention was paid to the wave form of the alternate current used.

It is known that if the arc is allowed to exert a preponderating influence at all on the alternate current circuit, it alters the wave form of both the current and the P.D. in a very marked degree.

As an illustration of the change produced in the wave form by the character of the external circuit, I give some curves for alternate current arcs taken from a paper which I read before the Manchester Literary and Philosophical Society in March 1894.

Here is shown, first the E.M.F. curve of the machine on open circuit. which is rather more peaked than a sine curve and involves the third harmonic largely. Next is shown the curve obtained under the same conditions, but with an arc lamp taking 10 amperes at 40 volts joined direct to the machine.

The lag recorded is due to the self-induction of the machine, which was a "Wilde" alternator, the armature of which contained iron.

It will also be noticed that the first ordinates of the curves are not quite equal to the last ordinates; this is due to the

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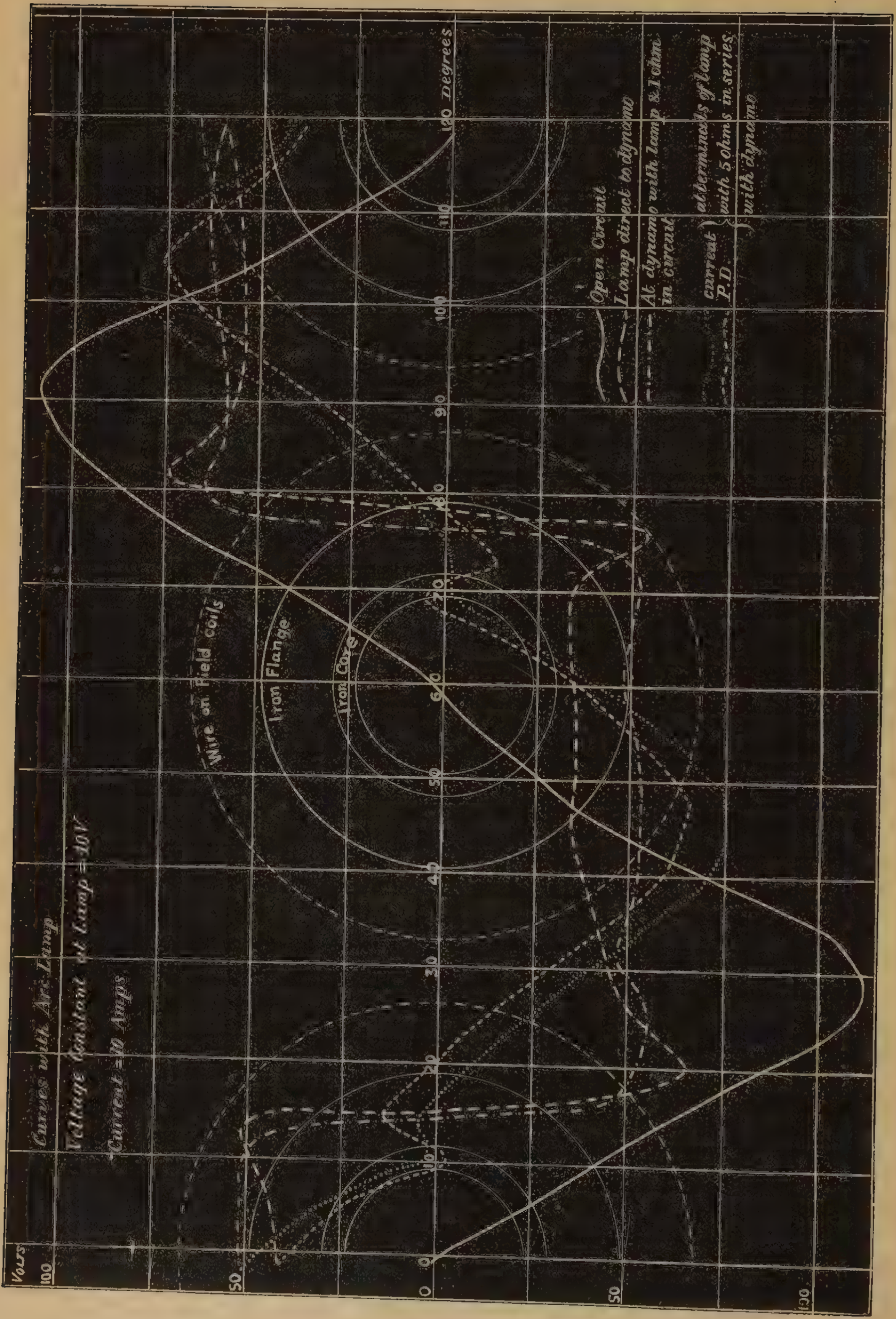
[*] Read April 24, 1896.

[†] *Supra*, p. 115.


Curves with Arc Lamp
Voltage constant at Lamp = 40V
Current = 10 Amps
Volts
100
50
0
50
100
0
10
20
30
40
50
60
70
80
90
100
110
120 Degrees
Wire on Field coils
Iron Flange
Iron Core
Open Circuit
Lamp direct to dynamo
At dynamo with lamp & 1 ohm in circuit
current } at terminals of lamp
P.D. } with 5 ohms in series with dynamo

curve being slightly asymmetrical. The whole period is only one third of the revolution of the alternator (shown as 120 degrees in the figure). In one complete revolution this lack of symmetry would of course disappear.

It is seen that the arc alters the wave form from a peaked curve to a very flat topped curve, changing sign almost instantaneously, and with two small maxima, which occur respectively just before and just after each reversal. The next curve shows the effect of adding a resistance of 1 ohm in series with the arc. This smooths out the two maxima, but otherwise does not affect the shape materially.

The current and P.D. curves with 5 ohms in series with the arc are next shown. Both waves now assume much more the form of the E.M.F. curve on open circuit, except at the zero line. Here the P.D. curve crosses the line twice in each direction, and the current curve runs parallel to the zero line for some way before crossing it.

These curves show, I think, the great effect that the arc has on the wave form, and also how this effect can be destroyed by placing resistance in series with the arc.

In a paper by Rössler and Wedding, which appeared in the 'Electrician' for August 31st, 1894, it was proved that an alternate current arc is more efficient, that is, gives a higher candle-power for the same electric power consumed, when the alternating current feeding it has a flat-topped than when it has a peaked wave form.

Rössler was, however, mistaken, I think, in assuming that the machine was making that curve. Indeed, this mistake runs through the whole of this otherwise most valuable paper. Rössler took three machines giving, as he thought, wave forms from the extremely peaked to the extremely flat wave, and determined the efficiencies of the same arc lamp for each of them. Whereas exactly the same results might have been obtained from one machine alone on causing it to give a higher voltage by increasing the field excitation and then absorbing the excess of voltage in resistance, exactly as in the case of the Wilde alternator above referred to.

I understand that in the experiments described by Dr. Fleming and Mr. Petavel there was always a resistance amounting to 7 ohms outside the arc, and hence a wave form was forced upon the arc which, as Rössler has proved, is not the most efficient one, and which the arc would convert into a form better suited to itself if it had been allowed to do so, as it is in commerce. This consideration must affect, not only the efficiency, but also the curves which Fleming and Petavel obtained for the variations in the luminous intensity.

Further, this action of the arc in modifying the wave form may throw some light on the discrepancy between the efficiency of alternate current arcs as determined in the laboratory and that stated to be obtained in practice.

When an arc is run in the laboratory, a large resistance is almost certain to be put in series with it to ensure that degree of steadiness which is essential to exact measurement, and hence the arc cannot alter the wave form. In the commercial use of arcs, on the contrary, the circumstances are widely different. In this case, for economic reasons, the arc must form a large percentage of the total "reactance" of the circuit, and therefore can easily alter the wave to the form required for the greatest efficiency.

It is interesting to note that the wave form giving the best result for the arc is almost exactly the opposite to that giving the best efficiency for transformers. In the former case a flat-topped wave is best, while for the maximum efficiency of transformers an exceedingly peaked wave is best, as lately found by Dr. Rössler.

This points to the building of alternators for use with transformers in such a way as to give peaked wave-forms.

In the case of the arc the building of machines to give the most efficient wave form is not so necessary, since, generally, the arc itself has the power of automatically converting *any* wave form into the one best suited to its requirements. Nevertheless, when the arc has to run in series with a large resistance it is of the utmost importance for obtaining the best efficiency that the machine should give a flat-topped wave.

City and Guilds of London Central Technical College,
April 2, 1896.

Discussion.

Mr. Blakesley said it seemed as if the more nearly the alternate current resembles a direct current, *i.e.* the longer in each period the current remains constant, the greater is the efficiency of the arc.

Mr. Price asked what was the cause of the reaction of the arc on the wave form ?

Mr. Tremlett Carter asked whether previous observers' results were vitiated by this action of the arc on the wave form ?

Prof. Ayrton (communicated) considered the author's suggestion of great importance, as bearing on the efficiency of the alternate current arc.

Prof. S. P. Thompson said that the dynamo employed by the author was one in which there was a large quantity of iron in the armature, so that the self-induction was large. Was it not on account of this large coefficient of self-induction, which would tend to keep the current constant, that the arc was able to alter the wave-curve ? If an arc is connected to the mains of a supply station in which a number of machines in parallel are feeding a number of lamps, would the arc still be able to affect the wave form of the current ?

Mr. Tremlett Carter asked if the author had tried the effect of replacing the arc by a resistance such that it would absorb the same volts as did the arc, and comparing the curves for the current and impressed P.D. with those obtained with the arc.

The Author, in his reply, said that the effect of the self-induction of the machine was shown in the curves. Current curves had not been taken with the arc straight on the machine. The current and self-induction were the same for all the curves, the voltage of the machine being increased by increasing the field when resistance was placed in series with the arc. When, as is commonly the case, special machines are used to supply arcs, and the load consists solely of arcs, the arcs could alter the character of the wave form. If the arc is replaced by a resistance, the wave form is of the same type as is obtained for the E.M.F. of the machine on open circuit.

XX. *On the Magnetic Field due to an Elliptical Current at a point in its plane within it.* *By* J. VIRIAMU JONES, *M.A., B.Sc., F.R.S., Principal and Professor of Physics in the University College of S. Wales and Monmouthshire, Cardiff*[*].

§ 1. IN a communication presented to Section A of the British Association[†] at Oxford in 1894, giving an account of measurements made to determine the value of the International Ohm in absolute measure by the method of Lorenz, I referred to a small error consequent on the fact that my standard coil is wound on a cylinder, the section of which at right angles to the generating lines is not a circle but an ellipse of small excentricity.

In considering the effect of this ellipticity on the value of the resistance calculated from the observations, it must be noted that the ordinary formula implies that the coil is circular. This formula is

$$R = Mn,$$

where R = the resistance in absolute measure,
M = the coefficient of mutual induction of the standard coil and disk circumference,
n = the number of revolutions of the disk per second.

But we are primarily concerned with the balance of the electromotive force between the ends of the resistance when the current is passing through it, and the electromotive force between the points of contact of the brushes on a radius of the rotating disk when the same current is passing through the standard coil; and this balance gives us, in the general case, the formula

$$R = 2\varpi n \int_{a_0}^{a_1} r\,H\,dr, \quad . \quad . \quad . \quad . \quad . \quad . \quad (A)$$

where a_0 and a_1 are the distances from the centre of the disk of the points at which the internal and external brushes are applied, and H is the magnetic-field intensity at a point on the radius through these points of contact at a distance r from the centre when unit current is passing through the standard coil. [If the coil is circular and coaxial with the disk, this formula simplifies into the formula first mentioned.]

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[*] Read May 22, 1896.

[†] Report of Electric Standards Committee, Appendix II., Brit. Assoc. Report, 1894.

§ 2. In the case of the coil used in my observations, the dimensions of which are given below, the excentricity of the elliptical section is so small that the value of the integral (A) differs only by a small quantity from the value it would have for a coil otherwise similar but of circular section with radius equal to the arithmetic mean of the semiaxes of the elliptical section ; and to a first approximation we may assume that the percentage correction to be applied to the value of the integral for the circular coil to obtain its value for the elliptical coil is the same as the percentage correction to be applied to its value for the circle in which the mean plane cuts the circular coil to obtain its value for the ellipse in which the mean plane cuts the elliptical coil. It will be sufficient for our purpose, therefore, to calculate the latter percentage correction.

§ 3. Let H_e be the value of the field intensity at a given point in the disk due to unit current in the ellipse, and H the value of the field intensity at the same point due to unit current in the circle coplanar and concentric with the ellipse and of radius c equal to the arithmetic mean of its major and minor semiaxes.

Then we have, in this case,

$$\begin{aligned} A &= 2\varpi n\int_{a_0}^{a_1} r\,H_e\,dr \\ &= 2\varpi n\int_{a_0}^{a_1} r\,H\,dr + 2\varpi n\int_{a_0}^{a_1} r\,(H_e - H_c)\,dr \\ &= n\,M_c + 2\varpi n\int_{a_0}^{a_1} r\,\sigma\,dr \\ &= n\,(M_c + B), \end{aligned}$$

where M_c = the coefficient of mutual induction of the circle and disk circumference,

and $\sigma = H - H_c$.

§ 4. To obtain the value of B, we must first find an expression for the intensity of magnetic field (H_e) due to an elliptical current at a point in its plane within it in terms of the semiaxes of the ellipse and the coordinates of the point.

Let the equation to the ellipse be

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,$$

and let ξ, η be the coordinates of the point in question.

The intensity of the magnetic field at the point (ξ, η) may be expressed by the formula

$$\mathrm{H}=\int_0^{2\varpi}\frac{d\theta}{\rho},$$

where ρ, θ are the polar coordinates of a point on the ellipse referred to the point (ξ, η) as origin.

Forming the polar equation to the ellipse with the point (ξ, η) as origin, solving for $\frac{1}{\rho}$, and substituting in the above equation, we have

$$\mathrm{H}=\frac{ab}{a^2b^2-b^2\xi^2-a^2\eta^2}\int_0^{2\varpi}d\theta\sqrt{f^2\cos^2\theta+2h^2\cos\theta\sin\theta+g^2\sin^2\theta},$$

where

$$f^2=b^2-\eta^2,$$
$$g^2=a^2-\xi^2,$$
$$h^2=\xi\eta.$$

Let ϕ^2, χ^2 be determined by the equations

$$\left.\begin{aligned}\phi^2+\chi^2&=f^2+g^2\\ \phi^2\chi^2&=f^2g^2-h^4\end{aligned}\right\}$$

and we have

$$\mathrm{H}=\frac{ab}{\phi^2\chi^2}\int_0^{2\varpi}d\theta\sqrt{\phi^2\cos^2\theta+\chi^2\sin^2\theta}$$

$$=\frac{4ab}{\phi^2\chi^2}\mathrm{E}(\phi,\chi),$$

where

$$\mathrm{E}(\phi,\chi)=\int_0^{\frac{\varpi}{2}}d\theta\sqrt{\phi^2\cos^2\theta+\chi^2\sin^2\theta},$$

an elliptic integral of the second kind. The value of $\mathrm{E}(\phi, \chi)$ for any values of ϕ and χ may be readily calculated by finding their arithmetico-geometrical mean (*v.* Cayley, 'Elliptic Integrals,' chap. xiii.).

§ 5. In the case of my standard coil

$$a=10{\cdot}5419 \text{ inches},$$
$$b=10{\cdot}5340 \text{ inches},$$

and the angle made by the radius of brush contact with the major axis is approximately 55°.

I have calculated the values of H_c and H_e for points on this radius distant 1 inch, 2 inches, 3 inches, &c., from the centre, with the following results :—

r.	$H_c \div 2\varpi$.	$H_e \div 2\varpi$.	$\sigma \div 2\varpi$.	$r\sigma \div 2\varpi$.
0	·094896	·094896	·000000	·000000
1	·095542	·095542	·000000	·000000
2	·097551	·097552	·000001	·000002
3	·101142	·101144	·000002	·000006
4	·106764	·106768	·000004	·000017
5	·115252	·115260	·000008	·000040
6	·128235	·128250	·000015	·000089
7	·149170	·149198	·000028	·000199

$$\sigma=H_e-H_c.$$

§ 6. We are now in a position to calculate the value of B. In the apparatus I used

$$a_0=\ {\cdot}0585,$$
$$a_1=6{\cdot}4949.$$

By numerical integration between these limits we have

$$\int_{a_0}^{a_1}\frac{r\sigma}{2\varpi}\,dr={\cdot}000163,$$

and

$$B=2\varpi\int_{a_0}^{a_1} r\sigma\,dr={\cdot}00643.$$

Also we have

$$a_1=\ 6{\cdot}4949,$$

and $$c=10{\cdot}5377,$$

and hence $$M_c=94{\cdot}014,$$

and therefore

$$B\div M_c={\cdot}0000684,$$

or the required correction is ·00684 per cent.

And by the argument contained in § 2 this may be taken to be to a first approximation the correction to be applied in the case of the coil. Now the coefficient of mutual induction of the coil and disk, calculated on the assumption that the section of the coil at right angles to the generating lines is a circle of radius c, is equal to 16613·75. Adding the calculated percentage correction to this value, we have finally for the apparatus used by me

$$R = n(16613{\cdot}75 + 1{\cdot}14) = 16614{\cdot}89\,n.$$

The value of the International Ohm in absolute measure previously given by me in the paper to which I have referred was

$$\cdot 99976 \times 10^9 \text{ absolute units.}$$

The value corrected for the coil ellipticity is

$$\cdot 99983 \times 10^9 \text{ absolute units.}$$

DISCUSSION.

Prof. SILVANUS THOMPSON said that this paper was of interest, not only on account of the application which others might make of the author's method, but also in that the correction, when applied to Prof. Jones' results, brought the international ohm more nearly into accord with the true ohm.

Mr. J. J. WALKER said he considered that the paper was better suited to the Mathematical Society. The integration which the author reduced to elliptic integrals might be more easily performed by another method.

Prof. AYRTON said that Prof. Jones' value for the true ohm was now 106·302 c.m. of mercury.

XXI. *Dielectrics.* *By* Rollo Appleyard *.

Some experiments upon the change of resistance of certain dielectrics with the duration of the testing-current, and with the testing-voltage, were described in a paper † which I read before the Physical Society two years ago. In continuation of this research some further tests have been made, the principal object being to determine the effect of *temperature* upon the dielectric resistance. For this purpose, mica and paraffined paper, in the form of condensers, have been chosen.

The resistances are measured by the "direct deflexion" method, and are expressed in megohms pro microfarad. The testing-voltage is the same throughout all the tests (450 volts), and each measurement is computed from the galvanometer-reading noted after the testing-current has been applied for one minute.

Two paraffin-paper condensers, each of one microfarad, and eight mica condensers, each of half a microfarad, were enclosed, separately, in water-tight cases, and submerged in a tank of water, the temperature of which could be continuously controlled. The platinum thermometers ‡ used for determining the temperature within the dielectric of the condensers have already been described.

Connexion between the condensers and the testing-apparatus was made by wires insulated with india-rubber; these were twenty-five feet long. The connexions within the condenser cases, where the wires were soldered to the respective sets of tin-foil sheets, were carefully sealed with paraffin wax. The ends of the leads were bared and cleaned in the usual way, for about six inches. Two such insulated wires were connected to each condenser, one to each set of tin-foil sheets.

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* Read May 22, 1896.

† "Dielectrics," Proc. Physical Soc. xiii. p. 155, 1895; Phil. Mag. Oct. 1894, p. 396.

‡ A "Direct-reading" Platinum Thermometer. Proc. Physical Soc. March 1896; and Phil. Mag. Jan. 1896.

Surface Leakage.

Suppose a current to enter one of these condensers at one of its insulated leads, which we will call A ; and the second lead, B, put to earth. It is clear that there are at least four paths—(*a*), (*b*), (*c*), (*d*)—for the current between A and earth :—

(*a*) Surface leakage at the end of A where the current enters the lead.
(*b*) Surface leakage at the end of A within the condenser.
(*c*) Surface leakage over the dielectric within the condenser.
(*d*) Through the dielectric of the condenser.

The leads can generally be so chosen that there is practically no conduction *through* their dielectrics. When measuring the resistance of the condenser by the "direct deflexion" method, the current to which the galvanometer responds is that due to the sum of the currents through (*a*), (*b*), (*c*), and (*d*). We have to determine what the deflexion would be if all the current went by way of (*d*). This can be done, approximately, by taking a preliminary test, with the end B disconnected from earth, and carefully cleaned ; noting the deflexion after the current has been applied at the end A for one minute. The deflexion thus obtained must be deducted from the deflexion observed during the second operation, which consists in putting the end B to earth, and again applying the current and noting the deflexion after one minute.

It sometimes happens that we want to separate (*a*) from (*b*) and (*c*) ; that is to say, we require to know what part of the preliminary deflexion is due to the exposed lead, and what part is due to internal leakage at the condenser. For this purpose I have applied a suggestion made to me by Mr. W. A. Price *, and have used a "guard-wire" to eliminate the leakage at (*a*). The method, which was used for its first trial in connexion with these experiments, consists in winding one or two turns of bare wire around the cleaned end of A, at some point not very far from the middle of the cleaned portion, intercepting the path of leakage. This wire is then connected to the testing-battery, as shown in fig. 1.

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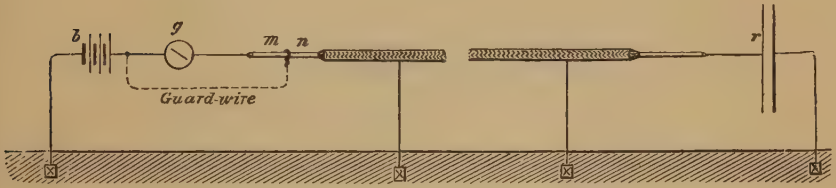
* This method has since been described by Mr. Price. See 'Electrical Review,' vol. xxxvii. no. 941, p. 702 (1895). In the appendix to the present paper I have evaluated the possible errors, and shown that they are negligible.

The efficacy of the "guard-wire" for the present tests was proved by the following experiment. A copper earth-wire was given several turns around the insulated lead A, just above the cleaned ends, to facilitate leakage; and the preliminary test was then made in the usual way. The observed deflexion, due to (*a*), (*b*), and (*c*), was 4 divs. The cleaned end was then smeared with a mixture of glycerine and blacklead, and the resulting deflexion was far off the scale; (*a*) had been increased enormously. A "guard-wire" was then applied at the middle of the smeared portion of the lead, and the deflexion was at once restored to its original value of 4 divs.

Fig. 1.



In practice it was always possible to avoid leakage (*a*) by giving a little care to the cleaning of the lead; the "guard-wire" was therefore used only as a check, and if it produced no change in the preliminary deflexion it was concluded that the preliminary deflexion was due to the effects of (*b*) and (*c*) alone, and that there was no leakage (*a*).

Results.

The results of the tests are given in Table I., which requires little explanation. The "Capacity," "Resistance," and "Electrification" of paraffin-paper and mica are there tabulated at the various temperatures.

Capacity.—As standards of capacity two condensers were used. These were mica of 1 and 0·5 microfarad capacity respectively. They were submerged in water and kept always

TABLE I.

Date.	Temperature.		Paraffin-paper. Mean values of two condensers.			Mica. Mean values of eight condensers.		
	F°.	C°.	Capacity. mfds.	Resistance. Megohms pro mfd.	Elec. p. c. between 1^m and 2^m.	Capacity. mfds.	Resistance. Megohms pro mfd.	Elec. p. c. between 1^m and 2^m.
11th March, 1896	32 8	0·4	0·98	17740	33	0·50	31286	40
12th " "	44·25	6·8	·98	11510	30	·50	28520	46
13th " "	54·5	12·5	·98	7216	28	·50	28427	47
26th Aug., 1895	65·7	18·7	·99	4196	24	·50	—	—
17th March, 1896	67·4	19·7	·97	3825	25	·50	25415	37
28th Aug., 1895	68·1	20·1	·98	3622	25	·50	22000	42
23rd " "	72·6	22·6	·98	2961	24	·50	22327	44
22nd " "	75·9	24·4	·98	2436	25	·50	—	—
29th " "	80·4	26·9	·98	1947	25	·50	16272	43
19th March, 1896	84·2	29·0	·97	1659	26	·50	16930	36
30th Aug., 1895	89·6	32·0	·97	1231	26	·50	17010	43
20th March, 1896	94·3	34·6	·98	1056	27	·50	18457	30
2nd Sept., 1895	100·1	37·8	·96	792	28	·50	15270	41
20th March, 1896	107·4	41·9	·95	611	32	·50	10340	33
3rd Sept. 1895	109·9	43·3	·95	551	32	·50	10521	38

at the same temperature, 27° C., which is just above the maximum air temperature. The object of using two standards was to avoid shunts and to simplify the work.

In the paraffin-paper condensers there appears to be an irregular variation of capacity with temperature. It is too small to affect them in practical telegraphy; but it is sufficiently large to prohibit their use as standards. We must suppose that the softening of the wax, and the consequent approach or recession of the tinfoil sheets, is responsible for part of these changes, to which must be added the variation of the specific inductive capacity of the dielectric. It is curious that with paraffin-paper condensers the capacity *diminishes* as the resistance diminishes. With gutta-percha or india-rubber core the result is nearly always the reverse of this.

Owing to the vibration of the building in which the tests were made, the capacities could not be measured with a greater accuracy than 1 per cent.; but within this limit it is safe to say that throughout the whole range of temperature mica retains a constant capacity.

Temperature Coefficients.

It is usual to assume * that the resistance R_τ of a dielectric at any temperature τ is represented by an equation of the form

$$R_\tau = R\alpha^\tau,$$

and that similarly, at some other temperature θ,

$$R_\theta = R\alpha^\theta;$$

so that

$$R_\tau = R_\theta \alpha^{\tau-\theta},$$

or

$$\log R_\tau = \log R_\theta + (\tau - \theta)\log\alpha. \quad . \quad . \quad . \quad . \quad (1)$$

From this equation, taking the fifteen observations of Table I, two at a time, in alternate order, and putting them successively as R_τ and R_θ in (1), we find as a mean value

$$\log\alpha = \bar{1}{\cdot}96344. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The temperature θ may be given some standard value, say 20° C.; and from a curve drawn from the fifteen observed values of resistance and temperature the corresponding resistance R_{20} may be obtained. The resistance at 20° C., found in this way, was, for paraffin-paper,

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* See Appendix II.

$$R_{20}=3670 \text{ megohms};$$

equation (1) in this case becomes

$$\log R_{\tau}=3{\cdot}56467+(\tau-20)(\bar{1}{\cdot}96344), \quad . \quad . \quad (3)$$

from which may be deduced the resistance corresponding to any temperature τ. It is interesting to see how nearly the results calculated from (3) agree with the observed values, as can be done by drawing the two curves which represent the observed and calculated resistances respectively.

The agreement between these two curves is so close, considering the conventions which always have to be adopted in tests upon dielectrics, that it has been suggested to me that a simple logarithmic curve may perhaps represent the true state of affairs, the observed results being assumed more or less at fault. This seems improbable for the following reason:—In equation (1) α is really the ratio of two resistances at an interval of 1° C., and it is presumed* that this ratio, *i. e.*

$$\alpha=\frac{R_{\tau}}{R_{\tau+1}},$$

is constant for all values of τ. This might be true for some ideal dielectric whose structure was perfectly definite at all temperatures, and which made invariable contact with its electrodes under all conditions; but there is no reason why α should be constant for a substance having the instability of wax.

Temperature Tables.

One use of these results is to enable the resistances of condensers to be reduced to the resistance at some standard temperature. A similar correction has always to be applied for gutta-percha and india-rubber cores. It is usual to draw up a table, assuming α constant over a given range, and to supply a correcting logarithm for each degree of temperature. This may be sufficiently accurate for small ranges of temperature; but I would suggest that a better way is to work from the *observed* results directly, making no assumptions whatever as to α. In order to do this, the "observed" results should be plotted as a curve, coordinating resistances and temperatures. The length of the ordinate corresponding to standard temperature on the "observed" curve is then to be taken as unity, and the lengths of the ordinates at every successive degree are to be expressed in terms of this unit. A table can then be drawn up giving a correcting divisor corresponding to each degree. With the aid of a slide-rule such a table can be completed in little more than an hour; and it is certainly more expeditious and less arbitrary than the ordinary tables of dielectric corrections. As an example, I have given in Table II. the divisors for paraffin-paper, from 33° F. to 110° F., deduced directly from the "observed" curve.

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* See Appendix II.

Table II. Paraffin-paper.

(To reduce the dielectric resistance at any temperature Fahr. to the corresponding dielectric resistance at 60° F., divide the observed resistance by the number opposite the given temperature.)

Temp. Fahr.	Divisor.	Temp. Fahr.	Divisor.	Temp. Fahr.	Divisor.	Temp. Fahr.	Divisor.	Temp. Fahr.	Divisor.
33°	3·19	49°	1·67	65°	0·775	81°	0·335	97°	0·169
34	3·07	50	1·62	66	·733	82	·319	98	·159
35	2·96	51	1·55	67	·699	83	·306	99	·153
36	2·87	52	1·46	68	·656	84	·292	100	·148
37	2·76	53	1·39	69	·625	85	·281	101	·141
38	2·66	54	1·33	70	·595	86	·270	102	·137
39	2·57	55	1·27	71	·562	87	·259	103	·133
40	2·48	56	1·20	72	·528	88	·247	104	·126
41	2·39	57	1·15	73	·501	89	·234	105	·119
42	2·30	58	1·09	74	·477	90	·225	106	·114
43	2·19	59	1·04	75	·450	91	·216	107	·108
44	2·09	60	1·00	76	·432	92	·207	108	·105
45	2·00	61	0·944	77	·411	93	·198	109	·101
46	1·91	62	0·901	78	·389	94	·189	110	·099
47	1·82	63	0·858	79	·375	95	·180		
48	1·76	64	0·820	80	·355	96	·175		

The resistances of *mica* condensers at the different temperatures are also given on Table I.; the curve drawn from them is not so smooth as that which represents the behaviour of paraffin-paper condensers; at the colder temperatures the resistance of the mica is so high that the preliminary deflexion, corresponding to (*a*), (*b*), and (*c*), is by no means small as compared to the second deflexion, due to (*a*), (*b*), (*c*), and (*d*) together. The results given are the best that could be obtained under the circumstances. In some ways it might have been advisable to connect the eight mica condensers in parallel; the deflexions to be observed would then have been greater, but the number of tests would have been fewer, and there would have been less chance of comparison one with another.

Electrification.—In Table I. the "Elec. p.c." is given for paraffin-paper and mica, at the various temperatures. This is derived from the galvanometer deflexions taken respectively at one minute and two minutes after the application of the testing current, thus:—

$$\text{Elec.\%} \text{ between } 1^{\text{m}} \text{ and } 2^{\text{m}} = \frac{1^{\text{m}}\text{reading} - 2^{\text{m}}\text{reading}}{1^{\text{m}} \text{ reading}} \div 100.$$

Paraffin-paper seems to acquire a minimum Elec. p.c. at about 20° C.; the reason is not very apparent.

Charge and Discharge.

The following figures (Table III.) may be useful as indicating the difference between the deflexions obtained in capacity tests, using "charge" and "discharge" deflexions, respectively, for paraffin-paper, mica, gutta-percha, and india-rubber. The deflexions represented by "charge" are not so uniform as the "discharge" readings, that is to say, a repe-

TABLE III.

	Charge.	Discharge.	Charge (repeated).
Paraffin-paper	196	196	195·5
Mica	223	223	223
Gutta-percha	208	214	207 210 210
India-rubber...........	198	200·5	201 197 198·5

tition generally gives a different reading, owing in part perhaps to some of the last charge having remained. This, however, is scarcely to be observed with mica. "Discharge" readings can generally be reproduced accurately.

Resistance during Melting.

If the temperature of paraffin-wax is raised much higher than is indicated in Table I., the change of state, as melting is approached, is characterized by a corresponding decrease in resistance. In order to examine this effect, a special condenser consisting of two horizontal circular plates, 11 in. diameter, was used. The lower plate was provided with a projecting rim to retain the wax, and was rather larger than the top plate. Three small wafers of thin ebonite sheet (16 mils) were used as distance-pieces. Melted wax was then poured in, filling the space between the plates.

There are so many disturbing influences affecting the resistance during the change of state, connected with the latent heat of solidification, the unequal melting, the absence or not of "electrification," and the contractions and expansions of the wax, that only qualitative results can be given.

Heating.—Starting at about 20° C. below the melting-point, the resistance rapidly diminishes until actual melting begins; there is then a definite fall to something like one third of what the resistance was just before melting. If heat is still applied to the condenser, the resistance keeps steady until the melting is complete; after which the resistance again diminishes steadily. So far as I am able to discover there is no "electrification" while the wax is in the melted condition.

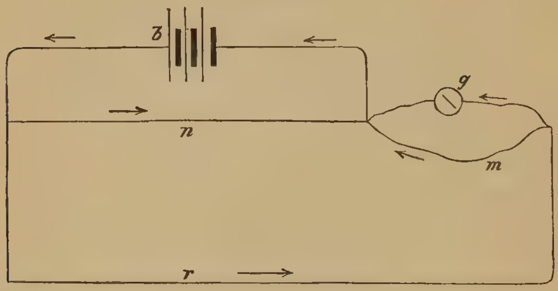
The spark-resisting power of the melted wax is at least one third that of the solid. The *melted* wax in this particular condenser broke down under 1200 steady volts; the insulation heals, as in the case of oil, to break down again in a few seconds.

Cooling.—A converse process occurs during cooling; and as the cooling is more uniform than the heating the effects can be more easily noted. The resistance of the melted wax gradually increases until the first crystals appear, at which stage it remains more or less constant until the process is nearly complete, when it very rapidly increases to about three times the value it had during the partial crystallization; it then behaves as solid wax.

APPENDIX I.

When an extra wire (fig. 1) is added from the testing-battery to some point near the middle of the cleaned end of the lead, intercepting the path of leakage (a) at that end, the circuits can be represented by fig. 2. The (a) leakage is here $m+n$, and the "guard" wire is supposed to come between m and n. Let b, g, and r denote the resistances of the battery, galvanometer, and the dielectric resistance of the condenser, respectively; let E be the E.M.F. of the testing battery, and G the current through the galvanometer when the "guard" wire is applied. It is clear that m shunts the galvanometer, and n shunts the battery; and we have to find the error which this introduces into the results. From fig. 2, by Kirchhoff's laws, we have

Fig. 2.



$$G = \frac{E}{(b+g+r) + \frac{1}{mn}(nrg+brm+brg+bmg+nbg)} \quad . \; . \; . \quad (1)$$

Let

$$\kappa = \frac{1}{mn}(nrg+brm+brg+bmg+nbg), \quad . \; . \; . \quad (2)$$

then κ may be regarded as equivalent to a resistance added to the simple circuit $b+g+r$.

To evaluate κ, we will put $m=n=r$; which means that the whole leakage, $m+n$, is assumed to be only twice the resistance of the dielectric of the condenser, and corresponds to a worse lead than any I have employed throughout these tests. In this case, (2) reduces to

$$\kappa=g+b+\frac{3bg}{m},$$

which has now to be compared with $(b+g+r)$ in (1).

Taking the very lowest observed value of r, which was 5×10^8 ohms, and putting $g=8000$, and $b=200$ ohms, we have

$$\frac{\kappa}{b+g+r}=\frac{1}{50{,}000},$$

so that κ is quite negligible in equation (1), and the error introduced by the "guard" wire is practically *nil*.

APPENDIX II.

Change of Dielectric-resistance with Temperature.

Let the resistance at some fixed temperature be R; and let R_1, R_2, R_3, &c. be the resistances at 1°, 2°, 3°... above R. The assumption is that the resistance diminishes $\frac{1}{n}$th of itself for every 1° rise of temperature. In this case,

$$R_1=R-\frac{R}{n},$$

$$n=\frac{R}{R-R_1}, \quad \ldots\ldots\ldots \quad (1)$$

and

$$R_2=R_1-\frac{R_1}{n}.$$

Or, putting in the value of n,

$$R_2=\frac{R_1^2}{R}.$$

Hence

$$R_1=R\left(\frac{R_1}{R}\right), \quad R_2=R\left(\frac{R_1}{R}\right)^2, \quad \text{\&c.};$$

and, generally,

$$R_t=R\left(\frac{R_1}{R}\right)^t. \quad \ldots\ldots\ldots \quad (2)$$

If τ and θ are any two values of t, we have

$$R_\tau = R_\theta \left(\frac{R_1}{R}\right)^{\tau-\theta}; \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

or,

$$\log\left(\frac{R_1}{R}\right) = \frac{\log R_\tau - \log R_\theta}{\tau-\theta}.$$

By putting observed values of R_τ, R_θ, τ, and θ in this last expression, the value of $\left(\frac{R_1}{R}\right)$ may be found, and substituted in (3), which then becomes the working formula. In the paper, $\left(\frac{R_1}{R}\right)$ is represented by α.

Comparison of α for Centigrade and Fahrenheit Scales.—Let R_1 be the resistance 1° C. above that at which R is measured. Then, by the preceding,

$$\alpha_c = \left(\frac{R_1}{R}\right),$$

and (2) becomes

$$R_t = R\alpha_c^t.$$

But 1° F. may be expressed as $\left(\frac{5}{9}\right)^\circ$ C., so that

$$R_{\left(\frac{5}{9}\right)} = R\alpha_c^{\left(\frac{5}{9}\right)}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

and $R_{\left(\frac{5}{9}\right)}$ will be the resistance at 1° F.

Hence

$$R_{\left(\frac{5}{9}\right)} = R\alpha_f. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Thus, from (4) and (5),

$$\alpha_f = \alpha_c^{\left(\frac{5}{9}\right)},$$

or,

$$\log \alpha_f = \left(\frac{5}{9}\right) \log \alpha_c.$$

DISCUSSION.

Prof. AYRTON said he could bear witness to the extreme value of Mr. Price's device, as it completely did away with the necessity for the extreme care previously required to prevent errors due to surface leakage. He regretted that he had not had an opportunity of comparing the author's numbers with some obtained some years ago by Prof. Perry and himself (Prof. Ayrton).

XXII. *On New Instruments for the Direct Measurement of the Frequency of Alternating or Pulsating Electric Currents.* *By* Albert Campbell, *B.A.**

In many experiments with alternating currents it is a very great convenience to be able to draw the supply from an ordinary lighting circuit, but the value of such a source of current is often lessened by the fact that the frequency of alternation is usually variable, the limits of variation being in some cases very wide indeed. It was for the purpose of getting rid of this uncertainty that, about a year ago, I devised the two instruments described below.

Most methods of measuring frequency depend either on Impedance measurements or on the production of Resonance (or Synchronism). The former I avoided, for unfortunately the wave-form as well as the frequency affects impedance, and I aimed at an instrument which would give trustworthy readings for any kind of pulsating current whatever.

Of the Resonance (or Synchronism) methods two are familiar. The first consists in running a small synchronous motor and measuring its speed by a speed-counter or indicator. This is a rather laborious way, and not very accurate if only a tachometer is used. In the second method † a stretched wire carrying the alternating current is placed in a constant magnetic field; by varying the stretching-weight the wire is tuned until it is set into strong vibration, and then the frequency is calculated from the tension, length, and mass of the wire. The original arrangement has been simplified, I believe, by Mr. Alexander Russell, of Faraday House, who uses a steel wire thrown into vibration by an iron-cored choking-coil placed near it. In both arrangements some uncertainty is introduced by the wire passing over a bridge or pulley.

In my first type of instrument I developed the last-mentioned arrangement. The working parts of the instrument are shown in fig. 1:—

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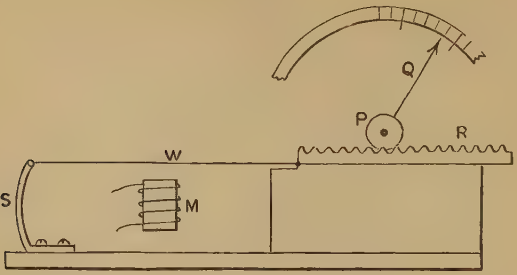
* Read May 22, 1896.

† Due to Professors Ayrton and Perry.

Fig. 1.



A steel wire, W, is fastened at one end to a spring, S, and at the other to a rack, R, sliding in guides. Near the wire is fixed an electromagnet M with a laminated core; this magnet is excited by the given alternating current. By turning the pinion P the wire can be tightened until it is thrown into vigorous vibration by the magnet. The pointer Q attached to the pinion then shows directly the frequency on a suitably graduated scale. Many varieties of this arrangement may be used; for example, the pointer may be fastened to S, or other kinds of magnifying-gear may replace the rack-and-pinion.

With careful use I believe that this type is accurate to within less than 0·2 per cent., but it is not so convenient as the second type, which I now proceed to describe. This more practical instrument is shown diagrammatically in fig. 2.

Here M is a choking-coil fixed near a steel strip A*, which can be moved back and forward through the clamp B by a rack-and-pinion, not shown. A's change of length is magnified by the rack-and-pinion D and E and the pointer F. The protruded length of A is altered until the alternating field due to M causes the maximum resonance, and the frequency is read off on the scale G.

By proper choice of the dimensions of the vibrator and the rack-and-pinion a very extended scale may be obtained. I have constructed one instrument in which the pointer goes more than twice round the circumference from 40 periods per sec. to 150 periods per sec., and at the middle part of the scale the accuracy of reading is within 0·3 per cent.

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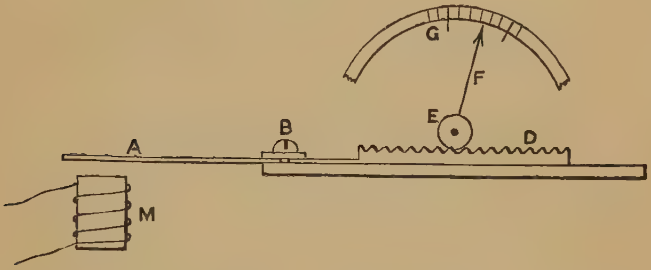
* I find, since writing the above, that in 1889 Professors Ayrton and Perry suggested the use of a variable magnetic tongue near an alternating electromagnet. Their idea, however, was never embodied in a direct-reading practical instrument.

Fig. 2.



The electromagnet M is usually of fine wire, and has a non-inductive resistance in circuit with it, so that the current taken is very small.

The reading is usually taken at the point of maximum resonance. This is observed by means of the sound given out or by watching the variation in amplitude either directly or by mirror, light-spot, and scale. Sometimes I fix near the vibrator an adjustable piece, against which it rattles or jars when the resonance is sufficient. This jarring piece may be made part of an electric circuit, including a lamp or an indicating instrument, in such a way that the circuit becomes broken when the greatest resonance occurs.

The instrument can be used as a speed-indicator for machinery by attaching to the rotating shaft a suitable commutator to make and break an electric circuit carried to the instrument, which may thus be at any distance.

If we keep to simple English we might call such an instrument a "Wave-teller," but to make the meaning clearer I propose to name it a "Frequency-teller."

In conclusion I may mention that since there is very little inertia in the moving parts the readings can be taken rapidly, and thus the instrument can be made to follow fairly quick changes in frequency.

Discussion.

Mr. Watson said he thought that in the case of the spring there would be a considerable temperature correction, and he suggested a method by which this might be compensated.

Mr. Blakesley asked if the author had found that the spring became magnetised and thus gave the octave.

Mr. Carter asked whether elastic fatigue influenced the results, and said that a synchronous motor and a speed indicator could be used to measure the frequency.

Prof. Silvanus Thompson suggested that it might be better to employ a polarized apparatus, since, to avoid the impression of forced vibrations on the spring, it was better, as was done in the case of tuning-forks, to make it massive. It had been found in other cases, such as in Hughes's telegraph and the telephone, that better results were obtained with polarized apparatus. He (Prof. Thompson) had used a telephone, placed anywhere near a magnet traversed by the periodic current, together with a tuning-fork, which gave beats with the note produced by the telephone, to measure frequencies. The variations in frequency ordinarily met with in practice were much greater than was generally suspected.

Mr. Blakesley said he considered that the advantage of the author's instrument over a telephone and tuning-fork was that it was continuously variable over a large range.

Mr. Enright asked if the author had been troubled by the spring or wire breaking into overtones. In some experiments, in which rather long wires were used, he had been troubled in this way.

Prof. Ayrton said that he did not think that it was possible to get the wire or spring to respond to the octave unless the alternating current contained a component of the frequency of the octave. In fact, he had himself used such a stretched string as a wave-analyzer. He had used a telephone to prove that the note given by a hissing alternate-current arc corresponded in frequency to that of the current. In the instrument used by Prof. Perry and himself a polarized arrangement was always employed, since the alternating current was passed either through a wire in a constant magnetic field, or through an electromagnet which acted on a wire through which a constant current was passed.

The AUTHOR, in his reply, said that the instrument responded, though feebly, to the octave, and this response might be made use of to check the accuracy of the scale. The total temperature correction would always be less than 0·1 per cent. and therefore may be neglected.

In the instruments shown the vibrator used was not a polarized one for three reasons :—

(*a*) in order to get a note whose vibration frequency should be *twice* the frequency measured, thus avoiding very low notes which are not so easy to observe accurately.

(*b*) to avoid the use of permanent magnets in alternating magnetic fields.

(*c*) to render unnecessary the use of an auxiliary direct-current circuit (for exciting electromagnets).

As to the question of elastic fatigue in the steel strip, I have never had the vibrator running for long enough time to detect this. In ordinary use the vibrator remains at rest except when an observation is taken.

XXIII. *Some Experiments with* Röntgen's *Rays.*

By Silvanus P. Thompson, *D.Sc., F.R.S.*[*]

The following is a brief narration of points observed by me during the past three months, and which are now brought before the Physical Society.

1. Many experiments have been made to observe polarization of x-rays, but no trace has been found. Tourmalines of several colours, and thicknesses varying from 0·1 millim. to 6 millim., have been used. Andalusite, mica, calc-spar cut in slices parallel to the axis, epidote, and ripidolite have been tried without result. The method pursued has chiefly been to cut the slice of crystal into three parts, laying two of them upon the third, one of the upper parts having its axis parallel to the axis of the under part, while the other had its axis laid at right angles. In this way equal thicknesses of crystal were compared side by side. Thinking that results might be obtained from dichroic crystals containing a metal of considerable atomic weight, slices were tried of crystals of sulphate of nickel, sulphate of nickel and potassium, sulphate of cobalt and potassium, sulphate of cobalt and ammonium, fluosilicate of cobalt, and fluosilicate of nickel, but no trace of polarization was seen.

Another method consisted in comparing the opacity of tourmaline in a direction parallel to the axis with that of an equal thickness in a direction at right angles to the axis. Not the slightest difference was observed either in the photographic shadows or with the use of a luminescent screen of barium platinocyanide.

2. For several weeks in the months of February and March experiments were made with many different forms of bulb to determine the source of the x-rays and the form of tube most favourable to their production. In common with the conclusions of so many other observers, it was found that the effective source was in every case a surface against which the kathodic discharge was directed. A form of tube which gave results superior to those of any Crookes tube at the time in

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[*] Read June 12, 1896.

the market is shown in fig. 1, in which the kathode consisted of a number of iron wires spread out from a centre, and the antikathode, which also served as anode, was a spade-shaped piece of iron or platinum. Another form of which many bulbs were tried was provided with an external kathode of foil, and an internal anode projecting to about the centre, terminating in a small spade of platinum-foil as antikathode (fig. 2). This also gave good results, but was liable to be pierced at high stages of exhaustion. With this form various experiments were tried as to the influence of the material of the antikathodic surface. Glass was found to work quite well, but to be more troublesome than metal. A phosphorescent enamel made by fusing together two parts of a soft lead-glass with one part of Balmain's luminous paint was also tried. The result of these experiments was to show that, contrary to the opinion then current that the presence of much phosphorescence or fluorescence was promotive of the production of the x-rays, the x-rays were most freely emitted when the conditions were such as to waste as little as possible of their energy in internal fluorescent effects: that in fact a metal surface was preferable to a surface of glass, enamel, or porcelain for receiving the impact of the kathodic discharge.

Fig. 1.

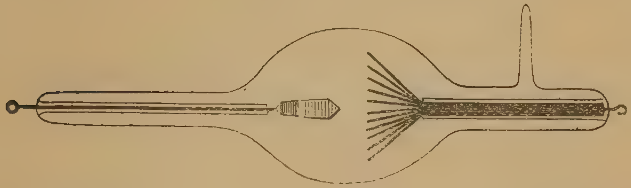
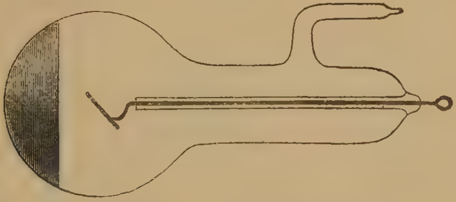


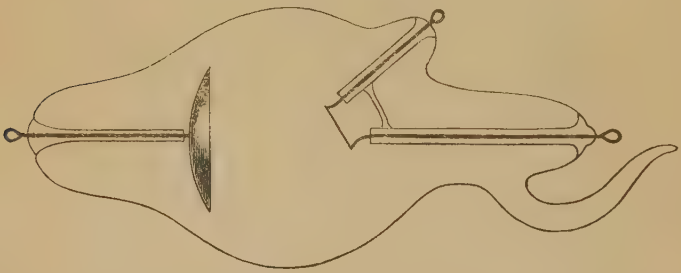
Fig. 2.



At about this time Mr. Jackson's perfected form of focus-tube was brought out, a tube which for photographic purposes was found superior to any other form, and has not yet been surpassed.

3. Observing in some of these experiments that the metal spade used as antikathode became red-hot, a special tube was constructed for me by Mr. Gardiner to test the question whether the high temperature of the antikathode was, or was not, prejudicial to the emission of *x*-rays. This special tube (fig. 3) was furnished with an antikathode of platinum-foil mounted so that while serving both as antikathode and as anode it could be heated by passing a current through it from an auxiliary battery. Observing the activity of the tube by means of a luminescent screen of platinocyanide of potassium, it was found that the heating of the antikathode, so far from being disadvantageous, decidedly promoted the emission of *x*-rays, and increased the continuity and brilliancy of the luminescence. Various amounts of current were sent through the platinum, the most effective result being obtained by currents which heated the surface to visible redness. Whether the effect is a direct one or an indirect owing to the driving out of occluded gases is not yet determined.

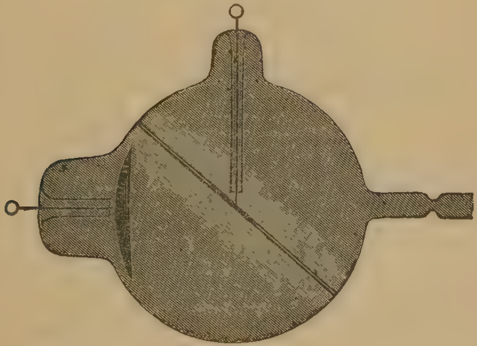
Fig. 3.



4. When watching with the luminescent screen the emission of *x*-rays within bulbs connected with the pump, some observations were made of direct importance with regard to the state of exhaustion that is best. The degree of vacuum which suffices for the production of kathodic shadows is known not to be sufficient for the production of *x*-rays. It is also known that when evacuation is pushed very far the internal resistance of the bulbs rises very high, so that they

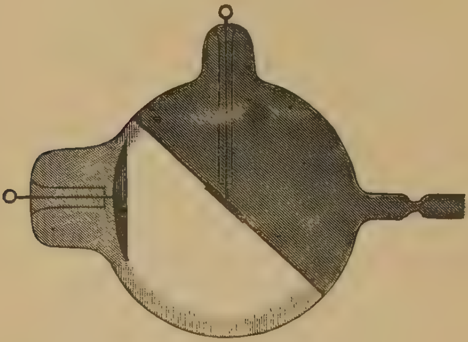
become almost non-conductive. If a bulb is exhausted, and heated during exhaustion, and the vacuum pushed almost to non-conductivity, and if a little air is again admitted and the tube again exhausted, the high degree of vacuum is again very soon reached, probably because during the first exhaustion the gases absorbed upon the walls of the bulb were mostly removed. After three or four repetitions of this process the transition from the low state of vacuum to the high state is exceedingly rapid. If a bulb in such a condition is examined by the luminescent screen while the pump is at work, scarcely any trace of x-rays can be noticed so long as the vacuum is such that the resistance is low. A pair of discharging-points arranged as a shunt to the tube serves as an approximate gauge. Kathodic shadows can be seen when the resistance is so low that the discharge-points do not spark even when placed 3 millimetres apart. When the resistance rises so that the spark-points must be put 20 or 30 millimetres apart x-rays begin to be given off; and are given off both from the back and from the front of the antikathode. The bulb, as seen upon the adjacent screen, shows two pale luminous regions divided by a fine oblique black line which is in the plane of the antikathode (fig. 4). If the pump goes on working, in two or three seconds, or while only a few cubic centimetres of mercury pass through the pump, the phenomenon changes. The luminosity behind the antikathode dies out, and that in front of the antikathode increases; so that there is seen simply a bright anterior region ending at the

Fig. 4.



oblique plane of the antikathode, beyond which all is dark (fig. 5). This oblique delimitation can also be seen in the yellow phosphorescence upon the walls of the bulb. This sudden transition occurs after the resistance of the bulb has passed its minimum. The brightest luminescence occurs when the spark-length exceeds 40–50 millimetres. The luminosity does not fall off much even at very small angles to the plane of the antikathode, proving that the emission of x-rays does not follow Lambert's law of the cosine by any means. Experiments on this point are still in progress.

Fig. 5.



5. The phenomenon of diselectrification by x-rays is very readily demonstrated. For this purpose I have found a very convenient instrument to be an electroscope consisting of two strips of aluminium leaf (which is lighter than gold leaf) suspended in a thin-glass jar entirely covered with a fine metal gauze. It is charged with a dry pile, and a metal cap is then placed over the charging knob, so that it is entirely electrostatically screened from external electrical influences. Positive and negative electrifications are both readily discharged, even at the distance of several feet from the bulb.

6. On the first announcement of the diselectrifying properties of x-rays, I attempted to obtain electric dust-figures as shadows of metallic objects by applying the x-rays to discharge electrified surfaces of glass or ebonite upon which mixed powders of red lead and sulphur were then dusted.

These were obtained almost at the first trial; but to produce them satisfactorily requires a little care.

The object whose shadow is to be obtained—a pair of scissors, for example—is laid upon a thin sheet of aluminium placed to stand on four feet at the height of about 20 millim. over the sheet of ebonite or varnished glass upon which the shadow is to be thrown. This sheet of ebonite is first carefully diselectrified by passing it over an alcohol flame, and then laid upon an earthed sheet of foil upon the table. The aluminium tray with the scissors upon it is placed over the ebonite. A guard-box of lead with a rectangular hole in its top is placed over all. Then the aluminium tray is charged electrically by a small influence-machine which has one pole put to earth and the other connected to the aluminium tray. In this state of things the ebonite plate lies in an electrostatic field, but is not electrified upon its upper surface. The x-rays are now caused to fall upon the aluminium tray, through which they pass save when obstructed by the metallic object, and, discharging the tray, virtually carry down the electrification upon the surface of the ebonite in straight lines, leaving the shadowed portions unelectrified. The influence-machine is disconnected, the aluminium tray removed, the sheet of ebonite lifted off the table, and the mixed powders are forthwith dusted over its surface, revealing the shadow. Both positive and negative shadows can be obtained. Several alternate dispositions are possible.

These observations were made early in February before the announcement by M. Righi of some similar cases of production of shadows by x-rays.

7. I have also made some observations upon the reflexion of x-rays. The production of diffuse reflexion by solid bodies is very easily observed; but hitherto I have no clear evidence of specular reflexion. Air unfortunately itself sets up diffusion, behaving as a semi-opaque fluid. If ordinary experiments on the reflexion of light had to be carried on in dense smoke or in milky water, a similar diffusion would interfere with specular reflexion. In one set of experiments a V-tube made of lead pipes set at right angles, and open at the bottom, was used, the x-ray source being made to shine down one limb, while a shielded photographic plate was placed at the

upper end of the other. The surfaces to reflect x-rays were placed at the open lower ends at 45° to the lines of incidence and of presumed reflexion. Reflexion of a sort was indeed obtained when surfaces of metal and of glass were placed across the bottom of the tubes. But an effect was also obtained even when nothing was placed across the open bottom. It seems exceedingly doubtful whether true specular reflexion has been observed in any case.

Discussion.

Prof. du Bois said that Galitzine had found that Röntgen rays were polarised by tourmaline, a special form of developer being employed. The behaviour of tourmaline to light-waves presents some curious features; for if the wave-length is increased, a point is at length reached where the ordinary and extraordinary rays are equally absorbed. For greater wave-lengths the ordinary conditions are reversed. If the Röntgen rays are not homogeneous, the contradictory results obtained by different observers might be due to the fact that they were working with rays which were differently absorbed by tourmaline.

Mr. Swinton said he had tried the effect of heating the kathode, and had obtained results similar to those which were obtained by the author. Mr. Swinton further said that he had found that the blue luminescence sometimes observed depended on the size of the kathode. With tubes in which the kathode was almost a complete hemisphere, it was impossible to eliminate this blue luminescense.

Mr. Appleyard suggested the performance of the experiments under the surface of a dielectric.

Prof. Gray said he had obtained some indication of regular reflection, but nothing definite.

The Author, in his reply, said that it had been found that if the Röntgen rays are reflected from a surface of sodium *in vacuo* the amount reflected is a minimum for normal incidence, and increases at oblique incidence. Comparing this behaviour with that of ultra-violet light, it supports the idea that the Röntgen rays consist of transverse vibrations.

XXIV. *On the Measurement of very large and very small Alternating Currents.* By ALBERT CAMPBELL, *B.A.*[*]

AIR-CORE transformers, although quite inefficient for ordinary lighting circuits, are yet much more valuable for testing purposes than most people are aware of. By the help of such transformers it is possible to extend almost indefinitely the ranges of many ordinary measuring instruments. If the secondary of an ironless transformer be kept in open circuit the secondary volts are accurately proportional to the primary P.D. if the frequency is constant, and hence by using an electrostatic voltmeter on the secondary we can transform either up or down, and thus measure voltages above or below the range of the electrostatic instrument. Of course the arrangement would have to be calibrated; this might sometimes be done by taking a reading for which both the primary and secondary voltages lay within the range of the voltmeter used.

The above way of using an air-core transformer was suggested to me some time ago by Mr. Hugh Erat Harrison, of Faraday House. I hear since from Mr. Mather that it has been also used at the Central Technical College[†].

If we attempt to measure *current* (in the primary) by observing the voltage on an open-circuit secondary, we find that for a given primary current the readings depend also on the *frequency*. It therefore occurred to me that the secondary circuit *should be closed* and should besides be *highly inductive*. In practice this was found to work with perfect success, the primary current being proportional to the secondary current, which latter was measured by an ammeter.

To find the most favourable conditions, let

I_1, I_2 be the primary and secondary currents respectively;

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[*] Read June 12, 1896.

[†] Since writing the above I have listened with interest to the paper on the "True Resistance of the Electric Arc," by Messrs. Frith and Rodgers. Their beautiful application of an air-core transformer to measure a small alternating current superimposed on a large direct current might, I think, be also employed to separate a *large* alternating current from a much *smaller* direct current.

R_2 the resistance of the secondary circuit;
N its inductance;
M the mutual inductance between primary and secondary.

Let $p = 2\pi n$, where n is the frequency of alternation.

Here M and N are constant, while R_2 may be variable with temperature.

At first let us assume that the P.D. follows a simple sine curve. Then

$$\frac{I_1}{I_2} = \frac{\sqrt{R_2^2 + p^2 N^2}}{Mp}. \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Now let the inductance of the secondary circuit quite swamp its resistance, so that we may neglect the term R_2^2 in comparison with p^2N^2. In this case we obtain

$$\frac{I_1}{I_2} = \frac{N}{M}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

i. e. *the ratio of the primary current to the secondary current is independent of the frequency of alternation and the resistances of the coils.* We shall see below to what extent this valuable result can be realized in practice. It is clear that in the above case, since $\frac{pN}{R_2} = \infty$, the secondary current I_2 is in exactly opposite phase to I_1.

Now suppose the primary P.D. to be no longer a pure sine function of the time but to follow any periodic curve. The curve of primary current can then be decomposed into a number of sine curves of various frequencies; each of these will produce a secondary of opposite phase, and the ratio of each primary component to its secondary will $= \frac{N}{M}$.

Accordingly the total secondary current curve will be of the same wave-form as the primary and in opposite phase to it, while *the ratio of current transformation will be the same as for a pure sine-curve alternating current**.

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* In this I have assumed that the Fourier series for the wave-form contains no constant term, *i. e.*, that there is no constant component in the current. If there is such a component, nothing corresponding to it will appear in the secondary circuit, and the ratio of current transformation will be by no means the same as if the current were purely alternating. In fact an air-core transformer affords a good means of separating out the purely alternating part of any periodic current.

Unfortunately, in practice we cannot quite attain to the condition $\frac{Np}{R_2} = \infty$, but we can get near enough to make the error quite small. In the following table are given (approximately), for different values of $\frac{R_2}{N}$, the values of the ratio $\frac{I_1}{I_2}$ for frequencies of $n=80 \sim$ per sec., and $n=40 \sim$ per sec., if the ratio $=1$ when $n=\infty$.

Table I.—Transformation Ratios.

$\frac{R_2}{N}$.	$n=\infty$.	$n=80$.	$n=40$.
100	1·00	1·02	1·08
50	1·000	1·005	1·02
25	1·000	1·001	1·005
10	1·000	1·0002	1·0008

By this it will be seen that if $\frac{R_2}{N}=50$, the error in the transformation ratio from $n=80$ to $n=\infty$ becomes insignificant; and that if $\frac{R_2}{N}=25$ the ratio is approximately constant down to $n=40$.

In practice it is easy to make $\frac{R_2}{N}=50$, but to get $\frac{R_2}{N}=25$ would require a rather lavish expenditure of copper wire.

Instead of finding the ratio of a particular transformer (with ammeter) once for all, a simple way is to find it each time for the frequency to be used, the strength of the primary current being chosen so as to lie within the range of the instruments available.

The following instance will make the method clearer. Suppose we want to measure currents up to 1000 amperes with an electrodynamometer and a Kelvin balance, each up to 100 amperes. Let the transformer have a "current ratio" of about 10 when the balance is the only load on the secondary. The electrodynamometer is inserted in the primary circuit, and a primary current of (say) 98·2 amperes is found to produce (say) 10 amperes in the secondary. If the electrodynamometer be now taken out of circuit, any primary current up to 1000 amperes can be measured by reading the balance in the secondary and multiplying by 9·82.

In an exactly similar way very small alternating currents may be measured.

In order to avoid errors due to stray magnetic fields the transformer may be wound with double coils placed side by side with their axes parallel and reversed in the well-known manner. It might also be well to have the position of one or more turns of the primary or secondary adjustable with a scale to show the proper position for each particular frequency.

For the purposes of calibrating at any frequency a second primary of a different number of turns from the first may be wound on the transformer, and the relationship of the two primaries may be determined once for all.

For example, if the first primary is for 1000 amperes the second may have 100 times the number of turns and be suitable for 10 amperes. The correction for any particular frequency or wave-form may then be found experimentally by using the second primary only.

I may mention that with very simple instruments I have used an air-core transformer to measure currents between 1000 and 2000 amperes.

[*Added June* 12*th*, 1896.]

In order to get a larger increase of secondary voltage a well-known method is to connect up a transformer with the primary and secondary in series as in fig. 1. By a somewhat similar method the ratio of current transformation may sometimes be increased. The connexions in this case are shown in fig. 2.

Transformers with Iron Cores.

I have recently investigated the case of transformers with *iron* cores whose secondary circuits were made highly inductive. This was done by short-circuiting the secondary in each case through a Kelvin 100-ampere balance.

Two transformers were thus tested—an iron-ring transformer with very small magnetic leakage, and a small transformer with open iron circuit. The first of these had a primary of 764 turns of No. 16 S.W.G. wire, and over this a secondary of 48 turns of 7/16 wire. The core of the second

Fig. 1.

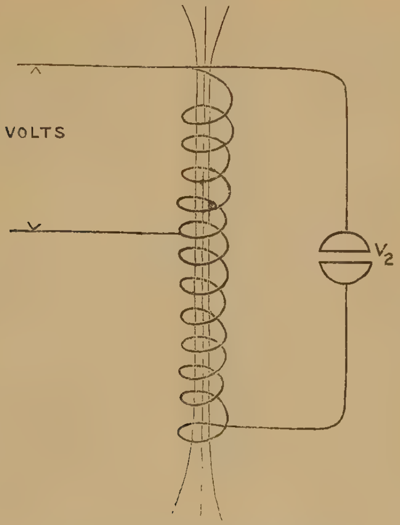
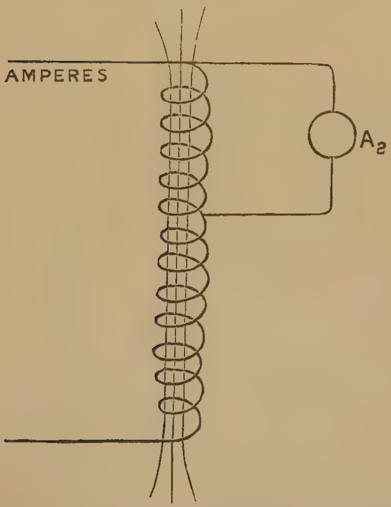


Fig. 2.



was a short bundle of iron wires, over which were wound the primary of 38 turns of No. 16 S.W.G. wire, and over it a secondary of about 3000 turns of No. 26 S.W.G.

The ring transformer was first tested with a Kelvin 10-ampere balance as secondary load, the frequency being 84 ~ per second. The balance had a resistance of about 0·55 ohm and an inductance of about 0·0016 henry. The results given in Table II. show that with this amount of resistance in the secondary the $\frac{I_2}{I_1}$ is by no means constant.

Table II.

Iron-ring Transformer. Secondary Load 10-ampere Balance.

I_1. Primary current.	Ratio $\frac{I_2}{I_1}$.
0·0365 amp.	8·12
0·0764	9·28
0·1205	10·12
0·201	11·48
0·336	12·28

Table III.

Iron-ring Transformer. Secondary Load 100-ampere Balance.

I_1. Primary current.	Ratio $\frac{I_2}{I_1}$.
1·0	15·695
1·5	15·67
2·4	15·67
4·1	15·73
6·0	15·69

Similar measurements were then made with a 100-ampere balance as secondary load. From Table III., which gives some of the actually observed ratios, it will be seen that *the ratio of current transformation is practically constant.*

With almost the same arrangement of secondary circuit experiments were made to find to what extent the ratio $\frac{I_2}{I_1}$ was affected by change of frequency. Table IV. gives the results of these and shows that *the ratio is almost independent of frequency through the range tried.*

TABLE IV.

Ring Transformer.

Frequency, ~ per sec.	$\frac{I_2}{I_1}$.	Error from 84 ~ per second.
40	15·48	0·7 per cent.
57	15·52	0·45 per cent.
84	15·59	

The results in Table V. of a similar experiment with the transformer of open magnetic circuit described above show how different its behaviour is.

TABLE V.

Transformer with open Iron Circuit.

~ per second.	$\frac{I_2}{I_1}$.	Error from 84 ~ per second.
44	28·1	24·3 per cent.
84	37·1	

It seems clear, therefore, that iron-ring transformers may in many cases be used in a similar way to that described above for air-core transformers; but care must be taken to have the resistance of the secondary circuit sufficiently small.

In conclusion I beg to thank Messrs. Lovell, Macalister, Sankey, and Norman for their kind help in some of the experiments.

Discussion.

Mr. Blakesley said that the author's arrangement could only be used for measuring the *current* in the primary. Only if the self-induction of the primary is large and its resistance small, the secondary circuit being open or of large resistance, can the voltmeter of the secondary coil give indications proportional to the P.D. of the primary. The reading will then be the P.D. of the primary multiplied by fraction representing the ratio of the coefficient of mutual induction to that of self-induction in the primary. This will be true independently of the period. But if the frequency is to be kept constant as well as all the other properties of the circuits, of course all effects in both circuits will simply be proportional to the P.D. of the primary, but in that case there is no necessity to make the self-induction of the secondary great, so that the advantage of doing so is not obvious.

XXV. *On the Use of Bare Wire for Resistance-coils.* *By* F. W. Burstall, *M.A., formerly Scholar of St. John's College, Cambridge**.

I. *Introduction.*

It is perhaps somewhat surprising that the form of resistance-coils as used in the ordinary box has, during the last twenty years, changed less than any other kind of electrical instrument. The most serious defects of the present form of resistance-box are, firstly, the difficulty of ascertaining the temperature of the wire forming the resistance, and, secondly, that the method of short-circuiting any required number of coils by means of divided brass blocks and taper plugs leads to contact-resistances which are often of very variable amount. A resistance-coil, the wire of which is covered with silk wound very closely and then covered with paraffin, of necessity conducts heat very badly, and therefore the temperature, as shown by a thermometer placed in the box, gives but little information as to the temperature of the resistance-wire. I have endeavoured to overcome the temperature difficulty by using manganin wire, but the uncertain nature of this material has forced me to abandon its use. The suggestion for the employment of a bare wire immersed in oil is due to Mr. E. H. Griffiths, and it is with the developments of this suggestion that the present paper deals. There can be no doubt that with a bare wire immersed in oil the heating by the testing current is much less than with covered wire, and further, the temperature of the wire can be accurately measured ; but perhaps one of the most important advantages in the use of a bare wire lies in the fact that the wire can be annealed in a most perfect manner by the passage of a sufficiently large current to heat it to a red-heat for a few seconds. It has long been known that well-annealed coils are much less liable to change than coils which have not been so treated. Dr. Lindeck, of the Reichsanstalt, has, in his various papers, laid great stress upon the necessity for annealing standard coils, which is effected by heating the coil in an air-bath to about 140°, the highest temperature a silk-covered coil can bear without material injury. The heating of the wire to a red-heat is, of course, much more efficient than the method adopted by Dr. Lindeck for freeing the wire from undue stress.

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* Read June 26, 1896.

The wire which I have employed for the whole of the resistance-box was drawn from one cast of platinum-silver, a material which experience has shown to be the most perfect for resistance-measurement.

The two forms of resistance-box usually employed are the series form and the dial form. In the former a number of coils are placed in series, and any required number can be short-circuited. This form, though it requires comparatively few coils, has the disadvantage that any fluctuation in one plug causes an error in the final measurement, and if any considerable number be used, it is almost impossible to prevent variation in the sum of the contact-resistances. For these reasons it seemed advisable to adopt the second form of box, the dial pattern, in which the number of plugs employed is a minimum, and remains the same whatever resistance is being measured.

It is not essential to make standard resistances in order to calibrate a resistance-box, as each dial can be measured in terms of the preceding dial. This method, however, perpetuates any errors that may have been made in the measurement of any of the smaller coils, and it is not possible to measure the coils in the box with the same accuracy that can be attained in the determination of the standard coils. I have therefore constructed standard coils for every dial, and their values have been obtained from a 1 ohm coil independently of the resistance-box.

II. *Construction of the Resistance-box and Standards.*

The resistance-box has five dials, each consisting of nine coils, ranging from $\frac{1}{10}$ of an ohm to 1000 ohms, and four pairs of proportional arms ranging from 1 to 1000 ohms.

The $\frac{1}{10}$ ohm coils have a length of 96 millim., the wire having a diameter of ·63 millim. Being so short, there is no difficulty in supporting the coils.

The wire of the 1 ohm coils has the same diameter and a length of 960 millim. These coils were at first wound into short spirals, and were allowed to hang freely from their terminals. It soon, however, became apparent that the convolutions of the spiral required to be fixed rigidly to the ebonite top. The method that has been finally adopted is to screw the spiral into holes pierced in a mica plate ; this plate being screwed top and bottom to a brass bar which is attached to the ebonite top.

In the coils of from 10 to 1000 ohms it is not easy to find a form of bobbin on which to wind the great lengths of bare wire required in such a manner as to avoid short circuits, and to bring the wire in contact as little as possible with the insulator. The form I have ultimately adopted is as follows (fig. 1):—To a central brass rod are attached, for the 10 ohm and 100 ohm coils, two small brass crosses at a distance of about 150 millim. apart ; to the ends of the arms of these crosses are screwed serrated strips of mica, the pitch of the serrations being about 1 millim. The resistance-wire is then bifilarly wound into these serrations. The 1000 ohm coils are of the same form, but from their greater length, 250 millim., it is necessary to employ a third cross in order to strengthen the mica, and even then difficulty has been experienced owing to the mica bending under the coil-tension, culminating in several cases in the breakage of the wire.

For the 10 ohm coils wire having a diameter of ·30 millim. and a length of 2400 millim. has been used. For the 100 ohm coils, a diameter of ·15 millim. and a length of 6200 millim. For the 1000 ohm coils, a diameter of ·075 millim. and a length of 14500 millim.

The top is of ebonite, 86 centimetres long, 28 centim. wide, and 19 millim. thick.

Instead of using divided blocks for the dials, conical plugs, having a maximum diameter of 9 millim., and a taper of 1 in 12, fitting into conical sockets have been employed *. The plugs are connected with the zero socket of the previous dial by means of heavy flexible leads, having a copper core, 5 millim. in diameter, composed of copper wires of diameter ·2 of a millim. It was found necessary to use comparatively large wire, as the first set of connecting leads, built up of wire ·07 millim. in diameter, parted with the constant turning and gave rise to a variable plug-resistance. The use of the conical plug in a solid conical socket is completely free from errors caused by the change of shape of the insulating material; further, such forms can be easily and accurately machined, a matter of no small importance to those who make their own apparatus.

The terminals are of large diameter (25 millim.), and are arranged with the screw in the moveable part, so that the top of the fixed part can be readily cleaned.

The proportional arms are fitted with the usual divided blocks.

The whole of the metallic part is made of phosphor-bronze, a material which is harder and less liable to oxidation than ordinary brass.

The containing box is lined with copper for the reception of the oil.

The form of bobbin and the wire for the standard resistance-coils are the same in the 1 and 10 ohm standards; but in the 100 and 1000 ohm standards the mica is stiffened by being slipped into a brass back, like the back of an ordinary tenon saw, which quite prevents any tendency to bend. The temperature is measured by means of a thermometer placed in a pierced tube which lies inside the coil. The coils are inclosed in an outer brass case and fitted with the usual copper terminals for use with mercury-cup connexions.

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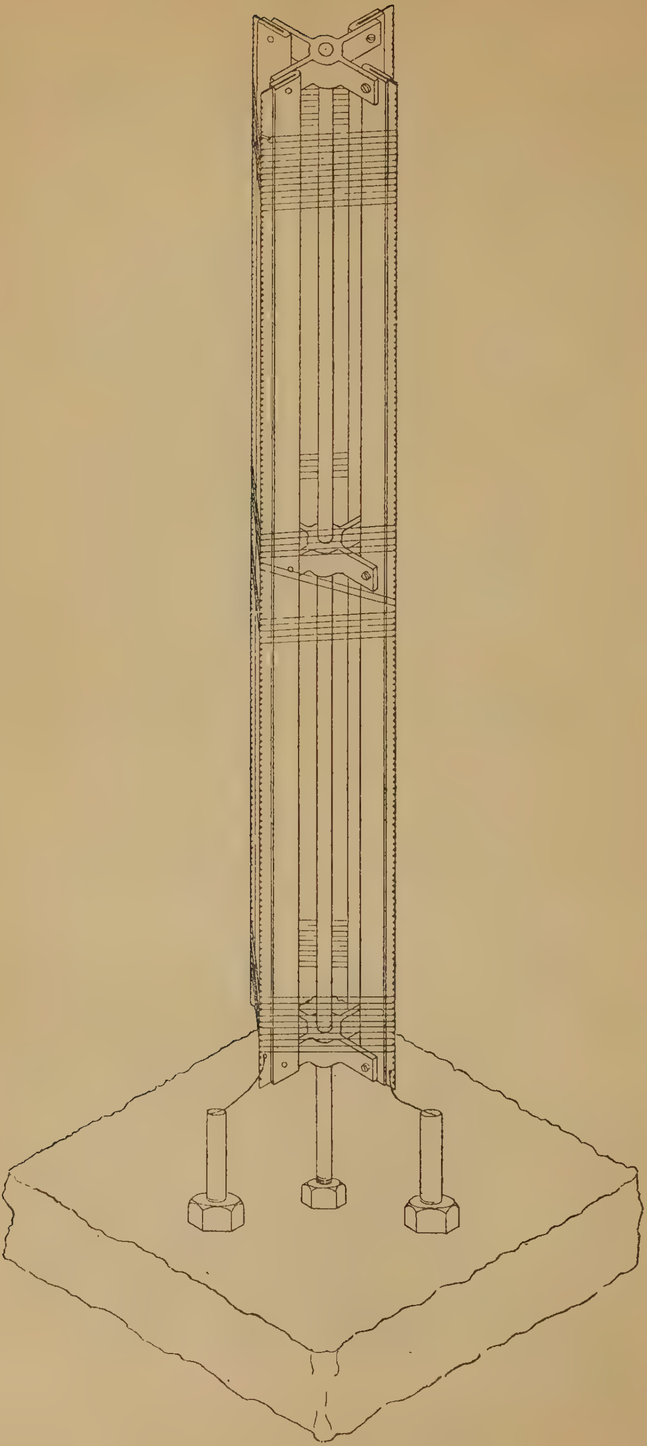
* This method of constructing the plugs is due to Mr. R. W. Paul, Proceedings of the Physical Society, 1892.

Fig. 1.



The $\frac{1}{10}$ ohm standard is of manganin, of the usual British-Association pattern.

The oil employed is a pure heavy hydrocarbon oil, obtained from Messrs. Price's Patent Candle Company; it is free from any trace of alkali or acid, and experiments conducted by Messrs. Price prove that it has no action on the wire.

III. *Construction of Bridge and Galvanometer.*

The bridge employed was of the usual equal-armed pattern, having a straight wire of platino-iridium 1·5 millim. in diameter and 500 millim. long. The wire is stretched immediately over the scale by means of two brass split chucks, these chucks terminating in circular brass rods, having a diameter of 6 millim. The rods slide in heavy brass blocks, and the wire can be tightened by nuts on the ends of the rods. Thus the wire is not soldered to the terminals, and can be removed for repair. The scale is divided into 1000 parts, and the divisions are then divided into 10 by estimation; this is easily done with the aid of a good magnifier. The slider moves on the scale itself, and is provided with a fine adjustment. The connexion to the galvanometer is made by a platinum-tipped spring, resting on a platinum wire which is laid parallel to the main wire. The equal coils are made of manganin, and have a resistance of about 10 ohms. They have several times been adjusted to exact equality, but have never preserved that equality for a long period. This inequality, however, is unimportant, since it is eliminated by double balancing, which has been done at least once during each set of determinations. The actual amount of inequality at the present time is shown by the facts that, with 1 ohm in each of the outer arms, the bridge centre (*i. e.* the mean of the readings when the coils are interchanged) is 500·25, whilst with 1 ohm in each arm it is 500·4, and with 10 ohms in each arm it is 500·5.

The wire is mounted on a stout marble base. To protect the wire as much as possible from heat radiated from the observer's body two blocks of wood have been hollowed in such a manner that they can be slid over the wire; the slider itself being wrapped in cotton-wool. These precautions reduce the effect of the thermo-currents to a few scale-divisions on the galvanometer. The effect of thermo-currents on the galvanometer was eliminated by the galvanometer-circuit being kept closed, the battery-circuit being made and broken as required. In order to connect the box-coils and the standards to the bridge, three pairs of heavy flexible copper leads have been constructed; one pair has mercury-cups, intended for the reception of the standard coils, at one extremity; a second pair has conical plugs, so that any particular coil in the box can be measured alone; the third pair and the other six extremities of the leads are provided with heavy copper spades, by means of which connexion can be made with the terminals of the box and the bridge. The three pairs of leads are adjusted to exact equality one with another, and each has a resistance of ·00966 ohms at a temperature of 11°·8. The values of these resistances do not enter into any determination as the leads were always in pairs, one in each arm of the bridge.

The galvanometer is a four-coil astatic instrument, the resistance of each coil being 60 ohms. The magnet system is supported by a fine quartz fibre 260 millim. long. In order to diminish as far as possible the effects of the vibration caused by London traffic, thick rubber blocks are placed on the pier supporting the galvanometer; this precaution has made it possible to work at any time except when large masses of iron are being moved in the neighbouring goods station. The scale is placed 3 metres from the mirror. The motion of the mirror is observed by means of a small electric lamp which projects the image of a fine wire on to the mirror, thence it is reflected on to a ground-glass scale. The observer is at a considerable distance from the galvanometer, a matter of some convenience since the zero is thus not so liable to be changed. The galvanometer gives a distinct movement for an alteration of ·00001 of an ohm, with 10 ohms in every arm, the battery being one Hellesen cell, together with an added resistance in series with it of 10 ohms. The galvanometer is connected to the bridge by 20 gauge rubber-covered high insulation wires; the rubber covering is necessary as I have found serious errors introduced unless the insulation of the galvanometer connexion is extremely good.

IV. *Determination of Standards.*

The temperature difficulties which are most liable to cause inaccuracy in resistance measurements have been greatly simplified by the remarkable uniformity of temperature of the room in which the measurements have been made. The room is almost entirely underground, and is protected on all sides by other portions of the building from the heat of the sun. It is lighted by one small electric lamp, which gives rise to no appreciable change of temperature. The presence of one experimenter causes the temperature of the air to rise about ·2 of a degree during the first half hour, and then it becomes constant; but the temperature of the resistance-box and standards, the latter being placed in earthenware vessels and surrounded by sawdust, takes several hours to rise as much as ·03 of a degree.

The bridge-wire was calibrated by a method due, I believe, to Mr. Griffiths. In one of the outer arms is placed, by means of mercury-cups, a coil of low resistance, called the gauge-coil; on the other hand, the gap can be short-circuited by means of a thick copper bar. In the opposite outer arms are placed three resistance-boxes in parallel. The object of this arrangement is to secure the necessary fine adjustment. Commencing with the slider at one end of the wire, the gauge-coil being in place, a balance is obtained by adjusting the resistance in the boxes, the slider remaining fixed. The gauge-coil is then removed and the copper bar substituted; the slider is then moved until a new balance is obtained. Proceeding thus, the whole wire is divided into a number of parts the resistances of which are equal. From these observations a chart can be prepared showing the deviation in resistance of the wire from a wire of uniform resistance per unit of length. One of the great advantages of this method is that, when an abnormal value has been obtained, it is a simple matter to come back to that particular portion of the wire for a check measurement. The gauge-coil was equal to the resistance of 22·293 mean scale-divisions (each 0·5 millim.), and the greatest error at any point of the wire amounted to ·4 of a scale-division. Throughout the whole of the comparison of the coils this calibration of the wire has been employed, and several apparently discrepant observations have been thus brought into line.

The temperature-coefficient of the bridge-wire has been found by raising the temperature of the room from 10°·4 to 18°·1, the bridge having in one arm a coil of known temperature-coefficient, and was found to be ·071 per cent. per degree.

The thermometer employed, which is of hard Jena glass, is divided into tenths of a degree, each tenth being approximately ·5 millim. in length. By means of a small reading telescope it can with ease be read to ·01 of a degree. It was tested at the Reichsanstalt and, over the range employed, was accurate to at least ·05 of a degree.

The temperature-coefficient of the platinum-silver wire, over a range from 10° to 25°, was determined by immersing a coil of about 1 ohm in paraffin-oil at 25°, and allowing it to cool very slowly. It was found to be ·000274 per ohm per degree.

The 1 ohm coil from which the standards have been built up is of manganin, and when tested in June 1894 by Mr. Glazebrook had a resistance of 1·00026 ohms at 13°·4. I was, however, so doubtful of the constancy of this standard that in December 1895 I constructed a bare platinum-silver 1 ohm coil. This was compared with the manganin standard, and the mean of eleven determinations gave its value as

$$1{\cdot}00000 + {\cdot}000274\,(t - 14{\cdot}96).$$

No doubt the manganin coil had changed its value in the eighteen months that had elapsed since it was tested, so that the above figures were not true ohms.

The 10 ohm standard was obtained by the use of three approximate 3 ohm coils *, first placed in parallel and balanced against a 1 ohm coil, and then in series with the 1 ohm coil and balanced against the 10 ohm coil. Instead of connecting the legs of the coils by mercury-cups, as is usually done, the fusible metal made by Mr. F. Thomas, of Cambridge, was used. This metal, which melts at 60°, gives very constant contact resistances, and does not seem to injure

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* This method is described in the B. A. Reports, 1883.

in any way the amalgamated terminals of the coils. I first determined the value of a manganin 10 ohm coil, which had been made for nearly a year. The value obtained on December 20, 1895, was 9·9987 at 9°·3. This coil was used as a standard to which to adjust the platinum-silver 10 ohm coil. The value thus obtained for the latter was 9·9895 at 10°·3. On stepping up to the platinum-silver 10 ohm coil, the value found on December 30 was 9·9905 at 10°·6. Taking into account the known temperature-coefficient, the latter at 10°·3 would be 9·9897, which differs by only two parts in 100,000 from its former value.

To obtain the value of the $\frac{1}{10}$ ohm coil, which was of manganin, a bridge was formed of two 1 ohm coils, a 10 ohm coil, and the $\frac{1}{10}$ ohm coil, and a balance obtained by shunting one of the 1 ohm coils*.

On January 12th, a value of ·099934 at 11°·2 was obtained.
„ „ 13th, „ „ ·099932 „ 10°·2 „ „

The temperature-coefficient of the wire was known to be ·000004 per degree. Between these two determinations the bridge had been dismounted. This coil was again tested on April 8th, and had then a value of ·099944 at 12°·1, whereas, if it had remained unaltered, its value at that temperature should have been ·099938. This change is, however, too small to cause any error in the box-coils.

For testing coils which are very nearly in the proportion of 1 to any power of 10 this method is extremely sensitive, and certainly more accurate than making the comparison by means of the wire bridge, provided that the resistances of three out of the four coils forming the bridge are accurately known.

In obtaining the values of the 1 ohm and 10 ohm standard coils, not more than 50 scale-divisions of the wire had been employed, hence a small error in the total resistance of the bridge-wire would not cause any inaccuracy in the values so obtained. The resistance of the bridge-wire was determined in the first instance by placing the standard 1 ohm coil in parallel with a coil of slightly smaller resistance than the bridge-wire. This gave the resistance per scale-division of the wire as ·00087 at 10°·1. After the value of the $\frac{1}{10}$ ohm coil had been found, the resistance of the bridge-wire was determined by balancing the $\frac{1}{10}$ ohm coil against a coil of slightly smaller resistance than the bridge-wire ; the $\frac{1}{10}$ ohm coil was then removed, and the gap short-circuited. The actual figures are of interest as showing the value of a calibrated wire.

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* This method is described by Mr. Glazebrook, B. A. Reports, 1894.

·1 ohm in A. Gauge in D. Temperature 10°·8.
Reading 394·0.
Correction for calibration —·3.

Short circuit in A. Gauge in D. Reading 967·9.
Correction for calibration, —·2.

Hence, if ρ be the resistance per scale-division,

$$\cdot 1 \text{ ohm coil} = 2 \times 574\,\rho = 1148\,\rho.$$

On interchanging the coils,—

·1 ohm in D. Gauge in A.
Reading 32·9. Correction for calibration, 0.

Short circuit in D. Gauge in A.
Reading 607·0. Correction for calibration, —·1.

This also gives

$$\cdot 1 \text{ ohm coil} = 1148\,\rho.$$

Whence $\rho = \cdot 00008705$ at 10°·8.

A and D being the two outer gaps of the bridge.

The 100 and 1000 ohm standard coils were determined on similar bridges. The 100 ohm coil,—

On April 14, had a value 99·913, at 10°·8.
" " 17, " " 99·964, " 12°·5.

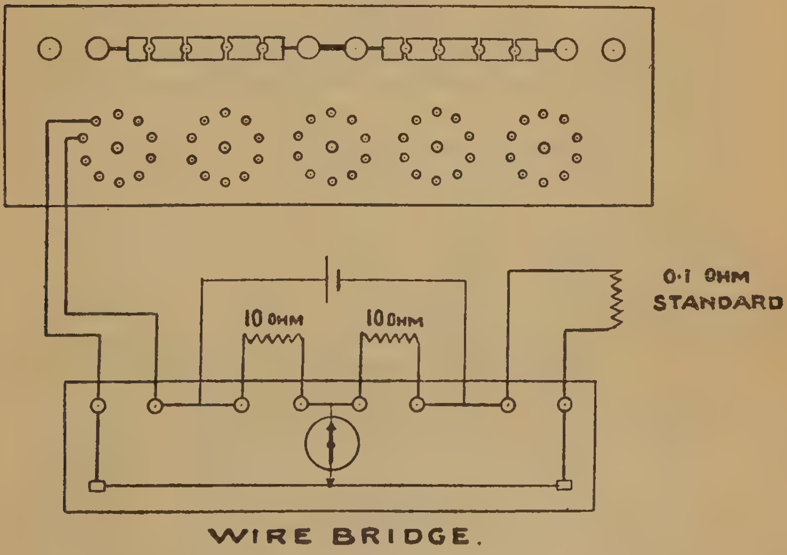
Whereas it should have been 99·960, according to the observation on the earlier date. This discrepancy is, I believe, due to the fact that the shunt required on one of the 10 ohm coils forming the bridge was greater than 12,000 ohms, which was the largest resistance at my disposal : in order to obtain a balance one of the 10 ohm coils had to be warmed, by means of a water-bath, about 2° above the temperature of the room, which led to some uncertainty as to the actual temperature.

The 1000 ohm standard coil was obtained by forming a bridge with the 1, 10, 100, and 1000 ohm coils, and had a value, on April 28, of 1000·24 ohms at 12°·8. No other determination has, as yet, been made. The coils were adjusted so as to be right at 15°.

V. *Calibration of the Resistance-box.*

For the first three of the dials the coils were balanced against standard coils (fig. 2), and the differences obtained by means of the wire bridge. In order to connect the conical plugs to the arms of the wire bridge, the flexible leads with conical ends were used; the standard in the opposite arm being connected by means of the leads with mercury-cup ends.

Fig. 2.



For the 1 and 100 ohms in the proportional arms spadeended leads were used, and attached to the terminals at the ends of the proportional arms (see diagram).

For the sake of convenience of comparison, I have reduced the resistances to their values at 11° C.

The values of the $\frac{1}{10}$ ohm dial are as follows :—

No. of Coil.	Jan. 12.	Jan. 20.	Jan. 22.	Jan. 24.	Jan. 25.	Jan. 26.	Mar. 11.	Mar. 14.	Mean.
1 ...	·10067	·10071	......	......	·10065	·10065	·10073	·10073	·10073
2 ...	·10012	·10012	·10009	·10008	·10008	......	·10012	......	·10010
3 ...	·10067	·10069	·10063	·10061	·10063	......	·10050	......	·10050
4 ...	·10008	·10014	......	......	......	·10011	·10010	......	·10011
5 ...	·09976	·09974	·09975	......	·09973	·09972	·09975	......	·09974
6 ...	·09982	·09987	......	......	·09985	·09985	......	......	·09985
7 ...	·10046	·10046	......	......	·10042	......	......	......	·10045
8 ...	·10003	·10004	......	......	......	·10003	......	......	·10003
9 ...	·10055	·10061	......	......	......	·10066	......	......	·10061

It will be noticed that, in coils 1 and 3, I have rejected certain values determined on dates before March 11; the reason being that, when the new flexible leads were attached in February, these two coils were touched and slightly bent, and had in consequence to be re-annealed.

The values of the 1 ohm coils are—

No. of Coil.	Jan. 8.	Jan. 11.	Jan. 26.	Feb. 12.	Feb. 16.	Mean.
1	·99929	·99929	......	......	·99931	·99930
2	·99956	·99956	......	......	......	·99956
3	·99569	·99568	......	......	......	·99568
4	·99964	·99969	·99970	......	......	·99968
5	·99776	·99774	......	......	......	·99975
6	......	1·00007	1·00007	......	1·00008	1·00007
7	......	·99976	·99979	·99978	·99980	·99978
8	......	·99878	·99875	·99875	·99878	·99877
9	......	1·00053	1·00051	......	1·00053	1·00054

Values of the proportional arms are—

	April 3.	Mar. 8.	Mar. 16.	Mean.
1 in X	·99863	·99860	·99860	·99861
1 in Y	·99645	·99641	·99642	·99643

Considerable trouble was experienced in the determination of the 1 ohm coils in the proportional arms, owing mainly to variations in the resistance of the plugs, which were fitted into the usual divided blocks. The amount of this error is of little moment when compared with the 10 ohm coil. The 1 ohm coils have not been employed in the determination of the larger resistances.

The values of the 10 ohm coils are :—

No. of Coil.	March 8.	April 3.	April 4.	Mean.
1 …………	9·9765	9·9764	………	9·9764
2 …………	9·9926	9·9925	………	9·9925
3 …………	………	10·0033	10·0033	10·0033
4 …………	9·9976	………	9·9973	9·9974
5 …………	9·9999	10·0000	10·0001	10·0000
6 …………	9·9574	9·9573	………	9·9573
7 …………	9·9558	9·9555	………	9·9556
8 …………	9·9863	9·9858	9·9864	9·9862
9 …………	9·9748	9·9753	9·9758	9·9753

The values of the proportional arms are :—

	Mar. 8.	Mar. 16.	Mar. 17.	April 3.	April 4.	Mean.
10 in X ……	9·9513	9·9506	9·9513	9·9507	9·9509	9·9509
10 in Y ……	10·0106	10·0099	10·0103	10·0098	10·0101	10·0101

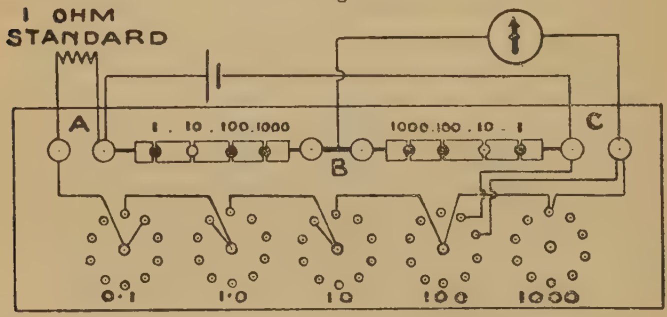
Desiring to obtain a check measurement, I balanced the nine ·1 ohm coils against the 1 and 10 ohm standards in parallel, and obtained the value ·90224 at 13°, while the first five coils of the dial, when balanced against the two 1 ohm coils in parallel, gave ·50125 at 13°·2. The actual values for the sums of the nine and five respectively, as found by the separate determinations, were ·90259 and ·50147. Of course any contact-error due to the plugs would appear when the values of the coils are added up. When the separate values of the nine coils are added up, the contact-error will have entered 18 times into the result, and for five coils 10 times ; on the other hand, when the coils are measured in series, the contact-error will only enter twice in each case, and the differences between the two sets of values in both cases are explained by the ascertained result that the contact-resistance of each plug is ·00002.

It is somewhat bold to assert that an error of ·00004 ohms has not entered into the determination of each of the $\frac{1}{10}$ ohm coils. The standard itself is certainly correct, as will be seen by reference to its values on the various dates ; the only possible explanation would be that the connecting leads of the two arms had not equal resistances. A subsequent test, however, proved that they were equal. I was, therefore, driven to the conclusion that the contact-resistance as above found was real, and not due to an experimental error.

The resistance of the box from terminal to terminal with each plug in zero was obtained in three ways. Firstly, by short-circuiting the mercury-cups in one of the arms of the bridge, and measuring the resistance of the leads and plugs on the bridge-wire. This gave a value of ·00967 ohm at 12°·8. Secondly, the first coil of the $\frac{1}{10}$ ohm dial, together with the leads, was balanced against the $\frac{1}{10}$ ohm standard; subtracting from this resistance the known resistance of the box-coil, we get a value of ·00964 ohm for the resistance of the leads, at 12°·4. Thirdly, a similar method was employed to measure the first coil in the 1 ohm dial, and this determination gave the resistance of the leads as ·00954 ohm at 10°·3. In order that these three observations may agree it is necessary to assume that the temperature-coefficient of the copper leads is about ·00005 ohm per degree. This is rather high, but the temperature range is too small for an accurate determination.

The values of the coils in the 100 dial were obtained on the box itself. In the gap A (fig. 3) the 1 ohm standard coil

Fig. 3.



was placed; this, together with the first three dials, gave a resistance of about 100·9 ohms; and constituted one arm of a Wheatstone bridge, the two 10 ohm coils in the proportional arms formed two other arms, the fourth arm was any one of the 100 ohm coils, which was connected to the gap C by means of the two flexible leads with conical ends. The plug from the centre of the 100 dial was in the zero of the 1000 dial so as to complete the circuit from the gap A to the gap C. The smallest resistance in the box was $\frac{1}{10}$ of an ohm, so that the last two figures of the resistance had to be obtained by interpolation from the swings of the galvanometer. The battery employed was one storage-cell, and the current was commuted for each observation. With the coils of the galvanometer placed in series an alteration of $\frac{1}{10}$ of an ohm in the box caused a total galvanometer swing of about 116 scale-divisions.

The values of the 100 ohm coils are: —

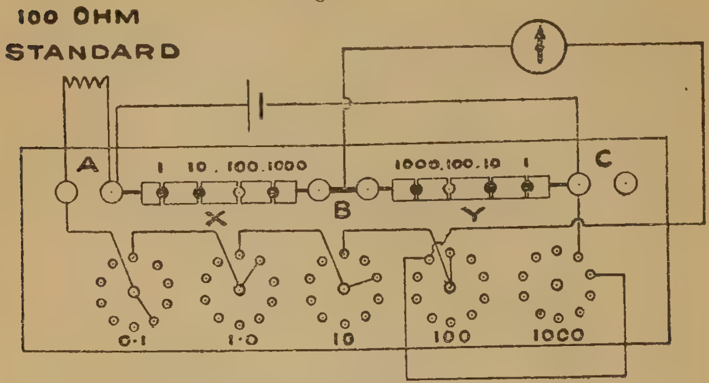
No. of Coil.	April 5.	April 6.
1	100·511	100·510
2	100·967	100·971
3	......	100·266
4	100·300	100·305
5	100·362	100·365
6	100·671	100·676
7	100·603	100·607
8	100·434	100·435
9	100·845	100·850

The 100 ohm proportional arms were determined by the same method as was used for the 100 ohm standard coils. Their values are :—

	April 17.	April 20.	April 21.	Mean.
100 in X .	100·224	100·219	100·220	100·221
100 in Y .	100·312	100·314	100·309	100·312

The 1000 ohm coils were also measured on the box itself. In the gap A (fig. 4) was placed the 100 ohm standard coil; this, together with the first four dials, formed one arm of a Wheatstone bridge; the two other arms were the 100 ohm coils in the proportional arms. A single flexible lead connected the last coil of the 100 dial with any required coil on the 1000 dial. Another lead connected the other terminal of the coil to be measured to the terminal of the proportional arm Y. The current was provided by two storage-cells.

Fig. 4.



The values of the 1000 ohm coils are :—

	April 25.
1	1023·35
2	1027·30
3	942·05
4	1022·04
5	1017·21
6	1032·04
7	1007·44
8	1003·45
9	1026·42

The values of the proportional arms are :—

	April 27.
1000 in X	947·92
1000 in Y	994·21

My object in giving the whole of the figures relating to the resistance-coils is to show to what an extent bare wire coils are to be trusted. Whether the coils retain their values as well as the resistance standards of the British Association, time alone can decide. There is no reason to suppose that, if the coils are kept continuously in an oil-bath, and the oil kept free from water, any considerable alteration can take place.

XXVI. *On the Theory of Moving Electrons and Electric Charges.* *By* J. LARMOR[*].

IN an interesting paper by Mr. W. B. Morton, communicated by the Physical Society[†] to the Philosophical Magazine for June, there is a criticism of a portion of my paper on "A Dynamical Theory of the Electric and Luminiferous Medium"[‡], which if valid would affect its whole tenor. As, however, the formulæ of that paper were to a considerable extent obtained by two independent trains of reasoning, it would have to be shown that both were wrong before an error could be fully substantiated. As a matter of fact, Mr. Morton's criticism arises from his reading into the analysis assumptions which are not there, but which had been used, with the proper limitations to secure accuracy, in another place in the previous part of the paper. As the point is really fundamental, and as the analytical statement in the memoir is no doubt too brief to convey at once a grasp of the procedure employed, without somewhat detailed consideration on the reader's part, I beg leave to offer the following general explanation.

The facts of chemical physics point to electrification being distributed in an atomic manner, so that an atom of electricity, say an electron, has the same claims to separate and permanent existence as an atom of matter. The fundamental question then is, how far the conception of separate isolated electrons, pervading the æther of free space, can provide an explanation of electrodynamic and optical phenomena. In the paper referred to I have gone further back, and have considered the question how far such a simple underlying scheme is able by itself to provide an explanation of physical phenomena in general; for it will obviously not be permissible to import into our dynamical notion of an atom of matter more than simple electric properties, unless these latter prove to be insufficient to include all actual knowledge of its relations. The conclusion arrived at in the memoir is that there is nothing in the ascertained laws of general physics which points to insufficiency in that scheme ; while there are some experimental results which somewhat militate against the existence of interatomic forces of any kind other than those included in it.

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[*] Read June 26, 1896.

[†] The present volume, *ante*, p. 180.

[‡] Phil. Trans. 1894 (A), pp. 719-822, and 1895 (A), pp. 695-743.

The main feature of the theory referred to is that the æther is not matter, as ordinarily assumed, nor in any way like matter ; it is the uniform substratum (analytical basis, if one is disposed to use that term, for it can never be the direct object of perception) in which the atoms of matter consist as permanent configurations of strain and motion. As was to be expected, the relations of inertia and elasticity of this uniform medium are simpler than those of matter, which is merely a molecular aggregate involved in its constitution. In fact, the only way to arrive at a scheme of the relations between æther and matter which shall be a complete dynamical theory and not merely descriptive, is to abolish the apparent duality in the phenomena, either by taking as here the molecules of matter to constitute singularities (in the mathematical sense of the theory of functions) in the uniform æther, or else by trying to make out the æther to be ordinary matter, and so giving up any attempt to explain why matter is molecularly constituted. This molecular constitution of matter is essential to the former theory, just as it is to all other theories or illustrations, like the vortex theories, which hypothecate a uniform underlying medium ; it is quite unintelligible—or rather quite unexplained—on the latter type of theory.

But however these things may be, the point criticised by Mr. Morton does not involve any considerations so refined, or—as possibly may be said—so ambiguous. The sections to which he objects claim to be a reconstruction of ordinary electrodynamic theory on the basis of permanent electrons associated with the atoms of matter. Whatever view one may entertain as to the presence of qualities other than electric in the atom, all are I think now-a-days agreed that the electron is there. And whatever view one may have as to the validity and sufficiency of an æther with simple rotational elasticity, the formal equations to which that theory leads *for free space* are just those equations of Maxwell which Hertz's experimental work has fully verified. The problem of electrodynamics is then that of the free æther, whose properties are represented analytically by these acknowledged equations, disturbed by the action of the electrons of material atoms moving about in it. The original Amperean electrodynamics, proceeding by consideration of elements of current, has not proved valid or sufficient in matters involving electric radiation, or even ordinary electrodynamic force. A most successful modification of it was that proposed by Weber, in which elements of current were replaced, as the fundamental object of consideration, by moving electric particles which acted on each other *at a distance* according to a law of force involving their velocities. This theory was, however, shown long ago by Lord Kelvin and Professor von Helmholtz to be untenable, on account of its violating the principles of the modern theory of energy; now, of course, direct action at a distance is altogether out of court. The present question is whether a theory of electrons which act on each other, not directly according to a law of force, but mediately by propagation of the effect across the intervening æther, suffices to avoid the discrepancies of earlier theories and give a consistent account of electrical and optical phenomena; and it is maintained that the answer is altogether in the affirmative. This question is, presumably, sufficiently important and fundamental to justify the present detailed explanation.

At the end of the first of the two papers referred to, building chiefly on the analytical results of previous theorists, the steady æthereal disturbance carried along by a moving electron had been investigated, and also the law of the force exerted on each other by two moving electrons through the intervention of the æther between them. This was on the hypothesis that each electron carried along with it a steady trail of æthereal disturbance, but that no sensible derangement of this steady motion ever occurred such as would lead to loss of the energy of the system by the starting of waves. If the velocities of the electrons remain always small compared with that of radiation, then, however their mutual influences alter their motions, this steady trail will instantaneously adjust itself to the new conditions without sensible excitation of radiation, and the theory will apply. But if any of the electrons are moving with velocities comparable with that of radiation, a change in velocity will involve derangement of this steady trail of æthereal strain and motion, giving rise to wave-motion which will carry off some of the energy by radiation. Accordingly in such a case it is altogether nugatory to speak of laws of action between electrons : the complete theory must then take account not only of the positions and velocities of each of the electrons at each instant, but also of the state of each volume-element of the surrounding æther. And the theory of mutual actions of electrons as expressed in the memoir was in fact thus restricted to cases in which their velocities were small compared with that of radiation : unless that condition is satisfied there is no such theory at all.

In the second paper (§ 15, *seqq.*) the general problem is attacked : it is now not a question of a set of electrons by themselves, each with a definite steady trail, but of the æthereal medium in general, *including* such electrons as exist in it. The analysis there given determines from foundations which all who adopt Maxwell's electrical scheme for free æther must allow, expressions for the force (P', Q', R') which acts on an element of volume of the free æther, and for the force $e\,(P, Q, R)$, ordinarily called electric force, which acts on an electron e ; and it uses these forces for further development of the theory. What Mr. Morton's computation virtually does is to assume that the trail of each electron is steady, and then to transfer to the electron itself the forcive due to (P', Q', R') acting on this æthereal trail. In the special case of no radiation, and of velocities small compared with that of radiation, this forcive can, as above explained, be transmitted through the æther to the electron itself, and be supposed there applied. But to so transmit it in general is to miss the point of the theory, and, as Mr. Morton himself remarks, to reach the absurdity that the force on a moving charge depends not only on the state of the surrounding æther but on the state of the æther at a distance.

As regards the main subject of Mr. Morton's paper, it may be of interest to state the following general theorem. Suppose a system of charged conductors is in steady translatory motion through the quiescent æther with velocity u, and let v represent the velocity of radiation in free æther : consider a correlative system of conductors obtained by uniform geometrical elongation of the actual system along the direction of motion in the ratio of $(1-u^2/v^2)^{-\frac{1}{2}}$ to unity, and find the electrostatic distribution of the same charges on this system supposed at rest: then the actual distribution of the charges on the moving system will be exactly correlative, viz., equal charges will exist on all corresponding elements of the two systems. This proposition is, however, limited to the case in which none of the bodies of the moving system are dielectrics, but all are conductors.

Finally, I take advantage of the present opportunity to draw attention to some special points in which the analysis of the second part of my memoir is incomplete. In §§ 28, 29, on the mechanical pressure of radiation, a statical forcive has been overlooked; when this is included the result practically agrees with that given by Maxwell. In §§ 34–6, on the material forcives in polarized media, the tractions on interfaces of transition remain to be developed, and the theory may be otherwise improved. I hope presently to treat this subject at length.

Cambridge, June 4, 1896.

XXVII. *On the Resistance of the Electric Arc.*

By JULIUS FRITH, 1851 *Exhibition Scholar, the Owens College, Manchester, and* CHARLES RODGERS, *B.Sc.*, 1851 *Exhibition Scholar, Firth College, Sheffield*[*].

35

[Plates I. to III.]

THERE seems to be some uncertainty as to what is meant by the resistance of the arc. Any given arc is a phenomenon which exists at a definite P.D. and current, and any attempt to measure its resistance must alter the state of the arc as little as possible or else we are no longer dealing with the same phenomenon. Hence it seems to us that the only way in which the resistance of the arc can be measured is by the ratio of a very small increment of P.D. applied, to the small increment of current produced. It must also be

[*] Read May 8, 1896.

borne in mind that the change in the current must be of so short a duration that the form of the carbons is not in any way altered.

Apparently the only method which fulfils these conditions is one in which a small alternating current is used, superimposed on the main continuous current. This has the effect of rapidly increasing and decreasing by a small amount the current passing through the arc. It seems probable that the effect on the arc of each small increase of current is annulled by the decrease of current immediately following. In fact a comparatively large alternating current may be superimposed on the main continuous current without producing any visible effect on the arc.

We are thus led to define the resistance of the arc as the ratio of a small increment of P.D. applied, to the small increment of current produced. This may be briefly written $\frac{dV}{dA}$. It is most important to distinguish this quantity, which we call the "instantaneous" $\frac{dV}{dA}$, from the tangent of the inclination of the tangent line of the curve representing the steady values of V and A, which we will call, for the sake of clearness, the "steady" $\frac{dV}{dA}$.

We have performed some experiments to exemplify the difference that exists between these two quantities; and also to show that in cases analogous with the arc, where, however, the result can be verified, the instantaneous $\frac{dV}{dA}$, found by superimposing an alternating current, gives correct values for the resistance.

In one of these experiments a glow-lamp, taking 10 amperes at about 8 volts, was placed in series with three I.E.S. 50 ampere accumulators, and a current sent through against the E.M.F. of the cells. This arrangement is just what is wanted to test the method, namely a resistance in series with a back E.M.F., both of which are functions of the current; and, further, the resistance can be separated from the back E.M.F. and measured, and the result compared with the value obtained for the resistance by the instantaneous $\frac{dV}{dA}$.

The result of this experiment is represented on Pl. I. Here are plotted the curves connecting the current and P.D. between the outside terminals of the arrangement and also between the terminals of the lamp and of the battery. This last gives the back E.M.F., since the resistance of the cells was small enough to be neglected. The P.D. at the lamp divided by the current gives the resistance; this is plotted on the same diagram; to make it clearer, however, the scale of ohms is multiplied by 10. We have measured* the instantaneous $\frac{dV}{dA}$ at various current strengths, and the values of this are plotted in a dotted line. It is seen that there is a very close agreement between these two measurements of resistance. On the same sheet are plotted values for the steady $\frac{dV}{dA}$. This differs considerably from both the resistance-curves. This leads us to see that the rapid excursions caused by the alternating current are not along the curve joining the steady values of V and A, but along a line which is everywhere more vertical than the tangent to the curve. This line is formed by joining the point on the curve to the instantaneous origin, which is distant from the origin of the diagram by an amount equal to the back E.M.F. at that particular current. Were the electrical excursions to travel along any intermediate path, the value obtained for the instantaneous $\frac{dV}{dA}$ would be dependent on the frequency. As will be seen later, this is not the case between the experimental limits of 250 and 7 complete alternations per second. If, therefore, the arc, as has been affirmed by various authorities, consists of a back E.M.F. and a resistance, we feel justified in applying this method for the measurement of its resistance, which has been found correct in closely analogous cases.

Now at very low frequencies indeed the electrical oscillations would travel along the curve connecting the steady values of V and A; and this is clearly the meaning of the critical frequency which we have observed with cored carbons (see p. 323), namely, that under the critical frequency the superimposed alternations travel on the steady value curve and become identical with the "steady" $\frac{dV}{dA}$.

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* By Method I. below.

Several experimenters have obtained values for the resistance of the arc which agree fairly well amongst themselves, and which seem to show that the arc has a positive resistance[*].

It was pointed out, however, by Prof. Ayrton at the Ipswich meeting of the British Association, that although there was a marked agreement between the values obtained by these experimenters, they were not at all in accordance with the conclusions drawn by himself from consideration of the curves obtained by Mrs. Ayrton. These curves connect the P.D. between the carbons with the current passing through the arc, for various fixed lengths of arc, and from them it is seen that for a given arc-length an *increase* of P.D. is always accompanied by a *decrease* of current. From this fact Prof. Ayrton concluded that if an attempt were made to measure the resistance of the arc by altering the P.D. between the carbons and finding the corresponding alteration of current produced, the resistance found by taking this ratio must be *negative*.

This conclusion was strengthened by some experiments made by Mr. Mather at Prof. Ayrton's suggestion. In one of these experiments two points of equal potential were found in a circuit consisting of an arc, a battery, and a resistance. Another battery, consisting of a few cells, of known E.M.F. and resistance was applied between these two equipotential points and the current flowing through the battery was noted. The resistances of the two parallel halves of the circuit, excluding the arc, were known, so that the current which, taking the arc resistance as zero, should flow through this battery could be calculated. Now the value of this calculated current was found to be less than the observed value, no matter in which direction the P.D. was applied, and this result was also obtained when an alternating P.D. was used. Hence the resistance of the arc was apparently less than zero.

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[*] An abstract of papers bearing on this subject was given by Mrs. Ayrton in the 'Electrician,' Sept. 13, 1895.

The other experiment consisted in running the arc at a steady P.D. and current, suddenly altering the resistance in circuit by a small amount, and noting the changes in the ammeter and voltmeter-readings so produced. The new conditions were maintained only long enough to allow of these readings being taken. The arc was then brought back to its former condition before taking another reading. It was found that a change of P.D. in *one* direction was always accompanied by a change of current in the *opposite* direction. The results of both experiments were, however, only qualitative.

All these experiments, together with the consideration of the curves found by Mrs. Ayrton, lead to the conclusion that the arc has a *negative* resistance, while former experimenters had all obtained a *positive* resistance.

It was in order to throw some light on this discrepancy that we undertook a series of experiments to determine with some degree of accuracy the resistance of the arc under various conditions.

Methods.

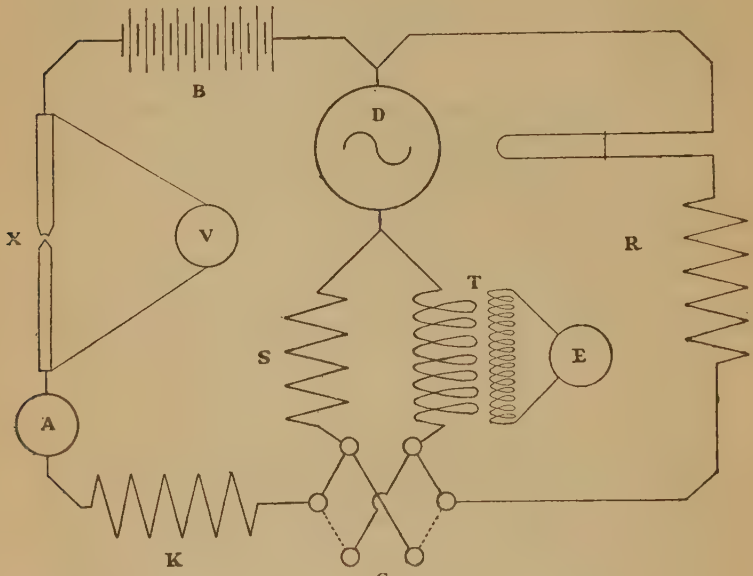
A number of methods were tried using alternating currents, of which the following were most successful :—

Method I. is represented diagrammatically in fig. 1. D is the armature of an alternator, the current from which passes round two circuits in parallel, one of which contains the arc X, and the other an adjustable resistance R. By adjusting R the alternating currents in the two halves can be made equal. When this is the case the impedances of the two halves to alternating currents must be equal.

In the diagram the continuous-current circuit is shown to the left. It consists of a battery of accumulators B, the hand-adjusted arc-lamp X, the resistance K, the ammeter A, and (with the commutator C as shown) the resistance S and the alternator D. It will be noticed that the alternator D carries the continuous current, but this of course does not prevent its acting as an alternator.

In order to measure the small alternating current independently of the continuous current flowing we used the air-transformer T, the thick wire coil of which was in series with the alternator D, the thin wire coil being connected with an electrostatic voltmeter E. The reading of the electrostatic voltmeter is thus unaffected by the continuous current, while it is, at a given frequency, a measure of the alternating current flowing.

Fig. 1.



By means of the commutator C, the air-transformer T can be thrown into either circuit, the resistance S being by the same operation thrown into the other circuit. The resistance S is equal to that of the thick wire coil of T, so that when S replaces T the continuous current is unaffected by the change.

Method of Experimenting.

The arc was run at the required current and P.D. by altering the number of cells in B, K being always kept the same. The current was kept constant by adjusting the arc by hand. Under normal conditions the current could be kept constant to within 1 per cent. R was now adjusted till the deflexion of E was the same when T was in either circuit. When balanced, a change of 0·01 ohm in R caused an appreciable difference in the deflexions of E. It was found to be useless to adjust more accurately than this, since the small variations in quality always found in carbons produced differences of this order in the resistance of the arc.

Let the value of R when a balance is obtained be R_1. This is equal to the resistance, to alternating currents, of the battery (b_1), the resistance (k), the arc lamp and connexions (l), and the arc (x).

$$\therefore \quad R_1 = k + b_1 + l + x. \quad . \quad . \quad . \quad . \quad . \quad (\text{i.})$$

The carbons are now firmly screwed together and the number of cells in B reduced till the continuous current is the same as before. R is again adjusted till the deflexions of E are equal ; and if R_2 is the new value thus obtained,

$$R_2 = k + b_2 + l; \quad . \quad . \quad . \quad . \quad . \quad . \quad (\text{ii.})$$

b_2 being the resistance of the portion of the battery now used. The cells are next cut out, the mains leading to them short-circuited, and a third value R_3 obtained,

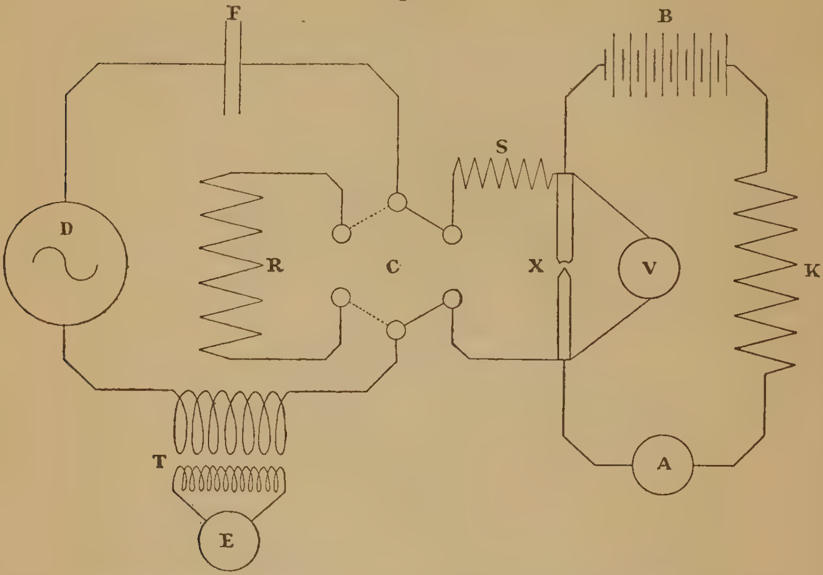
$$R_3 = k + l. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (\text{iii.})$$

From (ii.) and (iii.) we obtain the resistance of b_2, and by proportion of any number of cells. Putting these values in (i.) we obtain the value of x in ohms.

The advantages of this method are that no calibration of the electrometer is required, the speed of the alternator need not be constant for long periods together, various alternators giving different frequencies and wave-forms can be used without materially altering the circuit, and especially that the resistance of the arc can be obtained directly in ohms as the difference of two readings of the box R.

Method II. is shown diagrammatically in fig. 2 ; the arc circuit in this case being shown to the right. X is the arc, B the battery of accumulators, K the resistance, A the ammeter, and V the voltmeter across the arc. B and K were made large, K being about 11 ohms. The circuit shown on the left consists of the alternator D, the transformer T, which together with E is now used simply as a delicate alternate-current ammeter, a condenser F, and a commutator C. By means of C we can put in circuit either the resistance R, or the arc-lamp X in series with the resistance S. The alternating current flowing through K is negligible compared with that flowing through X on account of the high resistance of K. The condenser F prevents any continuous current due to the P.D. at the arc from passing through the alternator.

Fig. 2.



Method of Experimenting.

If L is the self-induction of the circuit and F its capacity, the impedance I of the circuit is given by $\sqrt{R^2+\left(\frac{1}{F\omega}-L\omega\right)^2}$, where $\omega=2\pi \times$ frequency. This is a minimum when $LF\omega^2=1$. The alternator was run at a speed corresponding to this frequency, at which the arrangement is most sensitive to changes of R.

An experiment consists in adjusting R till the deflexions of E are the same whether R or $S+x$ is in circuit. Then

$$R=S+x.$$

The experiment was first tried without a resistance at S, but it was found impossible when using solid carbons to obtain a balance even when R was zero. In order to make it possible to obtain a balance with R greater than nothing, a resistance S numerically greater than the negative resistance of the arc had to be put in series with the arc. In our experiments we kept S constant and adjusted R. The experiment might have been performed by making $R=0$, and adjusting S till the readings of E were equal. Then $S+x$ must be equal to zero, and therefore $x=-S$, or *the resistance of the arc is equal to the resistance in the box* S *with its sign changed.*

This method gives results which agree well with those obtained by Method I.; but on account of the necessity of keeping the speed of the alternator constant the arrangement is more troublesome to manage, and therefore was used only as a check method.

Description of the Apparatus.

The *arc-lamp* had a hand adjustment for each carbon with centering arrangements for the positive. The arc-length was measured by projecting an image of the arc on to a screen by means of a lens in the usual way.

The *resistance* K consisted of broad platinoid strips bent back on themselves so as to avoid self-induction*. It did not heat appreciably with the largest currents used.

The *battery* consisted of 50 10-ampere E.P.S. cells arranged in 4 groups of 10 cells each, 4 of 2, and 2 groups of 1. These groups could be connected up by means of mercury cups.

The *ammeter* and *voltmeter* were Weston instruments of the horizontal type.

The *air-transformer* T was made in two sections which were usually used in series. The constants were as follows:—

Section No. 1 :

Fine wire coil.

Diameter of wire (uncovered) = 4·3 mils.
" " (covered) = 6·8 "
Number of turns . . . . = 9730.
Resistance . . . . . . = 4707 ohms.

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* Described in Phil. Mag. February 1892.

Thick wire coil.

Size of wire No. 12 S.W.G.
Number of turns . . . . =128.
Resistance . . . . . . =0·143 ohm.

Section No. 2 :

Fine wire coil.

Diameter of wire (uncovered) =3 mils.
„ „ (covered) =5 „
Number of turns . . . . =29,500.
Resistance . . . . . . =32,120 ohms.

Thick wire coil.

Size of wire No. 12 S.W.G.
Number of turns . . . . =96.
Resistance . . . . . . =0·112 ohm.

The thick wire was double-cotton-covered. The thin wire was silk-covered.

The *voltmeter* was an Ayrton-Mather reflecting electrostatic voltmeter giving a deflexion of 700 scale-divisions at a distance of 2500 scale-divisions for 100 volts. This instrument was admirably adapted to our purpose on account of its very quick swing and excellent damping.

The *resistance* R consisted of a box wound with thick german-silver wire plugging to 0·1 ohm, in series with which was a german-silver wire (No. 10 S.W.G.) doubled back on itself and carrying a slider. Points were marked out at distances corresponding to 0·01 ohm. These spaces were about 10·7 centims. long.

The *Alternator.* Several different alternators were used. The one used while obtaining the curves given was a Pyke and Harris machine, which was especially suitable, since it had no sliding contacts, both armature and field being fixed. The other machines were—for high frequencies a Ferranti, and for low frequencies a Gramme alternator. The fields in all cases were excited by storage-cells.

The condenser had usually a capacity of about 60 microfarads.

Variation of Conditions.

The conditions of the experiments were varied as much as possible. We have studied the effect on the resistance of the arc of variations in the amount, frequency, and wave-form of the alternating current; the effect of different kinds of carbons and different P.Ds. and currents; the effect of using different combinations of cored and solid carbons, of carbons cored with substances other than carbon; and the effect of the relative size of the carbons.

Results.

The amount of alternating current between the limits we have used does not appear to have any influence on the value obtained for the resistance of the arc. The largest alternating current used had a R.M.S. (root mean square) value equal to about 10 per cent. of the continuous current. The smallest had about one-tenth of this value. In obtaining the curves shown, the R.M.S. of the alternating current was usually about 0·5 ampere.

The frequency also did not have any effect on the resistance of the arc between the limits 250 and 7 complete alternations per second. Frequencies above 150 were obtained from a Ferranti alternator; those between 150 and 45 from a Pyke and Harris; and those between 24 and 7 from a Gramme machine.

Besides these three machines a Mordey transformer of ratio 1:1 was sometimes used, with its primary in circuit with the Pyke and Harris alternator. On account of its having a closed iron circuit it was not so suitable, but it gave the same results as the alternators.

We have thus obtained considerable variety in the wave form as well as in the frequency. Hence the wave form does not affect our measurements of the resistance of the arc.

The values thus obtained for the resistance of the arc under various conditions are plotted in Pls. II. & III. Pl. II. shows the relation between the resistance of the arc and the current, the P.D. between the carbons being kept constant. Pl. III. fig. 1 shows the relation between the resistance of the arc and the P.D. between the carbons, the current being kept constant. Pl. III. fig. 2 shows the relation between the resistance of the arc and the arc length, the current being kept constant. In all these cases the resistance is measured from the central line marked 0, positive values above the line and negative values below the line.

Certain makes of carbon have been selected for experiment, namely, Apostle, Brush, Thomson-Houston, and Carré carbons, as representative of the various qualities of carbon now in use commercially.

The diameter of the positive carbon was in all cases 11 millim., the diameter of the negative was 9 millim. in the case of the Apostle and Brush carbons, 8 millim. in the case of the Carré carbons, and 11 millim. in the case of the Thomson-Houston carbons. These sizes were selected as being most suitable for the currents we were able to take from our cells. The sizes of the Apostle carbons were those used by Mrs. Ayrton in her experiments.

That the diameter of the carbons used does not have any great effect on the resistance of the arc for the same current and voltage is shown by the curves D and E, Pl. II. and fig. 1, Pl. III. These are the curves for two solid Apostle carbons – one set D having carbons 11 millim. and 9 millim. and the other set E having carbons 18 millim. and 5 millim. in diameter respectively, and it will be seen that they lie very close together.

On the curves each make of carbons is represented by a particular kind of line as explained in the figure, the Apostle carbons, for instance, are represented by a dotted line. Curves bearing the same letter refer to the same combination of carbons in the different makes. For each make there are four combinations :— + cored, – cored ; + cored, – solid ; + solid, – cored ; + solid, – solid ; represented respectively by the letters A, B, C, and D.

The general characteristics of these curves are that the ordinates of those for + solid – solid are always negative, and those for + cored – cored always positive, while the other curves all lie between these two extremes, those which have the positive carbon solid always being more negative than those which have the positive cored. The greatest uniformity is met with in the case of the + solid – solid combination, in which all the curves lie close together. For this reason the curves for the Thomson-Houston carbons have been omitted, as they lie completely on the other curves, and would tend to cause confusion. In the case of cored carbons, however, the uniformity is not so marked, as the material of the core varies largely with the make. In fact, in a single carbon the core may vary considerably, and for this reason the curves for cored carbons are more troublesome to take.

Pl. II. shows the relation between the resistance of the arc and the current that the arc is taking, the voltage at the carbons being kept constant. It will be noticed that in the case of those combinations which have a solid positive, the number expressing the negative resistance of the arc increases as the current decreases. With the other two combinations—those having a cored positive—the curves for Brush and Carré carbons have the same general characteristics, while those for the Apostle carbons cut these latter. This is probably due to peculiarities in the coring of the Apostle carbons.

Pl. III. fig. 1 shows the relation between the resistance of the arc and the P.D. between the carbons, the current being kept constant at 10 amperes in all cases. These curves are more instructive than those in Pl. II., as the character of the arc alters more with change of voltage than with change of current. As before, the curves for the solid carbons are all very close together. In this case they all show a minimum (maximum negative) value at about 55 volts. With combinations having a cored positive, this minimum becomes more strongly marked and occurs at a lower voltage.

It has been our practice to examine the image of the arc on the screen and note when any change in the appearance of the arc takes place. It was first noticed by Mrs. Ayrton that with cored carbons under certain conditions a dark space appeared somewhere near the centre of the arc, dividing the purple glow into two parts. This was never noticed in the case of solid carbons. Our observations have fully confirmed this, and, moreover, we find that the dark space, when only one carbon is cored, is nearer this carbon. We have noted the appearance or disappearance of the dark space, and we find a remarkable coincidence between this point and the minimum of the curves. In the curves for cored carbons in Pl. III., to the right of the minimum point the arc always shows a dark space which becomes less marked on approaching the minimum, and at the minimum finally disappears. To the left of this point, although the dark space has quite disappeared, yet the difference in colour between the two parts of the glow is still observable, that near the negative carbon being of a redder tint than that near the positive.

Pl. III. fig. 2 shows the relation between the length of the arc and its resistance at constant current. The general characteristics of these curves are the same as those of fig. 1, since the length of the arc is roughly proportional to the voltage.

Effect of the Core.

Since the presence of a core in either carbon has such a marked influence on the resistance of the arc, we tried the effect of boring out the core of a Carré carbon. The result of using this hollow carbon as the positive is shown by curve F in Pl. III. It will be seen that this curve follows very closely that obtained with both carbons solid. The effect was now tried of filling this hollow carbon first with plaster of Paris, and then with kaolin. In both cases the resistance of the arc so produced was positive. Another effect of these cores was to cause the arc to burn at a much lower voltage for a given current and arc length. The arc was, however, too unsteady to allow of a curve being taken.

Particulars of Cores.

The diameters of the various cores are as follows :—

Carbon.	Diam. of carbon. mm.	Diam. of core. mm.
Apostle	11	2·7
	9	2·33
Brush	11	2·73
	9	2·18
Carré	11	2·4
	8	1·9

Inverted Arcs.

In the experiments hitherto described the positive carbon has always been uppermost. Experiments were now made to ascertain if the resistance of the arc is dependent upon whether the positive carbon is the upper or the lower.

Using solid carbons, the resistance is not appreciably altered by inverting the arc. With cored carbons, however, the resistance of the arc is altered, and moreover the physical character of the arc is changed. As we have pointed out above, the distinguishing feature of the arc when using cored carbons is the appearance of a dark space situated near the centre of the purple glow, which appears to considerably increase the resistance of the arc. On inverting such an arc, however, we find that this dark space completely disappears, and, as we should have expected, the resistance is considerably diminished. At low currents or P.Ds., the difference between the resistances of ordinary and inverted arcs diminishes till the two values become nearly equal. These are the conditions under which the dark space ordinarily disappears. Hence when the dark space has disappeared, inverting the arc makes but little difference in its resistance, which is also the case with arcs using solid carbons, in which there is no dark space.

By placing the carbons horizontal, a value for the resistance of the arc can be obtained which is intermediate between the two values obtained by placing the positive carbon respectively above and beneath the negative carbon, the current and P.D. being the same in the three cases.

From consideration of the part played by the dark space, it would appear that the resistance of the arc is greatly affected by the state of the contact between the purple glow and the negative carbon. When the inside purple glow is in good contact with the negative carbon, the resistance of the arc is most negative. When, however, there is a dark space (caused by the presence of foreign matter or of carbon in a different physical state), and hence a lack of contact between this glow and the negative carbon, the resistance becomes increased.

Hissing Arcs.

An attempt was made, using Method I., to find the resistance of a direct-current hissing arc produced with accumulators, but it was found that even with the alternator at rest there was a large deflexion of the electrometer, showing that *the current through a hissing arc is oscillatory.* In order to determine the frequency of the oscillation, a condenser and telephone were put across the arc. The sound heard in the telephone was, however, of no definite pitch.

A method suggested by Mr. Campbell was used to measure the amount of this oscillatory current. The current taken by the hissing arc was passed through the primary of a transformer, the secondary of which was put in series with a reflecting twisted-strip ammeter, the deflexion being noted. The arc lamp was then disconnected, and a known alternating current was sent through the primary, of such magnitude that the deflexion was the same as before. This known current has the same R.M.S. as the oscillatory current passing through the arc, for if the impedance of the secondary is large compared with its resistance, the deflexion of the ammeter is the same for the same R.M.S. current whatever the frequency. The arc was run at 14·5 amperes and 35 volts, Apostle carbons +9 mm. −9 mm. being used. While the arc was burning the deflexion of the ammeter varied between 200 and 250 scale-divisions. Primary currents of 0·43 ampere and 0·53 ampere, as measured by another twisted-strip ammeter, produced these scale-readings respectively. Hence in this case the R.M.S. of the oscillatory current amounted to about 3 per cent. of the continuous current.

Low Frequencies.

On comparing our results with the steady $\frac{dV}{dA}$ taken from the slope of the curves published by Mrs. Ayrton, the agreement as to sign, which held in the case of solid carbons, was found to break down in the case of cored carbons; for while the general shape of the curves obtained by Mrs. Ayrton was the same for cored as for solid carbons, that is both curves except for short arc-lengths indicated a negative resistance *, we find that with both carbons cored the resistance is *always* positive. With a view to elucidating this point, we repeated Mr. Mather's experiments using both carbons cored, and found the $\frac{\delta V}{\delta A}$ negative as shown by the following table:—

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* Mrs. Ayrton does indeed find that for cored carbons at short lengths the curves slope slightly upwards, thus indicating a positive resistance.

Brush Carbons. Both cored.
+11 mm. −9 mm.

Arc taking 9 amperes at 43 volts.
Resistance in circuit about 5 ohms.

Resistance introduced.	Volts.	δV.	Amperes.	δA.
+·5	44·8	+1·8	8·10	− ·90
+·4	44·2	+1·2	8·30	− ·70
+·3	44	+1·0	8·45	− ·55
+·2	43·7	+ ·7	8·62	− ·38
+·1	43·3	+ ·3	8·75	− ·25
−·1	42·8	− ·2	9·20	+ ·20
−·2	42·4	− ·6	9·38	+ ·38
−·3	42·0	−1·0	9·60	+ ·60
−·4	41·6	−1·4	9·85	+ ·85
−·5	41·1	−1·9	10·08	+1·08

In the curves connecting the P.D. between the carbons and the current, each point is obtained by taking the readings of ammeter and voltmeter after the arc has been made to burn for a *very considerable time* at constant current and arc-length, so that one may say that the "frequency" is zero. In this case, the resistance, if deduced from the slope of these curves, would be negative both for cored and for solid carbons. In Prof. Ayrton's original method, although the readings are taken as quickly as possible, still the "frequency" is small compared with the lowest frequency we have hitherto used, and both the carbons and the arc may have time to alter before the ammeter and voltmeter can be read. This method would lead to the same conclusion as before, viz. that the resistance of the arc is always negative. Our measurements, however, with frequencies as low as 7·5 gave a positive resistance for arcs using cored carbons. Hence it seems that there must be a critical frequency, above which the resistance has a positive value which is independent of the frequency, and below which it appears to have a negative value, and further, that this frequency must lie between 7·5 and 0.

Our former methods were not suitable for working with a frequency lower than 7, for at this frequency the needles of the voltmeter and ammeter began to vibrate with the alternating current. This vibration of the needles was made use of in the following way, to indicate the sign of the $\frac{dV}{dA}$ at frequencies lower than 7. The Gramme alternator used in the previous method was run as a rotatory transformer. By adjusting the continuous current passing through this transformer, and hence its speed, any desired frequency could be obtained down to as low as one complete alternation in 2 seconds. By an arrangement of mirrors the needles and scales of both ammeter and voltmeter could be observed simultaneously. In this way it could be seen whether the two needles were at any instant vibrating in the same or in opposite directions. It is evident that if the needles are vibrating in phase, that is if an increase of P.D. is accompanied by an increase of current, then $\frac{dV}{dA}$ must be positive; while if they are vibrating out of phase, that is if an increase of P.D. is accompanied by a decrease of current, then it is negative. With this arrangement, *solid carbons at any frequency gave a negative resistance.* With *cored carbons*, however, we found as we expected, that *above a certain frequency the resistance was positive, while below this frequency the method gave a negative value for* $\frac{dV}{dA}$. The critical frequency was found by observing the point at which the swing of the needles was uncertain, being sometimes in phase and sometimes out of phase. At this point the frequency was 1·8, at 1·9 the sign was certainly positive, and at 1·7 was negative. This result was obtained with all arcs whose resistance as given by our curves was positive (with one exception mentioned below), and was not obtained with any whose resistance was negative. In the case of arcs whose curves cross the zero line, the transition from positive to negative, as shown by the disappearance of the critical frequency, was well shown and corresponded with the crossing of the zero line. As an example, we ran an arc with Brush carbons +11 mm. cored −9 mm. solid at 10 amperes, the curve for which is marked B in fig. 1, Pl. III. It will be noticed that this curve crosses the zero line twice, at 55 volts and again at 41·3 volts: at 60 volts we found the sign of the $\frac{dA}{dV}$, as shown by the swing of the needles, to be positive at frequencies above 1·8 and negative below this frequency, at 45 volts it was negative at all frequencies; at 35 volts, however, it was positive at frequencies above 1·8, and also at frequencies *below* 1·8. That the resistance would be positive at all frequencies seemed probable, since the curves connecting the P.D. and current at constant arc-length for such carbons show that with small arc-lengths an increase of P.D. is accompanied by an increase of current.

The experiments have been carried out in the Physical Laboratory of the Central Technical College, and our best thanks are due to Professor Ayrton and Mr. Mather for much kind advice during the course of the work.

NOTE.—*Fluctuation of the Current given by a Continuous-Current Dynamo.*

An attempt was made to replace the cells in Method I. by a dynamo, but it was found that even with the alternator at rest the electrostatic voltmeter showed a large deflexion. This was evidently due to the oscillation of the current owing to the commutator of the dynamo having a finite number of segments. The oscillation was found to increase as the brushes were moved out of the sparkless position. The dynamo was a 5 kilowatt 2-pole machine, and was giving 10 amperes at 70 volts. By adjusting the brushes to the sparkless position this oscillatory current, measured in the same way as in the case of the hissing arc above, could not be reduced below 2·5 per cent. of the continuous current, while by rocking the brushes out of this position it could be increased to as much as 9 per cent. without the sparking at the brushes being excessive. The commutator was in good condition. This oscillation may introduce errors in the measurement by polarized instruments of the currents given by dynamos owing to the differences between the mean and the R.M.S. of such a current.

DISCUSSION.

Prof. A. GRAY doubted whether it was right to give the name "true resistance" of the arc to the slope of the curve connecting the potential difference (V) and the current (A).

The authors' method of deducing $\frac{\delta V}{\delta A}$ was only true if the curve was a straight line; while in the case of the arc, E and a may both vary with the current.

Mrs. Ayrton said that with reference to the question of the existence of a back E.M.F., the evidence tended to show that it did not exist. By using an exploring carbon no constant back E.M.F. could be found.

Prof. Ayrton said that, considering the arc as consisting of a back E.M.F. and a resistance, it was necessary to separate these two. Simply obtaining one value of the P.D. and the current was of no assistance in solving this question, but a series of values had to be taken. By taking the changes in the P.D. and current sufficiently small, the curve over the range considered was practically straight. It was curious to note that as long as observers obtained a positive value for the resistance of the arc, no fault was found with the method, but that now a negative value was found, the accuracy of this method was questioned. If a back E.M F. does really exist, then it follows that the arc must have a negative resistance. Mr. Frith has shown why some people have got positive and some negative values for the resistance of the arc, and also that with an alternating current you may get either one or the other.

Mr. Carter asked if the fact that the arc had a negative resistance did not imply a back E.M.F. in order that the arc might be stable. If so, was a negative resistance such an absurdity?

Mr. Campbell said that he was very pleased to see that the authors had applied a method which he (Mr. Campbell) had suggested for measuring pulsating currents. If a pulsating current, such as could be obtained by means of a make and break, were passed through a thermopile you would get a back E.M.F., while if an alternating current were employed, you would not.

Mr. Frith, in his reply, said that he had defined the "true resistance" as $\frac{dV}{dA}$. Mrs. Ayrton has shown that an arc will not run unless a certain resistance is placed in series with it;

this resistance must be numerically equal to the negative resistance of the arc itself.

Prof. Ayrton said Mr. Frith's remarks as to the cause of the want of stability of an arc without outside resistance were most suggestive.

The Chairman said he did not like the expression P.D. He suggested the employment of photography to facilitate the accurate registration of the instrument readings.

XXVIII. *Admittance and Impedance Loci.*

By Frederick Bedell*.

38

The quantities chiefly considered in the discussion of alternating currents are electromotive forces and currents, the values of these being determined for different conditions. Electromotive forces and currents are commonly represented by vector diagrams; and the change in these diagrams, as some one quantity is varied, is shown by the loci of the vectors which are altered thereby. What may be termed electromotive force and current loci are thus determined. The numerical values for which these are constructed necessarily depend upon some condition involving an assumed value either of the current or of the electromotive force: thus we may assume a certain current to be constant (as the primary current of a transformer), and construct an electromotive-force diagram with loci showing the changes in the various electromotive forces as some part of the circuit is changed; or we may assume the impressed electromotive force constant, and ascertain current loci for the same variation. Let us limit ourselves to the transformer. In the first case above, it will be found that the constant assumed primary current I_1 is a factor in the value of every line representing the components of the primary electromotive force E_1. By factoring out I_1 we have an impedance diagram similar to the electromotive-force diagram, and without any assumption as to the value of the current or electromotive force. Similarly each line in a current diagram, constructed for a constant impressed electromotive force E_1, represents a current which is a multiple of an admittance (the reciprocal of an impedance) and the factor E_1. By factoring out E_1 an admittance diagram is consequently obtained, similar to the current diagram but with no assumption as to the current or electromotive force. Admittance and impedance diagrams accordingly correspond to current and electromotive-force diagrams respectively, differing from them only by a factor.

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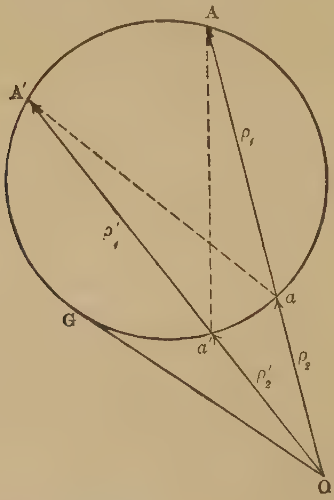
* Read June 26, 1896.

Impedance and admittance loci, or electromotive force and current loci, for the primary of a transformer will in general be arcs of circles for changes in any one of the constants of the primary or the secondary circuit.

Some interesting relations arise from the reciprocal nature of admittance and impedance. Let us note the following relations between reciprocal vectors :—

If any vector has an arc of a circle for its locus, a vector proportional to its reciprocal will have an arc of a circle for

Fig. 1.—Reciprocal Vectors, ρ_1 and ρ_2.



a locus. In fig. 1 let ρ_1 be any vector from the origin O, having its locus as shown upon the arc of a circle. The vector ρ_2, drawn in the direction of ρ_1 and proportional to its reciprocal, will have its locus upon an arc of a circle, which may be shown as follows. Let ρ_1 and ρ_1' represent the vector in any two positions, OA and OA′. The intercepts Oa and Oa' will represent the reciprocal vectors ρ_2 and ρ_2'; for in the similar triangles OA′a and OAa',

$$\rho_1 : \rho_1' :: \rho_2' : \rho_2.$$

Hence

$$\rho_1' \rho_2' = \rho_1 \rho_2 = \text{ a constant.}$$

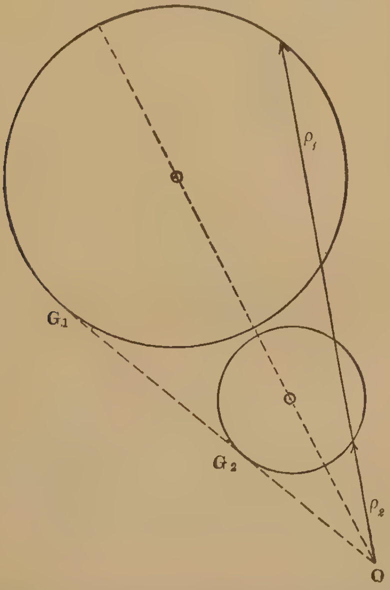
The value of this constant product of ρ_1 and ρ_2 is $\overline{OG}^2$.

By a suitable selection of scale, the loci of ρ_1 and its reciprocal ρ_2 may be represented as arcs of the same circle, as in fig. 1; or they may be represented by different circles, as in fig. 2. In the latter case,

$$\rho_1 \rho_2 = OG_1 . OG_2 = \text{ a constant.}$$

As the origin O approaches the circle which represents the locus of ρ_1, the centre of the reciprocal circle becomes more

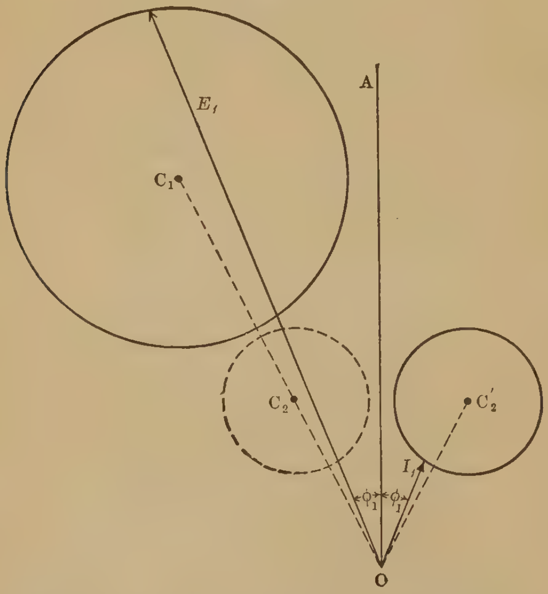
Fig. 2.—Reciprocal Vectors.



distant and its radius becomes greater. When the origin O is a point in the circumference of the first circle, the centre of the reciprocal circle is at an infinite distance ; that is, the reciprocal locus is a straight line.

Let us apply these principles to the transformer diagrams. The locus of the primary impedance, as some particular quantity is varied, is a portion of a circle. For example, this may be shown to be the case when the secondary resistance is varied. Since the admittance of the primary is the reciprocal of its impedance, the admittance may be represented by the vector ρ_2 in the above construction, if the impedance is represented by ρ_1. These loci may be drawn to scale for actual values. In a constant current transformer the primary electromotive force varies directly as the primary impedance. In a constant potential transformer the primary current varies directly as the primary admittance. But the admittance is the reciprocal of the impedance ; hence if we have an arc of a circle for the locus of the primary electromotive force when the primary current is maintained constant, we may employ

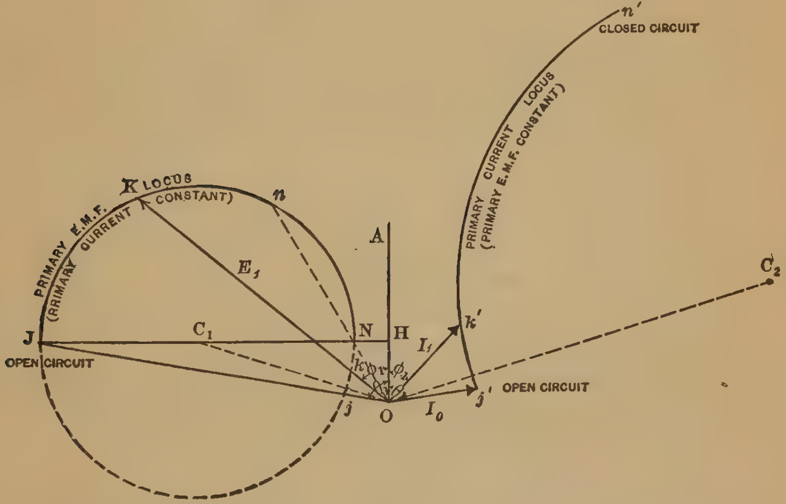
Fig. 3.



the above method to obtain the arc of a circle which will be the locus for the primary current when the transformer is supplied with a constant electromotive force. The converse operation may likewise be performed.

In fig. 3 let the circle C_1 represent the locus of the primary electromotive force E_1 during some particular change of condition, the primary current meanwhile being maintained constant and in the direction OA. The difference in phase between the current and electromotive force is the angle ϕ_1. The locus of the primary current under the same change of conditions, if the primary electromotive force is maintained constant, is the dotted circle C_2, which is reciprocal to C_1. If the constant electromotive force is drawn in the direction OA, the locus of the primary current is the circle C_2', drawn so that the angles AOC_1 and AOC_2' are equal.

Fig. 4.—Method of obtaining Primary Current Locus by the Principle of Reciprocal Vectors.



An application of the method of reciprocal vectors is shown in fig. 4. Positive rotation is counter-clockwise. The semicircle JKN represents the locus of the primary electromotive force of a transformer, when the primary current is constant and is in the direction of OA, and the secondary resistance is varied.

The electromotive force has the position OJ on open circuit and ON on short circuit. OH is the power electromotive force on open circuit, and includes the effects of primary resistance and the losses due to hysteresis and eddy currents; HJ is the electromotive force to overcome the primary self-induction. These are in the direction of the primary current, and at right angles to it, respectively. A line from J to K, at right angles to the secondary current, would show the reaction of the secondary upon the primary. It is to be noted that the line NH represents the effects due to magnetic leakage. It is desired to find the locus for the primary current when the primary electromotive force has a constant value, and is drawn in the direction OA. The angle of lag θ_1, between the primary electromotive force and current on open circuit, is JOH. Accordingly, with a constant electromotive force in the direction OA, the open circuit current I_0 is laid off lagging behind the electromotive force at an angle of $AOj' = \theta_1 = JOH$. The open circuit current I_0 may be laid off to any convenient scale. To construct the locus for the primary current proceed as follows:—Lay off the line OC_2 so that the angles AOC_1 and AOC_2 are equal. The point C_2 is located so that $OC_2 : OC_1 :: Oj' : Oj$. The primary current locus is then drawn as the arc of a circle with C_2 as a centre, passing through j'.

The limits of the primary electromotive force locus are the points J and N. The corresponding limits of the primary current locus are the points j' and n'. It will be noted that these points correspond to the points j and n on the circle C_1, which are reciprocal to the points J and N.

In the absence of magnetic leakage the points N and H coincide. The point n' would then lie in the line OA. The deviation of the primary current locus from the line OA is produced by magnetic leakage.

An experimental curve showing the primary current locus for a constant potential transformer, as affected by magnetic leakage, is shown in fig. 5.

The reciprocal relation between admittance and impedance vectors gives a simple method for determining the conditions for consonance and resonance in transformer circuits*.

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* "Resonance in Transformer Circuits," by Bedell and Crehore, 'The Physical Review,' vol. ii. p. 442.

Figure 6 is given as a particular instance in illustration of the statement given above that loci produced by the variation of any one constant are usually arcs of circles. The primary loci are always arcs of circles. The diagram shows the changes due to a variation in the secondary self-induction.

Fig. 5.—Primary Current Locus for Constant Potential Transformer; Determined Experimentally.

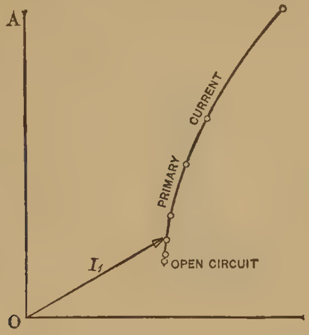


Fig. 6.—Effect of the Variation of the External Secondary Self-Induction in a Constant Current Transformer.

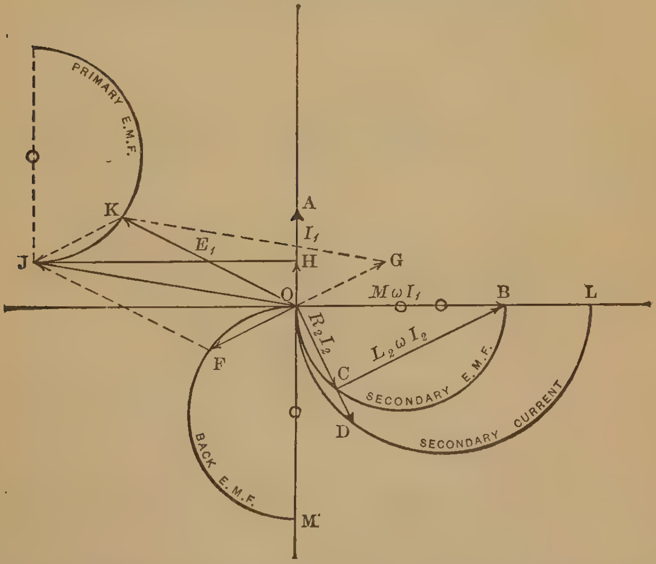
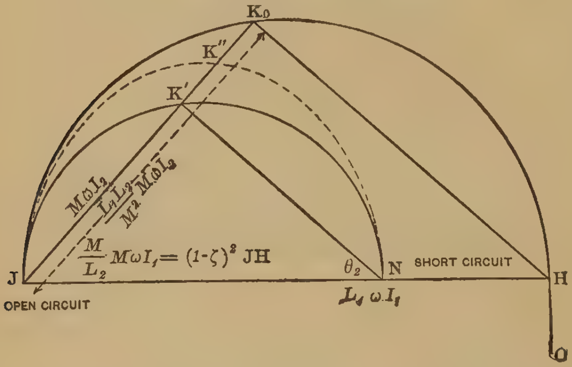


Fig. 7 shows the effect of magnetic leakage. The curves shown are loci for primary electromotive force, when the primary current is I_1. The primary electromotive force is composed of the components OH, to overcome ohmic resistance and supply open-circuit losses, HJ to overcome self-induction, and (with absence of magnetic leakage) JK_0 to overcome the back electromotive force of mutual induction.

The semicircle JK_0H is the locus for the primary electromotive force in the absence of magnetic leakage. The semicircle JK′N is the locus for the primary electromotive force

Fig. 7.—Effect of Magnetic Leakage.


K_0
K″
K′
$M\omega I_2$
$\frac{L_1 L_2}{M^2} M\omega I_2$
$\frac{M}{L_2} M\omega I_1 = (1-\zeta)^2$ JH
θ_2
J
N
H
O
SHORT CIRCUIT
OPEN CIRCUIT
$L_1 \omega I_1$

when the coefficient of magnetic leakage ζ is constant from open circuit to short circuit. In this case we have the relations

$$\frac{JK'}{JK_0} = \frac{JN}{JH} = \frac{M^2}{L_1 L_2} = (1-\zeta)^2;$$

$$1-\zeta = \sqrt{\frac{JK'}{JK_0}} = \sqrt{\frac{JN}{JH}};$$

$$\zeta = 1 - \sqrt{\frac{JK'}{JK_0}} = 1 - \sqrt{\frac{JN}{JH}}.$$

In an actual transformer the magnetic leakage is not constant, but varies with the load. The locus represented by the dotted curve JK″N is for such a case in which the magnetic leakage is zero on open circuit and increases to the maximum at short circuit. Where the magnetic leakage is variable, it is determined for any point as K″ by the ratio of JK″ to JK_0.

$$\zeta = 1 - \sqrt{\frac{JK''}{JK_0}}.$$

Thus let us suppose that the back electromotive force JK″, actually induced in the primary by the secondary current I_2, is eighty-one volts; and that JK_0, which would be the back electromotive force in the absence of magnetic leakage, is 100 volts. We then have the equation

$$1 - \zeta = \sqrt{\frac{81}{100}} = \frac{9}{10} = 0{\cdot}9\,;$$

which indicates that the mutual induction is nine-tenths of the value it would have in the case of no magnetic leakage. The coefficient of magnetic leakage is accordingly found to be 10 per cent.; thus:—

$$\zeta = 1 - 0{\cdot}9 = 0{\cdot}10.$$

Fig. 7 is drawn to represent the values of the various electromotive forces in the primary circuit of a transformer, for a given value of the primary current and for different values of the secondary resistance. If the magnitude of each line is divided by the primary current I_1, fig. 7 represents the values for the primary impedance (without any assumption as to constant current or electromotive force) for different values of the secondary resistance. The effect of magnetic leakage upon the primary impedance of any transformer is thus shown for different values of the secondary resistance.

The above construction affords a simple method of studying the conditions for a decrease of the primary impedance of a transformer when the secondary circuit is closed[*].

Curves corresponding to the dotted curve in fig. 7 have been experimentally determined[†] by the writer.

Cornell University, May 1896.

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[*] Discussed at length before the Physical Society by Mr. E. C. Rimington in his paper on "On Air-core Transformers," Proc. Physical Soc. vol. xiii. p. 1.

[†] 'Proceedings of the International Electrical Congress,' Chicago. 1893, p. 234.

Discussion.

Mr. Blakesley asked the author what was his test of resonance? Was it that the primary current and E.M.F. were exactly in the same, or in opposite phase? The term "resonance" was an acoustical one, and he did not see why it should be applied to one particular case in the electrical problem. The statement made by the author, that "impedance and admittance loci, or electromotive force and current loci, for the primary of a transformer will in general be arcs of circles for changes in any one of the constants of the primary or the secondary circuit," is only true under greater limitations than those cited by the author.

Mr. Inwards asked what degree of accuracy the author had obtained.

The Author, in reply, said that if the applied E.M.F. and the current were brought into phase by means of a condenser in the secondary, then he called that a case of resonance. The agreement between the experimental and theoretical results was generally within from one to three per cent.

PROCEEDINGS

AT THE

MEETINGS OF THE PHYSICAL SOCIETY

OF LONDON.

SESSION 1895-96.

February 8th, 1895.

Captain Abney, President, in the Chair.

The following were elected Members of the Society:—A. E. Briscoe, H. F. W. Burstall, E. H. Griffiths, A. A. Campbell Swinton, and W. E. Wilson.

Mr. W. B. Croft gave an exhibition of some Simple Physical Apparatus, of Photographs of Optical Phenomena, and of Clock Springs broken during frost.

Mr. S. Skinner read a paper "On the Tin-Chromic Chloride Cell."

February 22nd, 1895.

Captain Abney, President, in the Chair.

W. R. Cooper, J. H. Gardiner, and J. W. Rodger were elected Members of the Society.

A communication was made by Mr. G. H. Bryan, "On the Mechanical Analogue of Thermal Equilibrium between Bodies in Contact."

March 8th, 1895.

Mr. Walter Baily, Vice-President, in the Chair.

C. E. Ashford, H. Borns, S. H. Burbury, E. Catchpool, C. W. S. Crawley, N. Eumorfopoulos, S. G. Mostyn, P. E. Shaw, H. Stansfield, F. T. Trouton, and J. Watt were elected Members of the Society.

Mr. Naber exhibited "a New Form of Gas Voltameter."

Dr. Johnstone Stoney, F.R.S., exhibited :—

(1) "The Local Heliostat;"
(2) "An Improved Siderostat."

Mr. G. U. Yule read a paper "On a Simple Form of Harmonic Analyser."

Prof. Minchin, F.R.S., and Mr. H. N. Allen made a communication, "On the Energy Movements in the Medium separating Electrified or Gravitating Particles."

March 22nd, 1895.
(At the Royal College of Science, South Kensington.)

Mr. R. T. Glazebrook in the Chair.

H. E. Armstrong, D. K. Morris, W. B. Morton, and J. J. Thomson were elected Members of the Society.

Prof. A. W. Rücker and Mr. E. Edser read a paper "On the Objective Reality of Combination Tones."

Dr. Burton made the following communications :—

(1) "Some Acoustical Experiments;"
(2) "Objective Demonstration of Combination Tones."

April 26th, 1895.

Mr. Walter Baily, Vice-President, in the Chair.

A. Campbell, Eleanor M. Wynn Edwards, A. Hay, G. R. Kaye, S. A. Notcutt, W. Rheam, H. E. Soper, and E. Thomas were elected Members of the Society.

Prof. S. P. Thompson read a note "On a Neglected Experiment of Ampère."

Mr. W. G. Rhodes read a paper on "A Theory of the Synchronous Motor."

Mr. Bryan communicated a paper "On a Simple Graphic Interpretation of the Determinantal Relation of Dynamics."

May 10th, 1895.

Captain Abney, President, in the Chair.

C. A. Buckmaster, E. Rigby, G. L. Thomas, and S. C. F. White were elected Members of the Society.

Mr. Herroun read a paper "On the Iodine Voltameter."

Mr. A. Sharp read a paper "On a New Method in Harmonic Analysis."

May 24th, 1895.

Captain Abney, President, in the Chair.

Sir Frederick Bramwell was elected a Member of the Society.

Dr. Kuenen read a paper "On the Condensation and the Critical Phenomena of Mixtures of Ethane and Nitrous Oxide."

Mr. Burstall communicated a paper "On the Measurement of Cyclically Varying Temperature."

June 14th, 1895.

Captain Abney, President, in the Chair.

S. Beale and T. H. Littlewood were elected Members of the Society.

The reading of Mr. Burstall's paper, "On the Measurement of a Cyclically Varying Temperature," was completed.

Mr. N. F. Deerr made a communication on "The Thermal Constants of the Elements."

June 28th, 1895.

Dr. Gladstone, Vice-President, in the Chair.

Catherine E. Berridge, Agatha Leonard, Dorothy Marshall, and J. J. Stewart were elected Members of the Society.

Mr. Bowden read a note "On an Electromagnetic Effect."

Mr. Rhodes read a paper "On the Armature Reaction in a Single-Phase Alternating-Current Machine."

Mr. Shelford Bidwell read a paper "On the Electrical Properties of Selenium."

October 25th, 1895.

Mr. Walter Baily, Vice-President, in the Chair.

Sir Andrew Noble and F. W. Shurlock were elected Members of the Society.

Prof. J. Perry, F.R.S., and Mr. H. F. Hunt read a paper "On the Development of Arbitrary Functions."

Mr. Lanchester read a paper "On the Radial Cursor: an Improvement of the Slide Rule."

November 8th, 1895.

Prof. A. W. Rücker, Vice-President, in the Chair.

Mr. W. H. Everett read a paper "On the Magnetic Field of any Cylindrical Coil or Plane Circuit."

Mr. Griffiths and Miss Marshall read a paper "On the Latent Heat of Evaporation of Benzene."

Prof. W. Ramsay and Miss Marshall read a paper "On a Method of Comparing the Heats of Evaporation of different Liquids at their Boiling-points."

November 22nd, 1895.

Captain Abney, President, in the Chair.

This was a Special Meeting. The following resolution with reference to the Articles of Association was passed:—

> "In Article 33 to strike out the words 'by the payment of £10 in one sum,' and in place of this to insert the words 'the

> composition fee shall be for every member who shall not have paid ten annual subscriptions, fifteen times the amount of the annual subscription payable by each member; and for any member who shall have already paid ten or more annual subscriptions, ten times the amount of the annual subscription payable by such member.' "

At the Ordinary Meeting of the same date, G. M. CLARK, W. H. EVERETT, J. FRITH, W. LARDEN, R. A. LEHFELDT, and J. H. VINCENT were elected Members of the Society.

Dr. JOHNSTONE STONEY exhibited some Photographs of Spectra.

Mr. APPLEYARD made the following communications :—

(1) " A note on the Action of Sulphur-Vapour on Copper ; "

(2) " A Direct-reading Platinum Thermometer ; "

(3) " An Historical Note on Resistance and its Change with Temperature."

December 13th, 1895.

Prof. REINOLD, Vice-President, in the Chair.

The Resolution of the Special General Meeting held on November 22nd, 1895, was confirmed.

H. G. BELL, G. W. COBHAM, E. D. FRIDLANDER, L. G. RYLANDS, and R. C. SLATER were elected Members of the Society.

Dr. JOHN SHIELDS made a communication on " A Mechanical Device for performing the Temperature Corrections of Barometers."

Prof. RÜCKER communicated a paper " On the Existence of Vertical Earth-Air Electric Currents in the United Kingdom."

January 24th, 1896.

Captain ABNEY, President, in the Chair.

H. R. J. BURSTALL, W. CASSIE, and H. S. S. HARVEY were elected Members of the Society.

Mr. CAMPBELL SWINTON exhibited some Photographs taken by Prof. Röntgen's Method.

Mr. E. SCOTT exhibited and explained some Geometrical Instruments invented by himself and Signor MONTICOLO.

Prof. J. D. EVERETT communicated a paper " On Resultant Tones.'

Annual General Meeting.

February 14th, 1896.

Captain Abney, President, in the Chair.

The following Report of the Council was read by the Chairman :—

Since the last General Meeting of this Society, though no great change has been set on foot, the Members of the Society have had fuller opportunities for forming an opinion of those which were instituted in January 1895 than they possessed last February.

The monthly issue of 'Abstracts' and of 'Papers' in one cover has proceeded steadily for fourteen months. During the first twelve months 791 Abstracts have been made and printed, and the whole furnished with a Name Index, a Classified Index, and a General Index, all of the most copious and complete character.

Members are therefore amply able to judge of the advantages of this important step, and the Council believes that any movement in a contrary direction would now be viewed by the Members, individually and collectively, with very great regret.

The Council, therefore, takes this occasion to emphasize the fact that much additional outlay necessarily attends the publication of these Abstracts, and that the scale of payments hitherto in use is altogether inadequate to meet the expense.

The Society, at a Special Meeting held on November 22nd, 1895, has already raised the Composition Fee by 50 per cent. In any case, this was a matter that required adjustment, on account of the low rate of interest of appropriate securities at the present time.

But it is the intention of this Council to strongly recommend to their successors in office the introduction of much more far-reaching measures, the details of which have occupied much attention during the past year, and the chief features of which have already been presented to the Members of the Society in a circular letter. The Council commends this letter to the serious consideration of every Member.

Since the last General Meeting there have taken place fourteen ordinary business meetings, all (with one exception) held in the usual way at the rooms of the Chemical Society, Burlington House.

The Members of the Society now number about 420.

Among the deaths which have taken place in the ranks of the Society are those of Mr. T. H. Huxley and Mr. J. E. Gillett.

The following additions have been made to the Library during the past year:—

Newspapers and Magazines:—

Nature.
The Electrical Review.
The Electrician.
Electrical Plant.
Engineering.
The Philosophical Magazine.
Wiedemann's Annalen.
Beiblätter.
Journal de Physique.

Journals of Societies, British:—

Journal of the Society of Arts.
Proceedings of the Birmingham Philosophical Society.
Transactions and Proceedings of the Cambridge Philosophical Society.
Transactions and Proceedings of the Royal Dublin Society.
Journal of the Institute of Electrical Engineers.
Memoirs and Proceedings of the Manchester Literary and Philosophical Society.
Proceedings of the Institute of Mechanical Engineers.
Quarterly Journal of the Royal Meteorological Society.
Proceedings of the Royal Institution.
Proceedings of the Royal Society.
Proceedings of the Edinburgh Mathematical Society.
Journal of the Camera Club.
Proceedings of the London Mathematical Society.
Monthly Notices of the Royal Astronomical Society.

Journals of Societies, Colonial:—

Journal and Proceedings of the Royal Society of New South Wales.
Geological and Natural History Survey of Canada.
Hong Kong Observatory Observations.

Journals of Societies, Foreign:—

Proceedings of the Academy of Natural Sciences, Philadelphia.
Journal of the Physico-Chemical Society of Russia.
Verhandlungen, Berlin Physical Society.

Mémoires and Séances of the Physical Society of France.
Journal of the College of Science of Japan.
Transactions of the Seismological Society, Japan.
Séances and Bulletin of the International Academy of Sciences of Cracow.
Report of the Ohio Meteorological Bureau.
Report of the Ohio Weather and Crop Service.
Prace Matematyczno-Fizyczne, Warsaw.

Books, &c.:—

Œuvres de Laplace, Tome xi.
Stanley's Nebular Theory.

THE Right Hon. THOMAS HENRY HUXLEY, Professor of Biology and Dean of the Royal College of Science, London. Born at Ealing, Middlesex, May 4th, 1825; died at Eastbourne, Sussex, June 29th, 1895, in his 71st year; buried at the Cemetery of St. Peter's Marylebone, East Finchley.

As a man of science Huxley will best be remembered as the Champion of the doctrine of Evolution, under whose influence that permeated all branches of science not wholly mathematical, and revolutionized thought, and as having introduced rational methods into the teaching of Biology.

Huxley early desired to become an engineer, and the appreciation of the constructive side of nature runs throughout his scientific work. In connection with a course of lectures delivered at the London Institution, and afterwards in the South Kensington Museum, which led up to his Manual of "Physiography," he materially extended the limitations of that term, and laid the foundation of a system of elementary scientific education, which has done as much for the popularization of physical science as of geology and biology. In the year 1856 he accompanied his friend Tyndall on his first visit to the Glaciers of the Alps, the result being the presentation, during the following year, to the 'Philosophical Transactions' of a joint monograph in experimental physics, entitled "On the Structure and Motion of Glaciers."

Huxley was President of the Royal, Ethnological, Quekett Microscopical, and Geological Societies. In 1869 he was prompted by the writings of Lord Kelvin (then Sir W. Thomson) to address the latter body on the "Limitations of Geological Time," and he worked out a theory of the Zoic Period of the Earth's History, which, as to periods of years, was by no means in harmony with that of his distinguished antagonist. It will be within the recollection of physicists that during the last few months of Huxley's life, the work of Perry has led to the conclusion that on the physical basis the age of the planet has been underestimated, and that the greater time demanded by the palæontologists (whom Huxley represented) may well have elapsed.

Huxley was a Hunterian Professor of Comparative Anatomy at the Royal College of Surgeons, a Fullerian Professor of Physiology at the Royal Institution, a President of the British Association, a Rede Lecturer at Cambridge, and a Romanes Lecturer at Oxford. In 1870 he was elected a Member of the London School Board, and he served on several Royal Commissions. In 1892 he was made a Privy Councillor, having previously refused Knighthood. He was a Corresponding Member of the Institute of France, and a Riddare of the Pole Star of Sweden, and he was the recipient of Diplomas, Medals, and Hon. Fellowships of numerous Universities, Institutes, and Academies.

The adoption of the Report was proposed and passed unanimously.

The election of Officers and other Members of Council then took place, the new Council being constituted as follows :—

President.—Capt. W. DE W. ABNEY, R.E., C.B., D.C.L., F.R.S.

Vice-Presidents who have filled the Office of President.—Dr. J. H. GLADSTONE, F.R.S. ; Prof. G. C. FOSTER, F.R.S. ; Prof. W. G. ADAMS, M.A., F.R.S. ; Lord KELVIN, D.C.L., LL.D., F.R.S. ; Prof. R. B. CLIFTON, M.A., F.R.S. ; Prof. A. W. REINOLD, M.A., F.R.S. ; Prof. W. E. AYRTON, F.R.S. ; Prof. G. F. FITZGERALD, M.A., F.R.S. ; Prof. A. W. RÜCKER, M.A., F.R.S.

Vice-Presidents.—SHELFORD BIDWELL, M.A., LL.B., F.R.S.; Major-General E. R. FESTING, R.E., F.R.S.; Prof. J. PERRY, D.Sc., F.R.S.; G. JOHNSTONE STONEY, M.A., F.R.S.

Secretaries.—T. H. BLAKESLEY, M.A., M.Inst.C.E.; H. M. ELDER, M.A.

Treasurer.—Dr. E. ATKINSON.

Demonstrator and Librarian.—C. VERNON BOYS, F.R.S.

Other Members of Council.—WALTER BAILY, M.A.; C. V. BURTON, D.Sc.; L. FLETCHER, M.A., F.R.S.; R. T. GLAZEBROOK, M.A., F.R.S.; Prof. A. GRAY, M.A.; G. GRIFFITH, M.A.; Prof. G. M. MINCHIN, M.A.; Prof. W. RAMSAY, Ph.D., F.R.S.; Prof. S. P. THOMPSON, D.Sc., F.R.S.; Prof. S. YOUNG, D.Sc., F.R.S.

Votes of thanks were passed to the Chemical Society; to the OFFICERS; and to the AUDITORS.

The Treasurer in Account with the Physical Society, from January 1st, 1894, to December 31st, 1895.

Dr.	£	*s.*	*d.*	£	*s.*	*d.*
Jan. 1, 1895. Balance in the Bank				272	4	10
Entrance-Fees	53	0	0			
Life Compositions	255	0	0			
Subscriptions for 1891–94	36	0	0			
,, 1895	153	0	0			
,, 1896	4	0	0			
				501	0	0
Dividends				77	5	8
Grant from the British Association				100	0	0
Sales :—						
Taylor and Francis	39	0	5			
1 copy Van der Waals	0	4	6			
				39	4	11
Donation, Suspense Account				20	0	0
Balance due to the Treasurer				3	16	2
				£1013	11	7

Cr.	£	*s.*	*d.*	£	*s.*	*d.*
Jan. 1, 1895. Balance paid to the Treasurer				4	18	6
Taylor and Francis :—						
Separate Copies	28	1	6			
Miscellaneous	42	10	7			
Proceedings	253	16	9			
Abstracts	214	0	6			
				538	9	4
Editor	50	0	0			
Petty Cash	7	7	6			
Abstractors	72	8	10			
				129	16	4
Periodicals	23	15	1			
Binding	2	17	6			
				26	12	7
Cheques and Bank charges				0	9	4
Lavoisier Fund				5	0	0
Attendance				1	17	0
Reporting				17	5	8
Tea &c.				5	0	0
Rent of Name Plate				2	2	0
Petty Cash :—						
Mr. Blakesley	1	11	2			
Mr. Elder	4	15	6			
Dr. Atkinson	2	12	5			
				8	19	1
Balance in the Bank, Dec. 31, 1895				273	1	9
				£1013	11	7

EDMUND ATKINSON, *Treasurer.*

Examined and found to agree with the vouchers, books, and accounts.

RICHARD INWARDS, } *Auditors.*
CONRAD W. COOKE. }

February 1, 1896.

PROPERTY ACCOUNT OF THE PHYSICAL SOCIETY, December 31, 1895.

Assets.	£	s.	d.	Liabilities.	£	s.	d.
Subscriptions due, estimated	19	0	0	Subscriptions for 1896 in advance	4	0	0
£400 Furness 4 per cent. Debenture Stock	456	0	0	Donation, Suspense Account	20	0	0
£1000 Midland 4 per cent. Preference Stock	1194	0	0	Balance due to the Treasurer	3	16	2
£200 Metropolitan Board of Works 3½ per cent. Stock	217	0	0	Abstracts, Oct.–Dec.	35	0	0
£270 Lancaster Corporation 3 per cent. Stock	227	6	8	Printers	120	0	0
£254 2*s.* 9*d.* New South Wales 3½ per cent. Inscribed Stock	249	7	5	Rent &c.	23	0	0
Balance in the Bank	273	1	9	Balance	2429	19	8
	£2635	15	10		£2635	15	10

EDMUND ATKINSON, *Treasurer.*

Securities examined and found correct. For the New South Wales 3½ per cent. Stock no certificate is given. The Bankers have duly received the last dividend for October 1895.

February 1, 1896.

RICHARD INWARDS, } *Auditors.*
CONRAD W. COOKE. }

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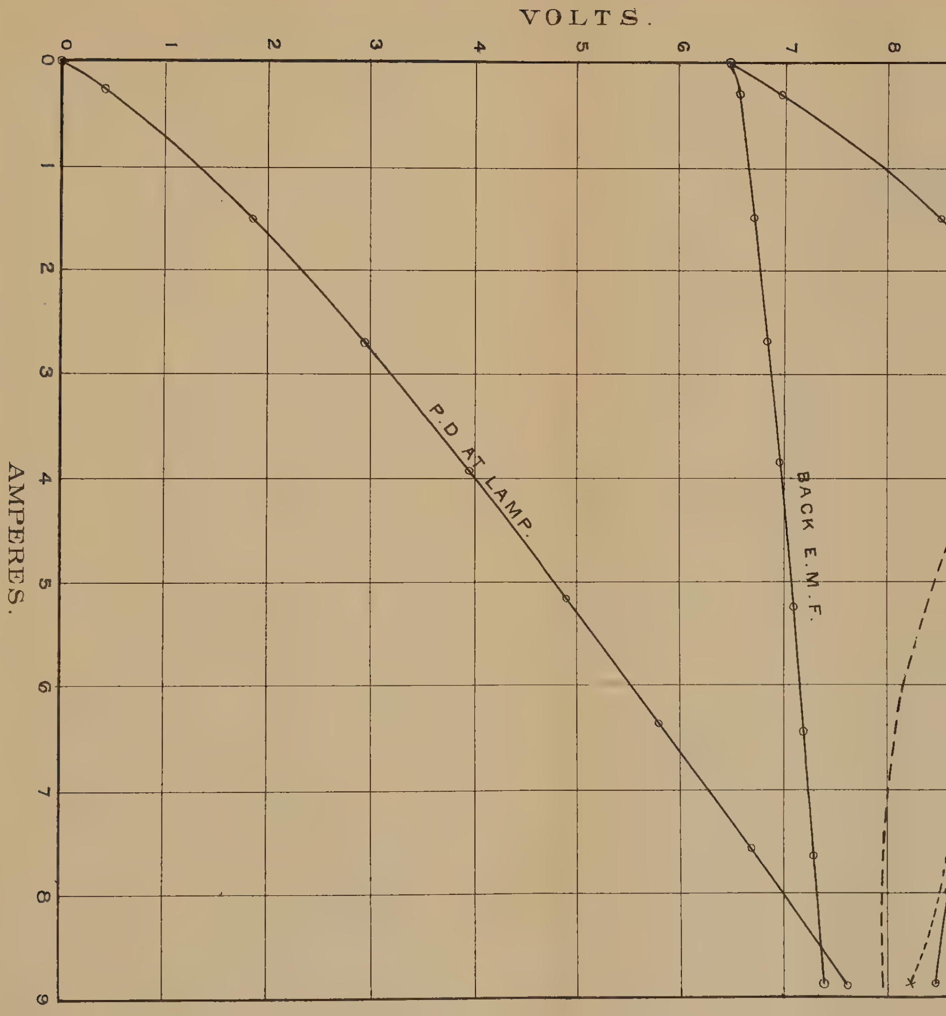
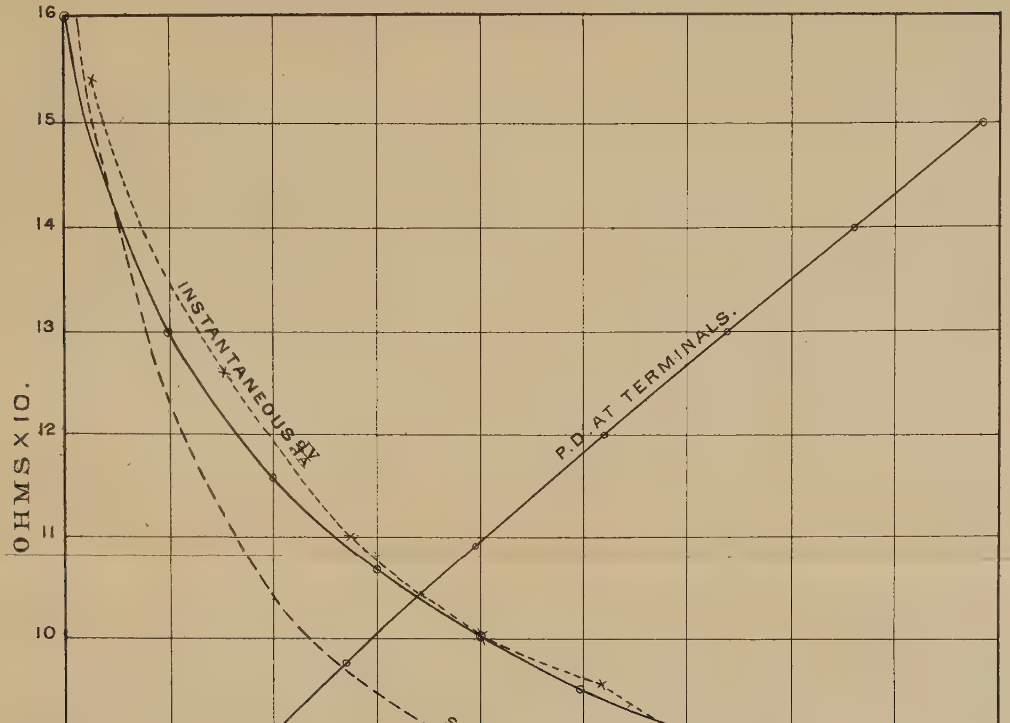
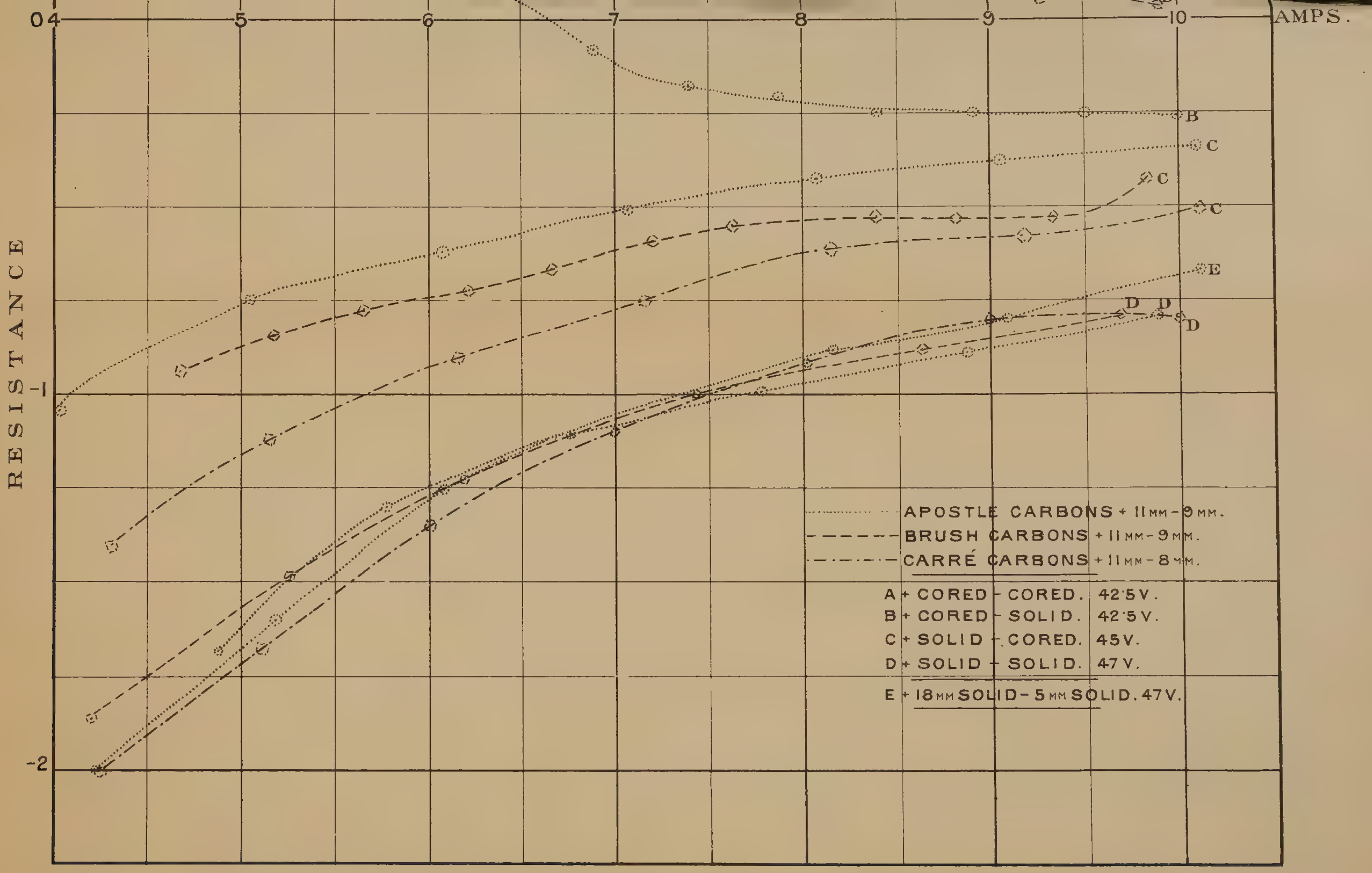


Mintern Bros. lith.

GLOW LAMP IN SERIES WITH BACK E.M.F.





Mintern Bros. lith.

Proc. Phys. Soc. Vol. 14. Pl. II.

OF THE ARC.

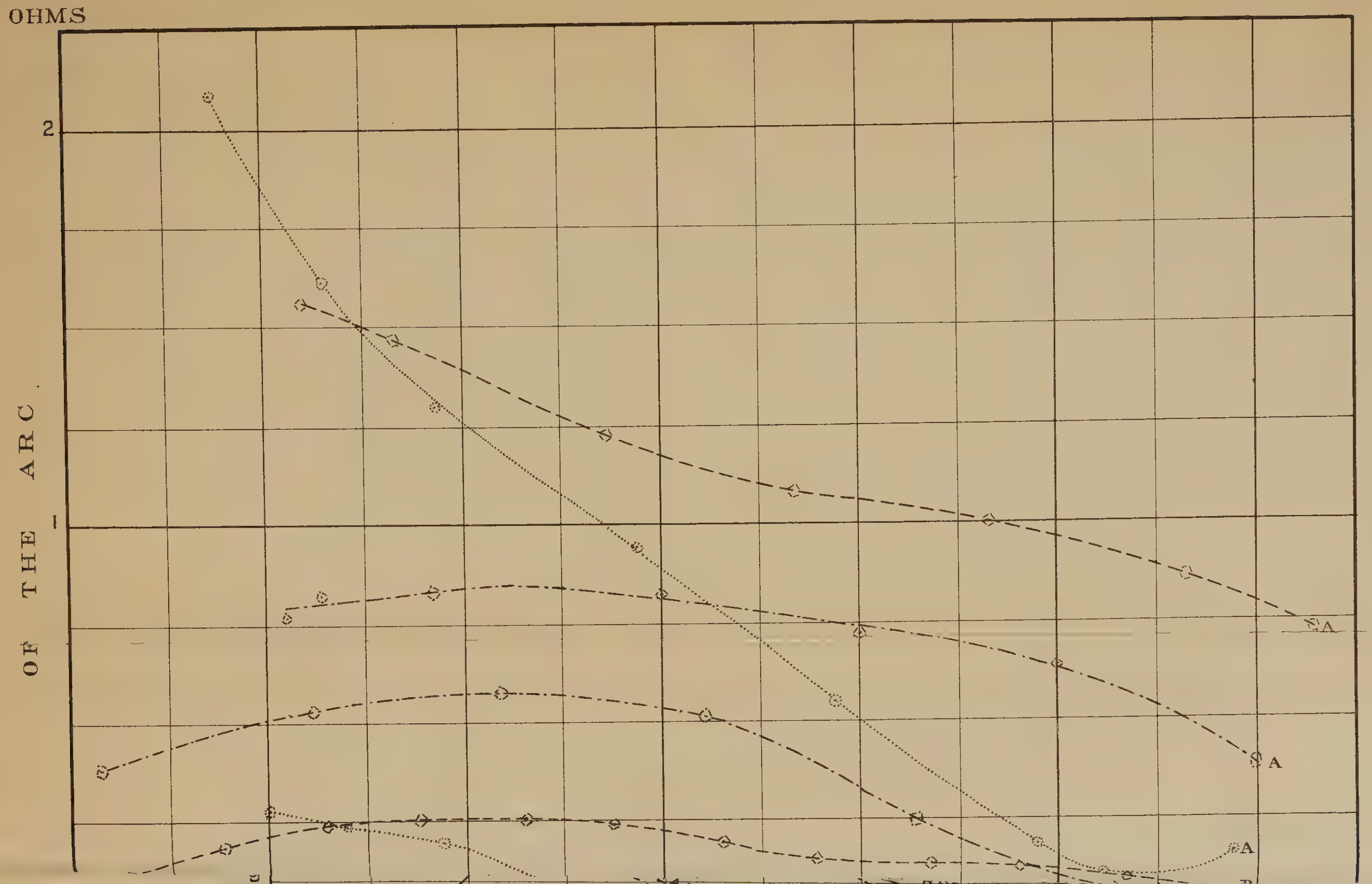
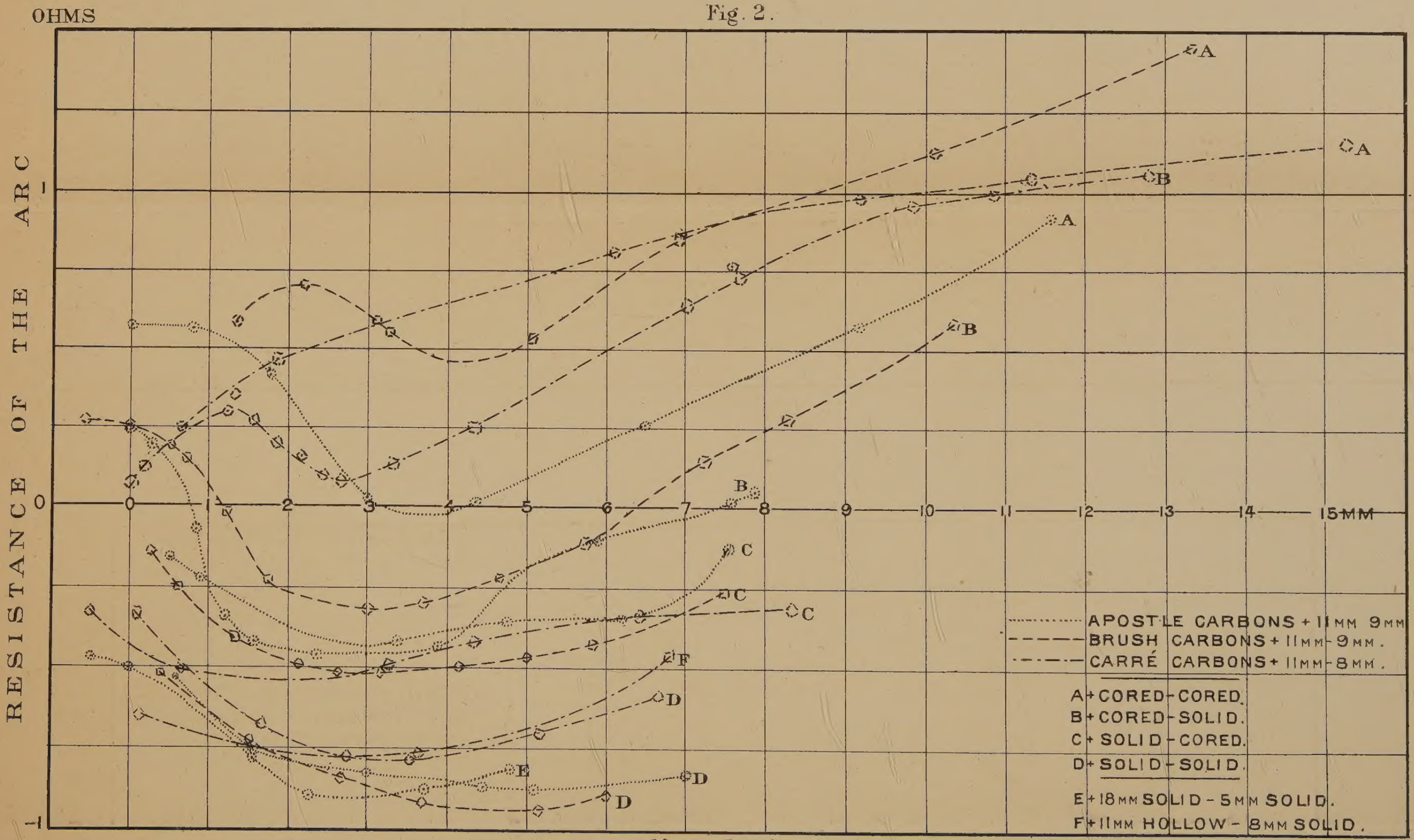
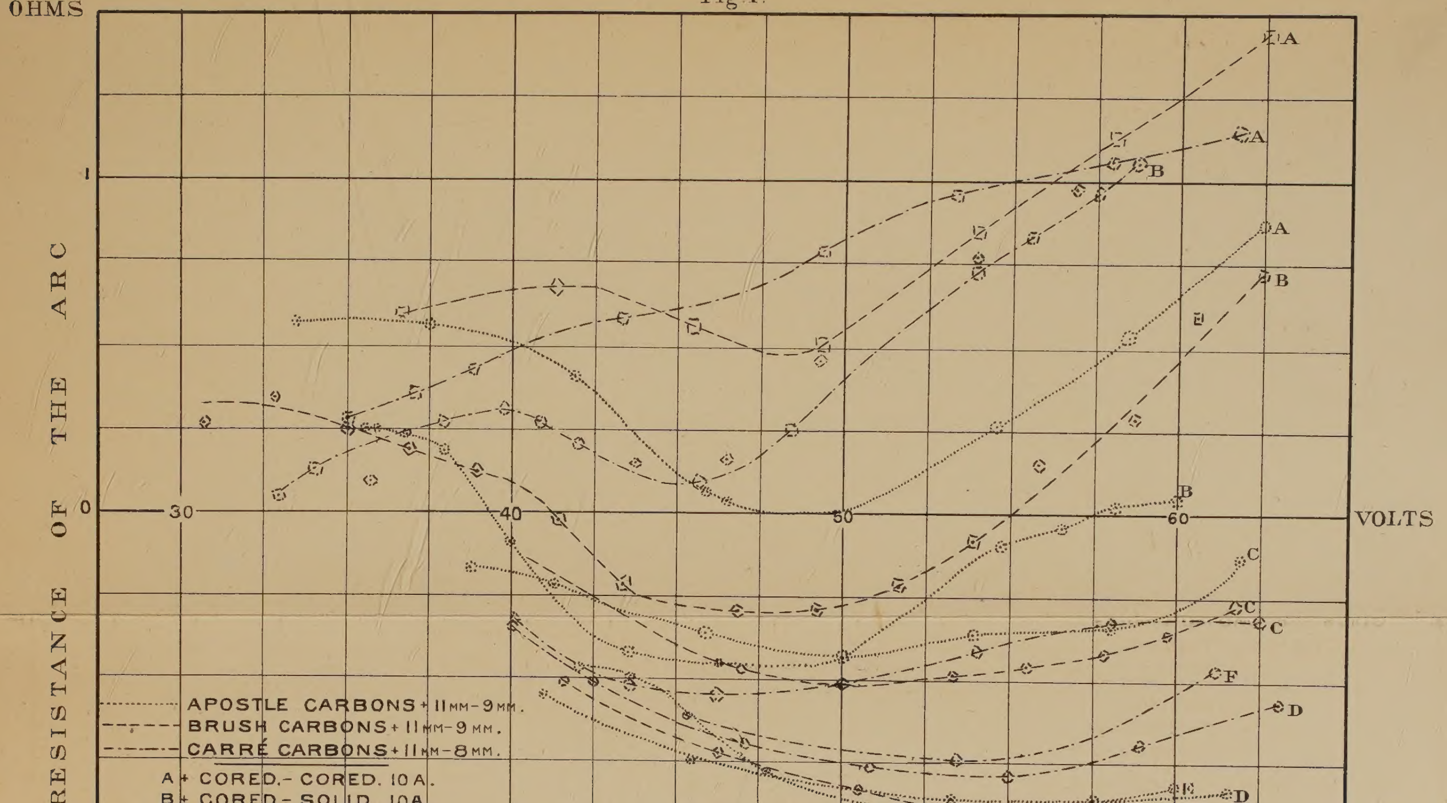


Fig. 2.



Mintern Bros. lith

Fig. 1.


OHMS
1
0
RESISTANCE OF THE ARC
30
40
50
60
VOLTS
A
A
B
A
B
B
C
C
C
F
D
E
D
APOSTLE CARBONS + 11MM - 9MM.
BRUSH CARBONS + 11MM - 9MM.
CARRÉ CARBONS + 11MM - 8MM.
A + CORED. - CORED. 10 A.
B + CORED - SOLID. 10A